INVESTIGATTON OF CQMPIJTATTON TECHNIQUES FOR SIMULATION OF LARGE SCALE POWER SYSTEM DYNAMICS

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ABSTRACT - This paper describes initial investigations towards arriving at a most suitable algorithm for real time simulation of power **system** dynamic.;. Even though the modelling and simulation aspects of real time simulation are based on ideas that have already been laid down for conventional transient stability analysis, the choice of the best algorithm for real time simulation require.; different guide lines.

In transient stability analysis, the stress is on accuracy. In real time simulation, computation time and numerical stability are more important. In this paper, various algorithms, obtained through different combinations of the choices of synchronous machine model, solution approach. angle corrector formula, and convergence criteria for the network solution, have been compared with respect to computation speed and solution accuracy.

Principal Symbols:

- E : general symbol for internal voltages of machines (phasor)
- E_{fd} : exciter output voltage
- P_e : generator power output
- P_m : mechanical power
- δ : position of q-axis with respect to network reference for phasor solution
- [Y] : bus admittance matrix
- [V] : bus voltage vector
- [I] : current injection vector
- Θ : generator terminal voltage angle
- W : angular frequency
- W_0 : synchronous angular frequency
- M : inertia constant
- h : step size of numerical integration
- ε : general symbol for convergence index

1. INTRODUCTION

Transient stability analysis and real time simulation of power systems both require the solution of the differentia-algebraic system of equations of a power system. In transient stability analysis, the stress is on accuracy, and a marginal increase in computational costs can be tolerated, in order to obtain the desired accuracy. In real time simulation, on the other hand, computation time and unconditional numerical stability are important. implicit integration methods are used to cope with the stiffness of the associated differential equations and thus to speedup the solution of these equations [1-7].

In this paper, the performance of various algorithms for the simulation of power system dynamics have been investigated, and compared with respect to computation speed and solution accuracy. These algorithms were obtained through different combination.; of the choices of the following features:

(i) selection of a synchronous machine model

(ii) selection of an implicit integration method for the solution of ths differential equations,

(iii) interfacing of the solutions of the algebraic and the differential equations,

(iv) better angle corrector formula, and

(v) better convergence criteria for network solution.

This paper will be useful in evaluating the trade offs between these various techniques, and also will be providing a basis for the development of a most suitable algorithm for real time simulation involving individual generator dynamics. Such an algorithm finds application in operator training simulators.

2. SYNCHRONOUS MACHTNE MODEL

Common representations for synchronous machines which have been proposed for transient stability simulation are

- model-0: Constant internal voltage behind transient reactance
- model-1: Variable internal voltages behind transient reactances
- **model-7:** Variable internal voltages behind subtransient reactances

Model-;! is the most accurate of the three models. **Also, as** the subtransient saliency is much smaller than the transient saliency , the number of iterations due to saliency would be less with the machine model-2, as compared with the model-1 . The model-;! is thus preferred for both transient stability and real time simulation.

3. SOLUTTON METHODS

Transient stability simulation involves three distinct tasks, namely

(i) network solution

(ii) numerical integration of differential equations, and

(iii) interfacing the solutions of the algebraic and the differential equations

3.1. Network Solution

The network solution is obtained by solving iteratively the matrix equation

$$[Y][\dot{V}] = [I] \qquad (3.1)$$

for bus voltages [V], given the current vector [I] at each time step. The elements of [I] are non-zero only for generator nodes. The computation of [I] depends on the Norton representation of the generators.

Optimal ordering, sparse matrix [8] and sparse vector methods [9] have been employed to speed up the solution of equation (3.1), and to achieve saving in storage requirements.

3.2. Numerical Integration

The general form of the ordinary differential equation for a variable 'y' with time constant 'T' is

$$py = (1/T)(Ax - y) \qquad (3.2)$$

where the input variable 'x' may also be a state variable. Discretization of equation (3.2) by an implicit integration method yields a difference equation of the form

$$y(t+h) = F(t) + Bx(t+h) \qquad (3.3)$$

where F represent the past history term of the variable y, and B is a constant. Both F and B are dependent on the method of integration.

In this study, algorithms which algebraize the differential equations on the basis of the Backward Euler formula and the Trapezoidal rule have been considered,

3.3. Interfacing of the Algebraic and Differential Equation Solutions

There are basically two approaches to the solution of the algebraic and differential equations: simultaneous solution method, in which the machine differential equations and the system algebraic equations are solved simultaneously, and partitioned solution method, in which these equations are solved alternately.

