Micromechanics of fracture and failure in concrete under monotonic and fatigue loadings

K. Keerthana ^{a,1} and J. M. Chandra Kishen ^{a,*2}

^a*Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012,
India, Phone: +91-80-2293-3117*

Abstract

In this research, the fracture processes in concrete subjected to monotonic and fatigue loadings are characterized and the differences in failure mechanisms are studied using acoustic emission (AE) and digital image correlation (DIC) techniques. Experiments are performed on notched plain concrete beams under three-point bending. The micro and macro structural activities are classified based on acoustic energy levels to differentiate between the formation of a fracture process zone (FPZ) under monotonic and fatigue loadings. It is observed that a FPZ within its conventional definition is formed under monotonic loading. On the contrary, the AE results under fatigue loading indicate isolated and dispersed micro cracks up to 95% of fatigue life. A damage index based on AE energy is proposed for concrete subjected to fatigue loading which could be used in health monitoring of structures.

Key words: Concrete, acoustic emission, b-value, fracture, micro cracking

* Corresponding author.

Email address: chandrak@iisc.ac.in (J. M. Chandra Kishen).

¹ Research Scholar

² Professor

Nomenclature

AE	=	acoustic emission
a	=	crack length
a_0	=	notch length
aJ	=	atto Joules
CAL	=	constant amplitude loading
CMOD	=	crack mouth opening displacement
D	=	beam depth
D_m	=	damage variable
D_{micro}	=	micro damage
D_{macro}	=	macro damage
D_{total}	=	total damage
DIC	=	digital image correlation
E_1, E_2, E_3 and E_4	=	AE energy levels 1, 2, 3 and 4
E_{tot}	=	total AE energy
FCZ	=	fracture core zone
FPZ	=	fracture process zone
L_{FPZ}	=	length of fracture process zone
N	=	number of loading cycles
N_{AE}	=	number of AE events
N_f	=	number of fatigue failure cycles or fatigue life
N_{max}	=	maximum number of AE events
P_{max}	=	maximum load amplitude of fatigue loading
P_{min}	=	minimum load amplitude of fatigue loading
P_u	=	peak load of monotonic loading
ROI	=	region of interest
TEZ	=	total energy zone
VAL	=	variable amplitude loading
W_{FCZ}	=	width of fracture core zone
W_{FPZ}	=	width of fracture process zone

1 Introduction

Concrete being one of the most widely used construction material, has decades of research orbiting around it in a quest to comprehend its intricate fracture and failure mechanisms. Cogent arguments put forward by the scientific community has led to a broad consensus regarding the existence of an inelastic zone in the fracture front called the fracture process zone (FPZ). This zone encapsulates toughening mechanisms such as micro-cracking, aggregate bridging, crack branching and crack deflection which characterises the quasi-brittle nature of concrete. Based on the type of loading, the fundamental features of concrete fracture show different behaviour. Concrete members subjected to monotonically increasing displacement exhibits an extensive softening after attaining its peak load. On the contrary, three phase progressive damage followed by sudden failure is observed when subjected to fatigue loading. This conflicting fracture response of concrete under different loading conditions warrants further exploration of the formation of FPZ. This paper aims to describe the similarities and differences in the fracture process zone formed in plain concrete subjected to monotonic and fatigue loadings through an experimental investigation.

Numerous experimental methods have been used to determine the size of FPZ and its influence on the fracture behaviour of concrete. Castro et al. [8] performed experiments with laser holographic interferometry combined with digital image analysis. Their experiments have revealed that the FPZ is of constant size regardless of the length of crack and it translates along with crack extension. Maji et al. [19] assessed the FPZ with acoustic emission technique and concluded that it has a considerable width when compared to other specimen

dimensions. The acoustic emission (AE) technique has been used by several researchers [17, 24] as it visually enables us to understand the evolution of micro mechanisms in the interior of concrete. Landis [17] related the micro and macro fracture mechanisms in concrete through AE technique. The experiments conducted by Otsuka [23] on concrete under tension with the aid of AE technique and X-rays using a contrasting medium showed the existence of total energy zone (TEZ) which encapsulates all the micro and macro events. The FPZ and fracture core zone (FCZ) constitutes almost 95% and 70% of total energy zone, respectively. Furthermore, Otsuka also studied the influence of maximum aggregate size, specimen size and ligament size on the FPZ of concrete. The FPZ and FCZ formed in concrete is shown in Figure 1.

Fig. 1: Fracture process zone in concrete

Grassl et al. [10] analysed the size effect on the fracture process zone in notched and unnotched three point bending tests of concrete beams by a meso-scale approach. They concluded that for unnotched specimen the width of FPZ increases with specimen size and for notched specimen the width of FPZ does not depend on its size. Yu et al. [33] proposed a three-dimension realistic numerical modelling method to study the fracture mechanisms of concrete

at micro or meso scale under complex loading conditions. They used X-ray computerized tomography and improved digital image processing (DIP) for the three-dimensional modelling. Alam and Ahmed [2] experimentally investigated the effect of micro-macro interaction on softening behaviour of concrete fracture using digital image correlation (DIC) and AE techniques. Based on their experimental observations they proposed an improved cohesive crack model where a damping transient stress due to micro-macro interaction is added to the cohesive stress.

