A GENERAL METHOD FOR POWER FLOW ANALYSIS IN MTDC SYSTEMS

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ABSTRACT

This paper presents a novel and general approach for the power flow analysis in multiterminal DC systems.A major feature of this approach is the clear demarcation of the controller and the DC network equations which enables the consideration of a variety of converter controls.The method can also handle the constraints on several variables such as DC current(Id),converter control angle(θ) and off nominal tap ratio(a).

1. INTRODUCTION

HVDC power transmission has several advantages over AC transmission particularly for long distance bulk power transmission and asynchronous interconnection of power systems.The major feature is the fast controllability of power which can be used effectively for improving the system security.However most of the HVDC schemes are planned for point to point power transmission with two terminals.Multiterminal(with more than two terminals)DC systems contribute to better economy and flexiblility of system operation thereby increasing the scope of application of HVDC links.MTDC operation has become a reality with the commissioning of Corsica terminal of the Sardinia link.A five terminal MTDC system(Hydro Quebec-New England) is under contruction and new systems are also being planned.

The planning of MTDC system requires detailed studies involving power flow analysis in AC/DC systems.There are several publications[1-4],which deal with the methods of modelling the DC systems in the power flow analysis.The incorporation of the DC system equations in the optimal power flow analysis has also been studied [5].The solution of the non-linear algebraic equations describing the AC/DC system can be obtained using Newton,Gauss-Seidal or other iterative methods.The approach to the solution of power flow problem in AC/DC systems is broadly divided into two categories 1)simultaneous or unified and 2)sequential method.The latter approach is widely used and involves the iterative solution of AC and DC systems separately and alternately until convergence is obtained.

There are several lacunae in the development of power flow analysis in MTDC systems as reported in the literature. A major weakness is the lack of adequate appreciation for the nature of the differences and similarities in the structure of AC and DC systems and their operation. For example,in a HVDC system there are several variables which are subjected to constraints, such as DC current(I_d), voltage(V_d),converter control angle(θ)and off nominal tap ratio(a). Also, while there are two control variables per terminal(θ and a) these are adjusted to schedule some other variables (current(Id), power(Pd), voltage(Vd) or reactive power(Qd)), at the specified (desired) values. In the case of AC systems, the control variables say,power(P_g) and voltage magnitude(V_g), at the generator buses are also the scheduled variables. The only constraints to be considered in the usual AC power flow analysis are the limits on the reactive power generation(Q_g).

The other shortcomings of the approach, used in [1-4] are given below.
1) The use of inflexible per unit system where the base AC and DC voltages are related. As only one DC voltage base can be used in a connected MTDC system, this implies a fixed AC base voltage which is inconvenient.2)The use of an overdetermined set of variables to characterise the state of operation in a DC system. For example in [1],[2] five variables(V_d,I_d,a,cosθ and ϕ where ϕ is the power factor angle) per terminal are used to describe the operating state. This can be confusing as all these variables are not independent. In analogy to AC system network, it is adequate to utilise the DC bus voltages,V_d(or some other related variables) to describe the state of the DC network.3)Lack of distinction between the control variables(a, θ) and the scheduled variables such as power,current, voltage and reactive power.

Also it is to be noted that out of '2n' scheduled variables in a 'n' terminal MTDC system,only 'n' variables are required to solve the DC network.The remaining 'n' variables such as the reactive power (Qd) or power factor (cosϕ) are to be used in solving the AC network equations.

4)Lack of a general framework for obtaining an optimal feasible solution when several constraints are to be considered.

In this paper a general method for power flow analysis in MTDC systems is presented which overcomes the shortcomings of existing methods described above. The basic approach is first described,followed by the formulation of the DC system equations and the outline of the algorithm for their solution. The handling of the constraints is also described. The method is illustrated with the help of a 4 terminal DC and 5 terminal-30 bus AC/DC examples.