In the simultaneous solution method, the algebraized differential equations and the network equations are combined to form a composite model. The algorithm for the Simultaneous Implicit (SI) solution approach consists of the following steps:

i. Form bus admittance matrix [Y]

ii. Predict rotor angles for all generators using the formula [1]

$$\delta(t+h) = 2\delta(t-h)-\delta(t-2h)+\frac{(h^2/M)}{(1+hD/2M)}(P_m-P_e(t-h)) \qquad (3.4)$$

iii. Form the right hand side of equation (3.1) using the computed rotor angles, and the terminal voltage vector at the previous iteration.

iv. Solve equation (3.1) for [V].

v. Correct the machine rotor angles from the solution of the swing equations.

vi. Compare the computed rotor angles with their earlier values. If they are not close enough, return to step (iii).

vii. Check for saliency convergence. If convergence has not been obtained, go to step (iii).

viii. Check for exciter output voltage convergence. If convergence has not been obtained, go to step (iii)

ix. Update the past history terms of the state variables.

x. Advance the time step, and go to step (ii).

In the algorithm which uses the Partitioned Implicit (PI) solution method [5,6], linear prediction and consequent correction of the internal voltages are incorporated, in order to avoid the problem of interface errors between the solutions of the algebraic equations and that of the differential equations. In this scheme, after getting the network solution, (i.e., after step (viii)), the differential equations are to be solved explicitly before the past history terms are updated.

4. ANGLE CORRECTOR FORMULA

In the algorithms developed, the solution of the swing equation is used for the machine angle correction. The equations for rotor acceleration and swing are

$$MpW = P_m - P_e \qquad (4.1)$$

$$p\delta = W - W_0 \qquad (4.2)$$

Angle corrector formulae obtained through various techniques have been investigated so as to study their effect on the number of iterations required for the network solution. They are :

4.1. The Use of Trapezoidal Rule

Application of the Trapezoidal rule to equations (4.1) and (4.2) leads to the solution

$$W(t+h) = W(t) + (h/2M)(2P_m-P_e(t)-P_e(t+h)) \qquad (4.3)$$

$$\delta(t+h) = \delta(t) + (h/2)(W(t+h)+W(t)-2W_0) \qquad (4.4)$$

4.2. Linear Interpolation of Pe

In this method, it is assumed that the variation of the electrical power output of each machine is linear over one time step. Then the equations (4.1) and (4.2) are solved analytically. The solution obtained by this method [10] is

$$\delta(t+h) = (-h^2/6M)P_e(t+h) + \alpha(t) \qquad (4.5)$$

where

$$\alpha(t) = \delta(t)+(W(t)-W_0)h+(h^2/2M)(P_m-(2/3)P_e(t)) \qquad (4.6)$$

4.3. Linear Interpolation of Pe Together with the Use of Newton's Method

With model-2, the generator power output is

$$P_e = (E_q''|V|/X_d'')\sin(\delta-\Theta) \quad (4.7)$$

The equation (4.5) which is obtained by linear interpolation on P_e can be rearranged to get

$$P_e = (-6M/h^2)\delta + \beta \quad (4.8)$$

Where

$$\beta = (6M/h^2)(\delta(t)+h(W(t)-W_0)) + 3P_m - 2P_e(t) \quad (4.9)$$

From equations (4.7) and (4.8), and using the Newton's method, it can be shown that the angle correction is

$$d\delta = \frac{(-E_q''|V|)/X_d'')\sin(\delta-\Theta)-(6M/h^2)\delta+\beta}{(E_q''|V|)/X_d'')\cos(\delta-\Theta)+(6M/h^2)} \quad (4.10)$$

and $\delta(t+h) = \delta(t) + d\delta$ (4.11)

4.4 The Use of Backward Euler Formula

Integration of the swing equation by the use of Backward Euler formula yields

$$W(t+h) = W(t) + (h/M)(P_m-P_e(t+h)) \quad (4.12)$$

$$\delta(t+h) = \delta(t) + h(W(t)-W_0) \quad (4.17)$$

5. SIMULATION RESULTS

Several combinations of synchronous machine model., solution method, angle corrector formula and convergence criteria have been examined. The distinguishing features of the corresponding algorithms, the results of which are reported in this paper, are given in Table-I.

Table I: Salient Features of the Algorithms

Feature \ Scheme	1	2	3	4	5	6	7	8
I. Solution Method: SI	x		x	x	x	x	x	x
PI		x						
II. Discretization of Differential Eqns.: Trapezoidal rule	x	x		x	x	x	x	x
Backward Euler			x					
III. δ Corrector: Trapezoidal rule	x	x						
Backward Euler			x	x				
Linear interpolation of P_e					x		x	
Linear interpolation of P_e&Newton's Methd						x		x
IV. Conv. Criteria: Rotor angle (δ)	x	x	x	x	x	x		
Generator power (P_e)							x	x

The performance evaluation of these algorithms has been done on the WSCC nine bus, three generator standard test system [11]. The disturbance simulated was a three phase fault on bus 7 at time t = 0, with the fault being cleared at time t = 0.087 seconds (5 cycles of 60 Hz) hy opening the line 5-7. Tests have been carried out with the three generators represented by different combinations of models. Typical simulation results have been presented of various schemes obtained with the generator 2 represented by model-2 and generators 1 and 3 by model-0.

