Fig. 3. The test image Barbara (672 $\times$ 560 with 8 b/pel).

(a)

(b)

Fig. 4. (a) Details of the low-resolution image recovered from the first 4×4 DCT coefficients of the 8×8 BW transformed Barbara. (b) Details of the low-resolution image recovered from the first 4×4 DCT coefficients of the 8×8 discrete cosine transformed Barbara. Aliasing effects are more disturbing than Figure 4(a) (see the scarf of Barbara).

Consider the test image Barbara (672 $\times$ 560 with 8 b/pel) shown in Fig. 3. The Barbara image is coded by both an implementation of the JPEG standard and a BWT coder. The BWT coder is actually the JPEG coder, which employs a BWT instead of the DCT. In both coding schemes, 8×8 image subblocks and the default weighting matrix of the JPEG standard [3] are used. The JPEG (IIR-BWT) [Daubechies-BWT] coder compressed the image to 1.349 (1.44) [1.487] b/pel. All of the coded images are visually indistinguishable from the original one (figures are omitted).

A 1-D DCT (Butterworth-IIR-BWT) [Daubechies-BWT] [SB-BWT] of size $N = 8$ can be implemented by performing 12 (8) [22] [62] multiplications.

In many image coding applications, scalability of the coded bit stream is a desirable property [9]. If a low-resolution signal can be recovered from the bit stream, it can be displayed in low-resolution display. The low-resolution images (336 $\times$ 280) recovered from the first 4×4 BWT coefficients are compared to the low-resolution signal that can be extracted from a DCT-based scheme. In Fig. 4 the low-resolution images are shown. It is clear that the low-resolution image obtained from BWT [Fig. 4(a)] is better than the one obtained from DCT [Fig. 4(b)]. Aliasing effects are more disturbing in Fig. 4(b). If a scalable coded bit stream is desired, then the BWT is more suitable than DCT. One can extract better quality low-resolution images in BWT-coded bit streams than in DCT-coded ones.

From any given PR filter bank one can construct a linear block transform. Since the class of PR filter banks is quite large, the corresponding BWT class is also large. The choice of BWT for a given application remains as an interesting problem.

REFERENCES

- [1] A. K. Jain, *Fundamentals of Digital Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [2] J. W. Woods, Ed., *Subband Image Coding*. Norwood, MA: Kluwer, 1991.
- [3] Didier Le Gall, "Digital multimedia systems: Digital image and video standards," *Commun. Ass. Comput. Mach.*, vol. 34, pp. 47-58, 1991.
- [4] I. Daubechies, "Orthogonal bases of compactly supported wavelets," *Commun. Pure Appl. Math.*, vol. 41, pp. 909-996, 1988.
- [5] S. G. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation," *IEEE Trans. Patt. Anal. Machine Intell.*, vol. 2, pp. 674-693, July 1989.
- [6] R. Ansari, C. Guillemot, and J. F. Kaiser, "Wavelet construction using Lagrange halfband filters," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 1116-1118, 1991.
- [7] M. J. T. Smith and T. Barnwell, "Exact reconstruction techniques for tree-structured subband coders," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 434-441, June 1986.
- [8] R. Ansari and B. Liu, "A class of low-noise computationally efficient recursive digital filters with applications to sampling rate alterations," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 90-97, Feb. 1985.
- [9] C. Gonzales and E. Viscito, "Flexibly scalable digital video coding," *Image Commun.*, submitted for publication.

Vector Quantization of Images Using Input-Dependent Weighted Square Error Distortion

Anamitra Makur

I. INTRODUCTION

The measure of quantization distortion in a vector quantization (VQ) [1] coder for image is vital to the perceptual coder performance. The distortion function should ideally quantify human visual discomfort towards quantization errors. Mean

Manuscript received July 7, 1992; revised March 1, 1993. This work was supported by Pacific Bell. Paper recommended by William B. Pennebaker.