2. OUTLINE OF THE BASIC APPROACH

The power flow analysis in AC or AC/DC systems involves the solution of nonlinear equations subject to certain constraints. In general, the equations can be written as

$$g(x,w) = 0 \qquad ...(1)$$

$$h^{min} < h(x,w) < h^{max} \qquad ...(2)$$

$$u^{min} < u(w) < u^{max} \qquad ...(3)$$

where x = state vector
w = vector of specified variables
u = vector of control variables
h = vector of variables which are constrained
(for example in AC system it can represent line flow or generator reactive power)

In an AC system, the control variables (u) and the specified variables (w) are identical (For example P_g,V_g at the generator bus). If 'w' is specified in advance (subject to the limits) and if inequality (2) is satisfied then the solution of equation (1) corresponds to the ordinary power flow analysis in AC systems. If 'w' is not specified in advance, they can be chosen by minimising a cost function subject to the constraints (1) to (3). Then this becomes the solution to optimal power flow.

The equations for the AC/DC system can also be defined in a similar fashion as described above. However it is convenient to separate the equations for the AC and DC systems by defining an interface at the converter AC buses. The real and reactive powers are specified at all the converter buses as a function of the DC system control specifications. The DC system can be represented by the DC network with converters modelled by their equivalent circuits. At each converter terminal, power or current can be specified except at the voltage setting terminal (VST) where the DC voltage is specified. As there are two control variables (a, θ) per terminal, the additional specification for the terminal could be control angle (θ) or reactive power Q_d or powerfactor(cosϕ). It can be shown that the real power at the AC/DC interface are functions of the first set of specifications (w_{DC1}) whereas the reactive power can be expressed as the function of the first and second set of specifications (w_{DC2}). In reference [3], Ong & Fudeh treat, DC bus voltages as the state variables and solve the DC network equations directly using Gauss-Seidal method. They represent the DC network by the bus resistance matrix formed by treating the VST as the reference terminal. This can be cumbersome if VST can change.

In the formulation given in this paper, the converter terminal is represented by a Norton's equivalent of a current source in parallel with a commutation resistance (R_c). This current source is proportional to the product avcosθ. For inverter if θ is assumed to be extinction angle(γ), then R_c is negative. Modelling the converter by its Norton's equivalent enables the DC network equations to be expressed with ground as reference node. The DC network equations based on bus resistance matrix are used to form the following controller equations.

$$w_{DC1}^{spec} = w_{DC1}(x_{DC}) \qquad ...(4)$$

where the state vector x_{DC} corresponds to the vector of current sources. Equation (4) can be solved using Newton or Gauss- Seidal method

The components of the second set of control specifications w_{DC2} can include θ or Qd or cosϕ at a terminal. However it can be shown that all these specifications are equivalent to specifying θ as in general cosθ can be expressed a function of w_{DC1} and w_{DC2}. Once θ is calculated reactive powers at the converter busses are specified and the AC power flow can be performed. The voltages at the AC busses are determined from the solution of AC power flow and the off nominal tap ratio(a) can be calculated as the product (av) is already determined from the knowledge of w_{DC1} and w_{DC2}

3.DC SYSTEM MODEL

3.1 Converter equations

At any given terminal(k) the average DC voltage can be expressed as:

$$V_{dj}=k_j a_j V_j \cos\theta_j - Rc_j I_{dj} \qquad ...(5)$$

where

$$k_j = \frac{3\sqrt{2}\, n_k n_{bk} V_{ab_k}}{V_{db}\, \pi}$$

$$Rc_j = \frac{3X_{ck} n_{bk}}{\pi Z_{db}}$$

n =the nominal turns ratio of the converter transformer.
n_b =is the number of bridges connected in series.
X_c =leakage reactance of the converter transformer on the secondary
V_{ab}=Base AC Voltage
V_{db}=Base DC Voltage
Z_{db}=Base DC resistance=$\frac{V_{db}}{I_{db}}$
I_{db}=Base DC current
(The subscript j refers to k^{th} terminal)