5.1. Comparison of Algorithms

Table IT gives the details of the number of iterations (average taken over 20 steps following the initiation of the disturbance) taken by these algorithms whrn h = 16.667ms. (1 cycle of 60 Hz) and when h = 110ms. Comparing the results of scheme 1 and scheme 3, computational advantage of simultaneous implicit approach over partitioned implicit approach can he seen.

The results of the studies indicate that with Backward Euler formula chosen for angle correction, both the alternatives for discretization of the differential equations provide higher damping. However, comparing schemes 1 and 3 (scheme 1 has Trapezoidal rule applied to all differential equations including the swing equation, whereas scheme 3 uses Backward Euler formula for the same), it, is seen that the Backward Euler formula usage takas more number of iterations. This indicates the choice of Trapezoidal rule for discretization of the system differential equations.

Table II: Comparison of Algorithms
[ε_δ=0.0001rd, ε_p=0.001pu., ε_v=0.01 pu.]

Scheme	h = 16.667 ms.		h = 110 ms.	
	$iter_{max}$	$iter_{avg}$	$iter_{max}$	$iter_{avg}$
1	2	0.70	11	6.90
2	3	0.75	14	8.40
3	2	1.50	15	8.95
4	2	1.50	15	8.95
5	2	0.70	7	5.30
6	2	1.20	7	4.55
7	3	1.75	7	5.20
8	3	1.65	6	4.15

In order to choose the angle corrector formula, results of schemes 1 4, 5 and 6 are compared. It is observed that for small step sizes (of the order of I cycle) schemes 1 and 5 take less number of iterations as compared to schemes 4 and 6. As the step *size* increases, scheme 6 shows better results as compared to schemes 1,4 and 5 From Table II, it can be seen that when the step size is one cycle, schemes 1 and 5 take the same number of terations for network solution However, the increase in the number of iterations with scheme 5, as the step size increases, is less than that with scheme 1. Thus scheme 5 turns out to be better than scheme 1 when the step size is more than 1 cycle.

In schemes 7 and 8, generator power output (P_e) is taken as the convergence criteria for network solution. When the results of these schemes are compared with the results of the remaining schemes that are based on angle convergence criteria, it can be inferred that when the step size is small, the schemes based on power convergence criteria are less attractive. However, when step size is increased scheme 8 shows a definite computational advantage.

Thus it can be concluded that while scheme 5 gives better results when the step size is small (of the order of one or two cycles), scheme 8 turnsout to he computationally the best at large step sires. However, when the step size is large, as the applied system disturbance can he seen by the program only after some delay, a higher overshoot may be expected in the relative swing.; of generators, and the solution accuracy would he affected. Thus relatively smaller step size of the order of one or two cycles is preferred for transient stability studies and hence the choice of scheme 5 for such simulation is obvious. On the other-hand, as the scheme 8 provides better scope for increasing the step size, this scheme is preferred for real time simulation. Thus further attention is confined to only schemes 5 and 8.

5.7. Effect of Sources of Network Solution Iterations

Because the machines do not exhibit pronounced saliency in the subtransient state, a saving in computation time ran be achieved by making the saliency correction noniterative. The effect of bypassing iterations due to saliency is given in Table III. From Fig.1 it can be seen that the accuracy is not affected.

Table III: Effect of Bypassing Iterations due to Saliency

Scheme	h = 16.667 ms.		h = 110 ms.	
	$iter_{max}$	$iter_{avg}$	$iter_{max}$	$iter_{avg}$
5	1 (2)	0.10(0.70)	6 (7)	4.40(5.30)
8	1 (3)	0.55(1.65)	5 (6)	3.30(4.15)

(Numbers in brackets indicate the observations obtained with the saliency iteration included)

In order to study the effect of the inclusion of the excitation system on the number of iterations for network solution, IEEE type-1 model is used [12], with the exciter saturation approximated by a two-slope piece-wise linearized curve [1]. From Table IV, it can be observed that the number of iterations for network solution increases with the inclusion of excitation system.

It is seen from Fig.2 that the iterations due to excitation system can also be bypassed without the accuracy being affected. Table V shows the reduction in the number of iterations that can be achieved by making the algorithms noniterative for saliency and excitation system.