The author is with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India. IEEE Log Number 9212579.

square error (MSE) distortion is most popular for image coding:

$$d_{\text{MSE}}(X, Y) = \frac{1}{K} [X - Y]^t \cdot [X - Y]. \quad (1)$$

Here X and Y are K -dimensional column vectors, and t indicates transpose. Unlike in speech coding, where more involved distortion measures such as the Itakura-Saito distortion function [1] have been used alongside the classical ones, there have been few investigations in image coding. It is known that MSE distortion is not a true measure of the psychovisual distortion. Earlier attempts to incorporate psychovisual effects in MSE consist of performing some complex operation before applying MSE, such as power-law, filtering [2], transform [3], etc., and performing the inverse operation after VQ. Few other measures of interest, such as weighted square error (WSE) distortion

$$d_{\text{WSE}}(X, Y) = [X - Y]^t \cdot W \cdot [X - Y] \quad (2)$$

where W is a weight matrix of size $K \times K$, may be found in [1].

II. INPUT-DEPENDENT WEIGHTED SQUARE ERROR DISTORTION

We suggest a general class of distortion measure by extending WSE such that W may depend on the input vector. Thus, the input-dependent weighted square error (IDWSE) distortion [1], [4], [5] is given by

$$d_{\text{IDWSE}}(X, Y) = [X - Y]^t \cdot W(X) \cdot [X - Y]. \quad (3)$$

Since the distortion function is no more commutative, we need to specify X as the input vector and Y as any codeword.

The generalized Lloyd algorithm (GLA) [1] to design an optimal VQ codebook requires that the centroid exists (preferably easily computable) for any given distortion function (for centroid condition and other necessary and sufficient conditions for optimality, see [1]). Computationally simple centroids such as mean (for MSE) or median (for mean absolute error) are possible if dimensional independence exists in the distortion function. If $W(X)$ is constrained to be diagonal, then dimensional independence is achieved in IDWSE, and the centroid simply becomes

$$C = E\{W(X)\}^{-1} \cdot E\{W(X) \cdot X\}. \quad (4)$$

By replacing the centroid expression of arithmetic mean (for MSE) in GLA with (4), it is possible to design a VQ codebook optimum in the IDWSE sense.

A. Codebook Design Using Image Activity

Image activity is a measure of the amount of visible detail present in an image block. Local image activity has been previously used to adaptively encode an image in order to maintain near-uniform subjective quality. For image VQ, it is known that an MSE codebook does not represent the edges (high-activity parts) of an image well. Therefore the training sequence (TS) is segmented into separate classes such as edge or shade, and subcodebooks designed for each class are concatenated to produce the final codebook [6]–[8]. This technique offers flexibility in reinforcing certain types of codewords in the codebook. However, TS segmentation, as well as the subcodebook sizing, is arbitrary and no longer optimal in MSE or any other sense. Only if the PDF of each class is known *a priori* may optimum sizing of subcodebooks be found asymptotically [8]. This leaves open any optimization for a practical codebook design.

In this case, if the encoding search for some input vector is limited to the corresponding subcodebook, the resulting distortion measure is

$$d(X, Y) = \frac{w(X, Y)}{K} [X - Y]^t \cdot [X - Y] \quad (5)$$

where

$$w(X, Y) = \begin{cases} 1, & \text{if } X \text{ and } Y \text{ belong to the same class,} \\ \infty, & \text{if } X \text{ and } Y \text{ belong to different classes.} \end{cases} \quad (6)$$

Thus, for a set consisting of vectors from more than one class, the centroid cannot be determined. Therefore it is not possible to design an optimum codebook for the entire TS. In order to achieve optimization, we have to allow the codewords belonging to some class to be influenced by training vectors from other classes. This is not undesirable. In fact, it actually compensates for an arbitrary segmentation of TS to some extent. If ∞ in (6) is replaced by a large but finite value, determination of the centroid is possible. However, this would always force a codeword of the same class as the input vector to be chosen as nearest. This may not be the case, since the segmentation is arbitrary. In order for us to allow flexibility in choosing a match across class boundaries, along with flexibility in reinforcing certain classes, IDWSE distortion may be used.