It is to be noted that the equation is written in per unit quantities. The per unit system used here is more general than that used in reference [3] as the base AC voltage V_{abk} can be chosen independently of the base DC voltage(V_{db}). In other words, the co-efficient(k) need not be equal to 1, in general. In addition, the value of k can vary depending on the terminal as the base AC voltage, the nominal turns ratio and the number of bridges can assume different values at different terminals. Base DC current is obtained from the relation

$$P_{db}=V_{db}I_{db} \qquad ...(6)$$

Although, it is not essential to choose the base DC power P_{db} equal to the base AC power P_{ab}, it is convenient to do so. Base DC voltage is normally chosen equal to the rated voltage in a parallel connected DC system. Base powers can be chosen to be a convenient number(say 100MW).

Equation (5) represents an equivalent circuit shown in Fig.(1),

$$\text{where } I_j = \frac{k_j a_j V_j \cos\theta_j}{R_{cj}} \qquad \ldots(7)$$

It is to be noted that for Rectifier terminal($\theta=\alpha$) whereas for the Inverter terminal is normaly chosen as the extinction angle (γ) in which case Rc is negative. However if θ is chosen as the angle of advance(β) Rc is positive.

3.2 DC.NETWORK EQUATIONS

The DC lines and the smoothing reactors are modelled as resistive networks. If the two poles of the bipolar MTDC system are balanced,it is adequate to consider only the network equations for a single pole. Combining the equivalent circuits for all the terminals and the DC network, the following equation is obtained

$$[G]\ V_d = I \qquad \ldots(8)$$

where [G] is a n x n conductance matrix for a 'n' terminal MTDC system (It is assumed that the non converter busses are absent or are eliminated).The equation (8) is always applicable, independent of the choice of the VST.

3.3 DC CONTROL EQUATIONS

At the voltage setting terminal,

$$V_d = \sum_{j=1}^{n} R_{ij} I_j = V_d^{spec} \qquad \ldots(9)$$

where R_{ij} is the element of the bus resistance matrix obtained by inverting [G].(To ensure the existence of R in all cases, it may be convenient to treat Rc as positive at inverters by putting $\theta=\beta$). At the remaining terminals either power or current is specified.

$$P_d = V_d I_d = P_d^{spec} \qquad \ldots(10)$$

$$Id = I_d^{spec} \qquad \ldots(11)$$

In the above equations both Pd and Id can be expressed in terms of I. For example,Id_k can be written as

$$I_{dk} = I_k - \frac{V_{dk}}{Rc_k} \qquad \ldots(12)$$

There are n control equations for n terminal system of the form given in equations (9),(10) or (11).They can be grouped together and written as

$$f_1(I) = w_{DC1} = w_{DC1}^{spec} \qquad \ldots(13)$$

where w_{DC1} is the vector of first set of scheduled varibles mentioned earlier.Equation (13) can be solved by Newtons method for the state victor 'I'.Once 'I' is solved,the other variables V_d,I_d and P_d are directly obtained.

The second set of control specification w_{DC2} involve specifying either θ(α or γ),Qd or cosϕ.At the k^{th} terminal

$$Q_{dk} = |P_{dk}| \tan\phi_k \qquad \ldots(14)$$

$$\cos\phi_k = \frac{V_{dk}}{k_k a_k V_k} = \frac{V_{dk}\cos\theta_k}{I_k Rc_k} \qquad \ldots(15)$$

In general w_{DC2} can be expressed as

$$w_{DC2} = f_2(I,\theta) = w_{DC2}^{spec} \qquad \ldots(16)$$

Since I can be solved from equation (13),it is possible to solve for θ as a function of w_{DC1} and w_{DC2}.