Table IV: Effect of Inclusion of Excitation System (Generator 2 with Exciter)

Scheme	h = 16.667 ms.		h = 110 ms.	
	$iter_{max}$	$iter_{avg}$	$iter_{max}$	$iter_{avg}$
5	3 (2)	0.90(0.70)	9 (7)	6.00(5.30)
8	4 (3)	1.70(1.65)	7 (6)	5 40(4.15)

(Numbers in brackets indicate the observations without excitation system)

Table V: Effect of Bypassing Iterations due to Both Saliency and Excitat-ion System

Scheme	h = 16.667 mn.		h = 110 ms.	
	$iter_{max}$	$iter_{avg}$	$iter_{max}$	$iter_{avg}$
5	1 (3)	0.10(0.90)	6 (9)	4.25(6.00)
8	2 (4)	1.00(1.70)	5 (7)	3.50(5.40)

(Numbers in brackets indicate the observations with the saliency and excitation system iterations included)

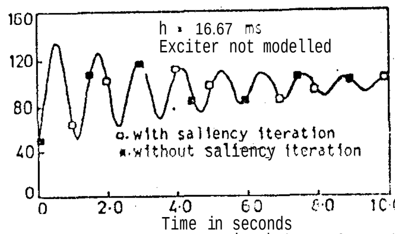
5.3. Effect of Increase in the Step Size of Numerical Integration

As the rate of increase of the number of iterations with the increase in step size is the least with scheme 8, this scheme is chosen to find the effect of further increase in the step size of numerical integration. Table VI shows the details of iterations of this scheme with a step size of 160 ms. It is observed that even at this large step size, the bypassing of the iterations due to both saliency and excitation system does not affect the accuracy of results appreciably. Fig 3 shows the swing curves of this case, with the saliency and the excitation system iterations disabled.

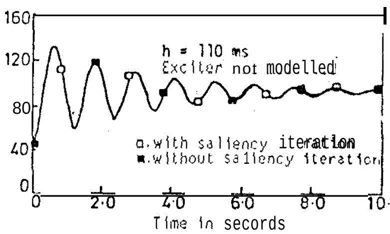
Table VI: Scheme 8 at h = 160ms

Exciter not Modelled				Gen.2 With Exciter			
With Saliency Iteration		Without Saliency Iteration		With Saliency & Exciter Iteration		Without Saliency and Exciter Iteration	
Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.
9	5.95	8	5.00	11	6.90	8	5.30

Numerical instability is observed when the step size is increased to 170 ms. It may be possible to alleviate this problem of numerical "blow up" by introducing a small damping on machine rating in the swing equation. This demands further investigation.

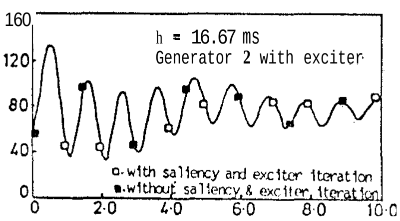


a) Scheme 5 - Relative swing between Generator 2 and Generator 1

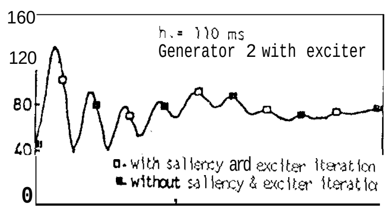


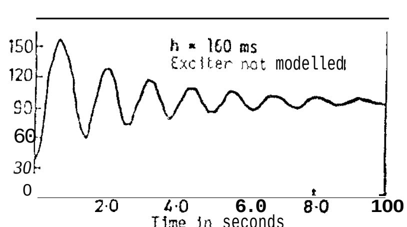
b) Scheme 8 - Relative swing between Generator 2 and Generator 1

Figure 1: Sensitivity of saliency iteration.

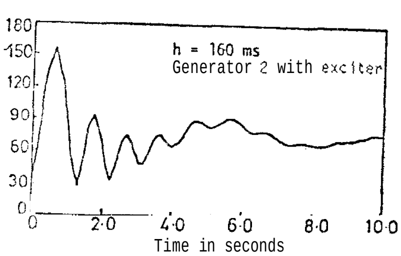


a) Scheme 5 - Relative swing between Generator 2 and Generator 1





a) With saliency iteration avoided



b) With saliency and exciter Iterations avoided

Figure 3: Scheme 8 - Relative swing between Generator 2 and Generator 1

6. CONCLUSIONS

This paper reports the results of investigations towards arriving at a most suitable algorithm for real time simulation of power system dynamics. The selection of a synchronous machine model, the solution approach, and a convergence criteria for the network solution have been dealt with. Effect of various angle corrector formulae on computation time has also been examined. Further work is needed to make the algorithms fully noniterative and to increase the step size of numerical integration, while keeping the solution numerically stable.

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