Consider IDWSE distortion of (3) having the weight matrix of the form

$$W(X) = a(X) \cdot I \quad (7)$$

where $a(X)$, a positive scalar, is the activity of X . Negative or zero value of $a(X)$ may lead to inconsistent distortion measure. Ideally, $a(X)$ should reflect the relative importance or visibility of X . Thus, if an edge block is assigned a higher activity than a shade block, in effect less distortion is tolerable for edges, which produces a codebook having plenty of edge codewords. Since the centroid for IDWSE is defined across class boundaries unlike (6), a single codebook may be generated on entire TS and the encoder has to search the entire codebook. The resulting codebook is optimized for a given $a(X)$. When $a(X)$ is optimized for perceptual quality, the codebook is also subjectively optimal. Since IDWSE measure itself depends on $a(X)$, it can not be optimized using IDWSE distortion unless some iterative algorithm is used. Since no explicit classification is performed, the number of (activity) classes is unconstrained. By choosing a continuous activity, infinitely many classes may also be realized. Finally, since X is the input vector, IDWSE encoding and MSE encoding are equivalent for (7), and no extra encoding computation is needed.

B. Encoding Using Boundary-Sensitive Distortion Measure

While a coding error may not be noticeable at the interior of a block, it may become annoying at the block boundary, especially if the adjacent block has an error of opposite sign. In other words, distortion of peripheral pixels is more serious than that of interior pixels. Proposed schemes, such as [9], improve this impediment for a few extra bits. We propose incorporation of positional sensitivity in the IDWSE distortion measure, which requires no extra bit. Replace W in (3) [or I in (7) if used along with activity] by

$$D = \{d_{ij}\}, \quad d_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } n < i = j < mn - n + 1, i \bmod n > 2 \\ q & \text{if } i = j = 1 \text{ or } n \text{ or } mn - n + 1 \text{ or } mn \\ p & \text{otherwise} \end{cases} \quad (8)$$

for an $m \times n$ block ($K = mn$). Here, elements corresponding to interior pixels, geometrical boundaries, and four vertices, have

values 1, p , and q respectively. Naturally $1 \leq p \leq q$, since distortion in a vertex is most annoying and that in the interior is least annoying.

If D is independent of X , the centroid is simplified from (4) to the mean of the cluster. However, extra computation is required during both codebook design and encoding. The number of elements not equal to 0 or 1 in D is $2(m+n)-4$. Since m and n are of the order of $\sqrt{K}$, the additional multiplication is of $\mathcal{O}(N\sqrt{K})$ per vector for a codebook of size N . These multipliers could be avoided in hardware by choosing p and q as integer powers of two and implementing them as shift operations.

III. SIMULATION RESULTS

Though IDWSE may be used on 1-D signal, we present only image coding results, since the suggested schemes of activity and boundary sensitivity are image specific. In order to better demonstrate the effects of using IDWSE distortion on the decoded image, separate simulation results for these two schemes are presented. The MSE codebook performance for same TS and same input image is taken to be the basis for comparison. Initialization is done by splitting and optimizing every size of GLA until the final size of $N = 1024$ on a TS having about 20 training vectors per codeword, and block size taken is 8×8 .

A proposed multiplierless function for computing activity is shown to offer subjective improvement. Consider the absolute difference in value between a random pair of pixels in an image block. For a high-activity block, this value is likely to be high. For better estimation, the absolute difference from m such random pairs is summed up. These m random positions are generated beforehand and stored for use in every block. If n classes are desired, the activity $a(X)$ is produced by applying an n -level quantizer (or a look-up table) on this value. Fig. 1 demonstrates the effectiveness of our scheme to extract edges of all orientations and high-detailed parts of an image. While Fig. 1(a) is the original image of Leena, in Fig. 1(b) the activity for each block is computed using eight pairs of pixels, and an appropriate shade is chosen for the entire block such that higher activity produces darker shade.

Four activity classes, equiprobable in TS, have been defined by nonuniformly quantizing the activity. Thus, $a(X)$ takes only one of four possible values a_1, a_2, a_3 , and a_4 after quantization. Two sets of such quantized values have been considered. Firstly, using arithmetic progression, we've chosen $a_i = 1 + (i-1)\delta$ for some positive δ . Secondly, using geometric progression, we've chosen $a_i = (1 + \delta)^{i-1}$ for some positive δ . Note that $\delta = 0$ is equivalent to MSE distortion in both cases. The simulation results show that with increasing value of δ , the edges and the contrasting blocks are indeed encoded better. Fig. 2 shows the average MSE/pixel of the decoded image from the IDWSE codebooks for various values of δ at a rate of 0.156 b/pixel. IDWSE distortion is not shown along the Y-axis since δ itself is changing along the X-axis, resulting in different IDWSE measures for each point. Slightly smaller distortion than $\delta = 0$ case (MSE) has been achieved for small values of δ . This is not a contradiction, since GLA promises minimum average distortion only for TS. Though the MSE distortion is higher, improved subjective quality is observed for higher values of δ . Fig. 3 shows the coding results for the image of Fig. 1. While Fig. 3(a) is the decoded image for $\delta = 0$ (MSE), Fig. 3(b) is the result for $\delta = 2$ with arithmetic progression (IDWSE). The edges are better represented in the second image than in the first one.