3.4 CONVERTER BUS EQUATIONS

At a converter bus (say k) P_{dk} and Q_{dk} are treated as loads given by

$$P_{dk} = V_{dk} I_{dk} = P_{dk}(w_{DC1}) \qquad \ldots(17)$$

$$Q_{dk} = |P_{dk}| \tan\phi_k = Q_{dk}(w_{DC1}, w_{DC2}) \qquad \ldots(18)$$

The equations (17) and (18) are used along with the AC power flow equation.Newton's method or its variants such as fast decoupled load flow (FDLF) method can be used to solve the AC power flow

4 INCLUSION OF CONSTRAINTS

It is to noted that unlike in AC systems, the control variable(a and θ), are different from the specified variable w_{DC1} and w_{DC2}.As there are limits on the control variables one must alway s check that the limit violations do not take place while specifying w_{DC1} and w_{DC2}.In addition to the control variables, it is also necessary to check for the limits on the DC current at all the terminals particularly at the VST.

The published literature on MTDC power flow [1-4] do not discuss procedures for systematically taking care of the various constraints.In reference (3), only constraint violation of the tap ratio (a) are considered. It is said that if the limits on 'a' are violated on any terminal,the dc voltage has to be rescheduled and the DC and AC power flow have to be performed iteratively until feasible solution is obtained. The limit violations of the DC currents can be checked at the stage of solving DC controller equations (13). This will establish the lower bound on the DC voltage specified at the VST subject to meeting the power demand at other terminals.If the feasible solution requires the operation at lower voltage than the lower bound based on the DC current constraints,the power specifications at one or more terminals have to be changed to current specifications(at their upper limits).

As the limits on the control angle (θ) are specified, it is possible to compute the limits on the vector w_{DC2} which are also fuctions of w_{DC1}.By calculating the limits in advance,it is possible to check,if the specified values of w_{DC2} are feasible.

From the knowledge of I obtained from solving controller equations(13),the product $a_k V_k \cos\theta_k$ is specified. From the specifications on w_{DC2} θ_k is calculated. From the knowledge of limits on 'a_k' the feasible range of bus Voltage 'V_k' can be calculated which will not result in rescheduling of w_{DC1} or w_{DC2}.If it is not possible to maintain the AC bus voltage within this range from the AC system control variables (reactive power generation) it would be necessary to modify w_{DC1} and w_{DC2}. It is to be noted that the specification on w_{DC2} are easier to modify than w_{DC1}. If w_{DC1} requires modification, it is desirable to consider first the DC voltage spefication, rather than changing the power specifications. The adjustments to w_{DC1} and w_{DC2} required for obtaining a feasible solution can be performed from sensitivity analysis.

5. EXAMPLES

5.1 Four Terminal MTDC system

This is taken from reference (5). The data are given in appendix I. The single line diagram of the system is shown in Fig.2. At each terminal the AC system is represented by a simple Thevenin's equivalent. It is assumed that the AC system at each terminal is isolated from the remaining systems. The results of the power flow are shown in table.1. The solution of I, done by Newtons method took two iterations to converge to the solution within the specified mismatch of 0.00001. If the maximum current limits at all the terminals are specified, then the minimum voltage to which the voltage at VST can be reduced is that corresponding to the maximum of the voltages obtained from Fig. 3 corresponding to the current limits. In Fig.3 the maximum current limit at each terminal is assumed to be 1.2 p.u.(at its own base). The minimum limit on the voltage V_d is determined by the maximum limit on terminal 2(VST) current. If the current limit at any of the terminals is violated, the power control at that terminal has to change over to current control with current specified at the maximum limit.

The variation of (avcosθ) as a function of DC voltage(specified at VST) is shown in Fig.4. This shows that this variation is essentially linear in nature. The rescheduling of this voltage in order to obtain a feasible solution can be calculated with the help of this figure. The variation of (avcosθ) with the variation of power(as per unit of the rated load at all the terminals) is shown in Fig.5. This also shows a linear variation.

5.2 5 Terminal 30 bus MTDC system

This is taken from reference (7). The :ingle line diagram of the system is shown in 'ig. 6. The results of the power flow are shown n Table 2. The solution for I done by Newton's ethod took two iterations to converge to the olution within the specified mismatch of 10^{-5} u. The solution for AC power flow was done by DLF due to Stott and Alsac [8] which took 5 :erations for a power mismatch of 0.001 pu.

:ble 1. Results of 4 terminal MTDC system

rminal antity	1	2	3	4
	0.9997	1.000	0.9762	0.9719
	6.0019	9.4146	-3.8415	-11.5755
	6.001	9.4146	-3.75	-11.25
	3.5281	4.6528	2.2587	6.9335
	1.0006	1.0002	0.9999	0.9978
	0.9983	0.9605	0.9990	0.9858
n deg)	18	6	21	19

Table 2 Results of 5 Terminal MTDC system

terminal Quantity	1	2	3	4	5
V_d	1.0626	1.0310	1.0368	1.0369	1.0494
I_d	1.3176	-0.5820	-0.6751	-1.2538	1.1933
P_d	1.4000	-0.6000	-0.7000	-1.3000	1.2523
Q_d	0.5380	0.2570	0.3050	0.6140	0.3460
V	1.0840	1.0020	1.0220	0.9820	1.1000
tap	1.0540	1.1180	1.1030	1.2140	0.9940
(θ in deg)	16.2	21.2	20.7	26.2	8.7

6. CONCLUSIONS

A novel and general approach for power flow analysis in MTDC systems is presented. This overcomes the several drawbacks of the methods described in the literature. This analysis permits the consideration of a variety of control specifications. The per unit system used in the formulation is general and not restrictive. The paper also describes the approach to obtaining a feasible solution.(satisfying constraints on a number of variables)

The power flow analysis is illustrated with the help of two examples.

7. REFERENCES

1) J.Arrillaga and C.P.Arnold, B.J.Harker, Computer Modelling of Electrical Power Systems, John Wiley, 1983.
2) J.Reeve, G.Fahmy and B.Stott, "Versatile Load flow method for multiterminal HVDC systems", IEEE Trans., Vol.PAS-96, No.3, 1977, pp.925-933.
3) C.M.Ong and H.Fudeh, "A simple and efficient ac-dc load-flow method for multiterminal DC systems", IEEE Trans., Vol.PAS-100, No.11, 1981, pp.4389-4396.
4) J.Mahseredjian, S.Lefebvre and D.Mukhedkar,"A multiterminal HVDV load-flow with flexible control specifications",IEEE Trans. on Power Systems, Vol.PWRD-1, No.2, 1986, pp.272-282.
5) P.C.S.Krishnayya, S.Lefebvre, V.K.Sood, N.J.Balu, "simulator study of a multiterminal HVDC system with small parallel tap and weak ac systems",IEEE Trans., Vol.PAS-103, No.10, 1984, pp. 3125-3132.
6) C.M.Ong, C.N.Lu and S.S.Chen, "The incorporation of HVDC equations in optimal power flow methods using sequential quadratic programming methods", IEEE Trans. on Power Systems, Vol.3, No.3., 1988, pp.1005-1011
7) G.Opuku and C.M.Ong, "Coordination of Reactive Power Sources to correct Bus voltage violations in an AC-DC system", IEEE Trans., Vol.PAS-103, No.6, 1984, pp.1128-1133.
8) B.Scott and O.Alsac, "Fast decoupled load flow", IEEE Trans., Vol.PAS-93, pp. 859-867, May/June 1974.