To incorporate boundary sensitivity, we have chosen $p = q = \delta$ for some $\delta > 1$. Here $\delta = 1$ is equivalent to MSE distortion.

(a)

(b)

Fig. 1. Image activity of 8×8 blocks: (a) original; (b) image activity.

Fig. 2. MSE distortion for image activity schemes.

(a)

(b)

Fig. 3. Image activity coding result: (a) decoded image for $\delta = 0$ (MSE) at 0.156 b/pixel, SNR 26.88 dB; (b) decoded image for $\delta = 2$ with arithmetic progression (IDWSE) at 0.156 b/pixel, SNR 26.81 dB.

Fig. 4 shows the average MSE/pixel of the decoded image for various values of δ . Though for small values of δ the IDWSE codebook shows a lower distortion than the conventional codebook, the subjective improvement becomes noticeable for higher values of δ . Fig. 5 shows the coding result for $\delta = 2.5$ (IDWSE), which should be compared to Fig. 3(a). The blockiness has indeed been reduced from earlier results. Moreover, the edges are better represented now. The codebook generated by the IDWSE distortion incorporating boundary sensitivity has another advantage over the MSE codebook. It is desirable from entropy considerations that all members of a codebook are utilized nearly equally often. It has been observed that the utilization of codewords for these IDWSE codebooks is closer to uniform compared to that for MSE codebook. This is because

Fig. 4. MSE distortion for boundary sensitivity schemes.

Fig. 5. Boundary sensitivity coding result: Decoded image for $\delta = 2.5$ (IDWSE) at 0.156 b/pixel, SNR 26.81 dB.

choosing a codeword now depends more on the boundaries, which are distributed more evenly than entire image blocks.

IV. CONCLUSION

The IDWSE distortion, which is computationally simple and usable with GLA, has been shown to offer subjectively better decoded images than the MSE distortion. With the image activity, better edge representation has been achieved for low-bit-rate VQ image coding without sacrificing anything. This strategy produces optimal codebook in the IDWSE sense and is therefore preferable to the equivalent subcodebook method. A simple way to compute the image activity has been proposed and shown to be quite effective. Introducing block boundary sensitivity in IDWSE distortion has produced a decoded image with less blockiness and better edge representation, while the encoder computation is marginally raised. Other modifications to incorporate human visuality in the IDWSE distortion can also be thought of. Moreover, it is easy to use the IDWSE codebooks with the improved VQ coders suggested by others in order to gain the advantages of both.

ACKNOWLEDGMENT

The author acknowledges the contribution of E. C. Posner for his guidance. The author also thanks a reviewer for pointing out a mistake in the earlier version.

REFERENCES

- [1] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Dordrecht: Kluwer, 1992.
- [2] K. S. Thyagarajan, S. Parthasarathy, and H. Abut, "A matrix quantizer incorporating the human visual model," in *Proc. IEEE ICASSP*, 1985, pp. 141-144.
- [3] N. M. Nasrabadi and R. A. King, "A new image coding technique using transforms vector quantization," in *Proc. IEEE ICASSP*, 1984, pp. 29.9.1-29.9.4.
- [4] A. Makur, "Low rate image coding using vector quantization," Ph.D. dissertation, California Inst. of Tech., October 1989.
- [5] P. Cosman, K. Oehler, A. Heaton and R. M. Gray, "Tree-structured vector quantization with input weighted distortion measure," in *SPIE Proc. Visual Communication Image Proc.*, 1991, pp. 162-171.
- [6] A. Gersho and B. Ramamurthi, "Image coding using vector quantization," in *Proc. IEEE ICASSP*, 1982, pp. 428-431.
- [7] B. Ramamurthi and A. Gersho, "Image vector quantization with a perceptually-based cell classifier," in *Proc. IEEE ICASSP*, 1984, pp. 32.10.1-32.10.4.
- [8] B. Ramamurthi and A. Gersho, "Classified vector quantization of images," *IEEE Trans. Commun.*, vol. COM-34, pp. 1105-1115, Nov. 1986.
- [9] A. Z. Meiri and E. Yudilevich, "A pinned sine transform image coder," *IEEE Trans. Commun.*, vol. COM-29, pp. 1728-1735, 1981.