Appendix I

A.1.1 Data for the 4 terminal MTDC system [5]
Fig.2
No. of DC lines:4
No. of DC terminals:4
No. of DC nodes:5

Dc line resistances:

from	to	(Resistance in p.u.)
1	5	0.000093
2	5	0.000093
5	3	0.001488
3	4	0.000372

Commutating Resistances:
$R_{c1} = 0.0172$
$R_{c2} = 0.01161$
$R_{c3} = 0.02283$
$R_{c4} = 0.00929$

DC System specifications:

terminal	specification	remarks
1	power 6.0	rectifier
2	voltage 1.0	rectifier
3	power -3.75	inverter
4	power -11.25	inverter

AC system details:

Qc	SCR	Z angle	Qd	e
3.31	3.5523	79.9613	3.5275	1.0968
4.95	3.5526	79.9784	4.6518	1.0839
2.0625	3.0405	79.9754	2.2586	1.0149
6.1875	3.7846	79.5603	6.9327	1.0051

A.1.2 Data for the 5 Terminal MTDC System.[7] Fig.6

Number of DC lines:4
Number of DC terminals:5
Number of DC nodes:5

DC line resistances:

from	to	Resistance(in p.u.)
1	5	0.01
5	4	0.01
5	3	0.01
3	2	0.01

Commutating Resistances:
R_c=0.0262 at each terminal

DC System specifications:

terminal	specification	remarks
1	power 1.40	rectifier
2	power -0.60	inverter
3	power -0.70	inverter
4	power -1.30	inverter
5	voltage 1.0494	rectifier

Ac system details:

The details used for AC systems are the same as that used in reference [7].

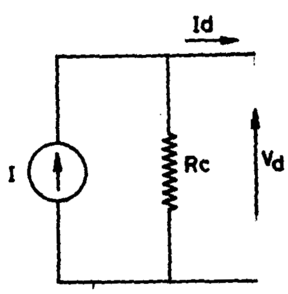

FIG. 1 -NORTON'S EQUIVALENT REPRESENTATION OF A CONVERTER

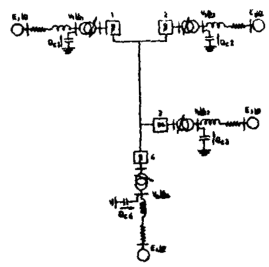

FIG 2-A FOUR TERMINAL MTDC SYSTEM CONNECTED TO ISOLATED AC SYSTEM

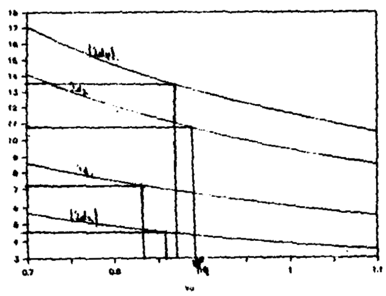

Fig 3 plot of Id vs Vd

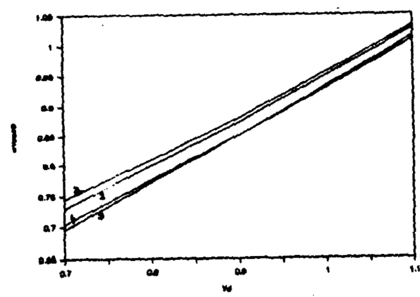

Fig 4-plot of (avcosθ) vs Vd

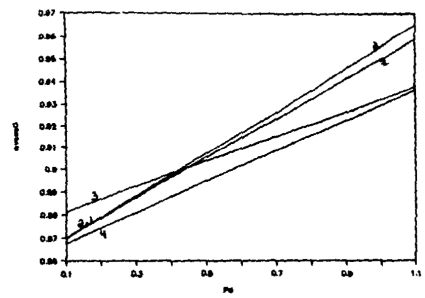

Fig 5 plot of avcosθ vs Pd

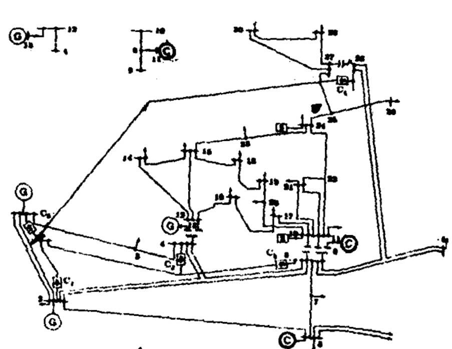

Figure 6 Sample 30-AC bus, 5-DC terminal system.