Comments on "Iterative Procedures for Reduction of Blocking Effects in Transform Image Coding"

Stanley J. Reeves and Steven L. Eddins

I. INTRODUCTION

In a recent paper, Rosenholtz and Zakhor [1] proposed an effective method for reducing blocking in transform coded images. Their method uses two projection operators and the theory of projection onto convex sets (POCS) to guarantee convergence of the iteration. One projection operator is defined from the known quantization levels used to code the transform coefficients. The other projection is based on the set of band-limited images with a given cutoff frequency. Unfortunately, the algorithm they actually implemented is only tenuously related to the theory of POCS. We offer a different basis for justifying their algorithm, one that provides an exact formal basis for establishing convergence and a more flexible theory for elucidating the possibilities of the algorithm.

II. NATURE OF THE ALGORITHM

As the authors indicated, a projection onto the set of band-limited images is equivalent to an ideal low-pass filter. However, the authors chose to approximate the ideal low-pass filter with

Manuscript received February 26, 1993; revised April 16, 1993. Paper was recommended by Associate Editor Ming-Ting Sun.

S. J. Reeves is with the Department of Electrical Engineering, Auburn University, Auburn, AL 36849-5201.

S. L. Eddins is with the Department of Electrical Engineering and Computer Science, University of Illinois, Chicago, IL 60607-7053.

IEEE Log Number 9209872.

the following convolution kernel:

0	0	0.0751	0	0
0	0	0.1239	0	0
0.0751	0.1239	0.2042	0.1239	0.0751
0	0	0.1239	0	0
0	0	0.0751	0	0

As Fig. 1 shows, this kernel is far from an ideal low-pass filter. If it were a reasonable approximation of a projection operation, repeated application of the filter would change the image only slightly or not at all. However, the difference between 5 and 20 iterations of the low-pass filter is dramatic, as shown by Fig. 4(c) and (d) in their paper.

POCS is clearly inadequate as a basis for justifying the algorithm with the convolution kernel proposed. Instead, the algorithm can be justified as an iterative solution of a constrained minimization problem. We can write the iteration proposed by Rosenholtz and Zakhor as

$$x_{i+1} = P_Q Lx_i \quad (1)$$

where P_Q is the projection operator onto the quantization set Q and L is the (nonideal) low-pass filter. We show that this iteration actually converges to the solution of a constrained minimization problem.

Consider the constrained minimization problem defined as follows:

$$\text{Minimize } x^T Hx \quad (2)$$

$$\text{subject to } x \in C, \quad (3)$$

where C is a convex set. This problem can be solved by a constrained steepest descent iteration of the form

$$x_{i+1} = P[x_i - \beta_i Hx_i] \quad (4)$$

for P a projection onto C and β_i chosen appropriately [2]. If we choose $P = P_Q$, $H = I - L$, and $\beta_i = 1$, then we obtain the following iteration:

$$x_{i+1} = P_Q[x_i + (L - I)x_i] \quad (5)$$

$$= P_Q Lx_i \quad (6)$$

This iteration can be shown to converge for any low-pass filter L whose magnitude response is less than or equal to one. The iteration is equivalent to the one proposed by Rosenholtz and Zakhor. Therefore, their algorithm actually solves the constrained minimization problem

$$\text{Minimize } x^T (I - L)x \quad (7)$$

$$\text{subject to } x \in Q. \quad (8)$$

This formulation of the algorithm shows that it minimizes frequency-weighted energy in the image while maintaining fidelity to the known quantization constraints. For a low-pass kernel, the algorithm tends to minimize high-frequency energy more than low-frequency energy. This formulation has the following advantages over POCS:

- **Guaranteed convergence:** This iteration is guaranteed to converge to a solution of the constrained minimization problem regardless of the cutoff frequency of L . (This explains why Rosenholtz and Zakhor always experienced convergence of their algorithm.) The POCS iteration, on