All the qualitative and quantitative studies on concrete fracture has confirmed the existence of FPZ and its influence on the fracture properties of concrete under monotonic loading through various experimental investigations. However, for concrete subjected to fatigue loading the formation of FPZ has not been confirmed and it is still an ongoing research problem. Fatigue is a phenomenon wherein structures such as bridges, pavements, airport runways are subjected to repetitive loads which causes progressive and permanent internal damage in the material leading to sudden failure. Due to variability in loading and the material heterogeneity, the damage quantification of concrete under fatigue loading poses a daunting challenge for researchers. Fatigue experiments on concrete have shown three stages of damage evolution [3, 7]. The first stage occurs due to the coalescence of internal micro cracks present in the pristine state of concrete. The second stage is a dominant phase illustrating a stable and gradually increasing state of damage due to the formation of micro cracks and this stage constitutes 90% of the total fatigue life. The third stage of damage shows an unstable state due to the coalescence of micro cracks to form a dominant macro crack. The theories of fracture and damage mechanics have been widely utilized to model the fatigue behaviour of concrete [3, 15, 21, 25].

The crack size during fatigue crack propagation in concrete is computed indirectly through the compliance method [30, 31] which is based on the concepts of linear elastic fracture mechanics (LEFM). The fatigue failure criterion is often established based on the concept of static envelope curve [14, 16]. As the experimental evidences depict a conflicting fracture behaviour in concrete under fatigue and monotonic loadings, the fracture process zone formation in concrete under fatigue has to differ from the one that forms under monotonic loading.

The main objective of this paper is to address different fracture mechanisms involved in the formation of FPZ in concrete subjected to flexural monotonic and fatigue loadings through an experimental investigation with the aid of acoustic emission (AE) and digital image correlation (DIC) techniques. Experiments are conducted on geometrically similar plain concrete notched beams of three different sizes. The specimens are tested under a simulated variable amplitude loading and constant amplitude loading. Surface displacements and crack lengths are computed using the DIC technique. The evolution of micro cracks in the interior of the specimen is obtained through AE technique. The work presented in this paper is structured as follows: The experimental program detailing the materials used for specimen preparation and the test set-up are described in Section 2. Comparison of mechanical results of monotonic and fatigue experiments are presented in Section 3. Damage evolution analysed through AE and DIC techniques are presented in Section 3. Characterisation of FPZ is presented in Section 4. The AE based b -value analysis for monotonic and fatigue load cases are discussed in Section 6. Damage index for concrete under fatigue is proposed based on the AE data and presented in Section 7. Finally, in Section 8, the summary and the main conclusions drawn

from this research are presented.

2 Experimental Program

2.1 *Materials and test specimen used*

The concrete specimens are prepared according to mix design procedure given in the Indian standard code of practice. Two mix designs are considered in order to compare the test results of concrete of varying strength. The first mix with cube compressive strength (f_{ck}) of 56 MPa, is used for preparation of specimens of three different sizes - small, medium and large and the second mix with cube compressive strength of 38 MPa is used for the preparation of only medium sized specimens. The mix proportion details and properties of various ingredients used in the preparation of concrete for both type of concrete mixes are summarized in Table 1. The small, medium and large sized beams are geometrically similar with uniform thickness (B) of 50 mm and varying depth (D) of 75, 150 and 300 mm respectively. All the beam specimens have span to depth ratio, $S/d = 4$ and relative notch to depth ratio, $a_0/d = 0.2$ with notch width (a_w) of 2 mm introduced at the time of casting.

2.2 *Experimental set-up and loading details*

The beam specimens are tested in a closed loop servo-controlled testing machine having a capacity of 35 kN. The crack mouth opening displacement (CMOD) is measured using a clip gauge mounted across the notch and load point vertical displacement is measured using linear variable differential trans-

Table 1: Details of concrete mix and material properties

Type of cement (C)	: OPC 53	
Type of fine aggregates (FA)	: River sand	
Size of FA	: Passing through sieve 4.75 mm	
Specific gravity of FA	: 2.67	
Type of coarse aggregates (CA)	: Crushed granite stones (angular)	
Maximum size of CA	: 12.5 mm	
Specific gravity of CA	: 2.74	
	Mix 1	Mix 2
water/cement ratio (w/c)	: 0.45	0.54
Mix proportion (C:FA:CA)	: 1 : 2.08 : 1.71	1 : 2.3 : 3.0
Poisson's ratio (ν)	: 0.12	0.12
Compressive strength (f_{ck})	: 56 MPa	38 MPa
Standard deviation of f_{ck}	: 4.96 MPa	9.06 MPa
Tensile strength (f_t)	: 3.6 MPa	2.6 MPa
Standard deviation of f_t	: 0.76 MPa	0.28 MPa
Modulus of Elasticity (E)	: 35305 MPa	33766 MPa
Standard deviation of E	: 155 MPa	185 MPa

Fig. 2: Test set-up and its schematic diagram

former (LVDT) mounted at the mid-span of the beam. The experimental set-up along with the schematic diagram of beam under three point bending are shown in Figures 2 (a) and 2 (b), respectively. The beam specimens pre-

Fig. 3: Variable amplitude loading

pared from both the mixes are initially tested under monotonically increasing CMOD with a rate of 0.001 mm/s to obtain the peak load and the post peak behaviour. Further, nine specimens from mix 1, three of each size are tested under variable amplitude loading (VAL). VAL is simulated in the form of increasing steps with sinusoidal waveform of 1 Hz frequency as shown in Figure 3. The minimum load amplitude is maintained at 0.2 kN to ensure a contact between loading device and the test specimen. The maximum load amplitude is increased in the steps of 0.25 kN for every 250 cycles starting with 0.25 kN. Five medium specimens made of mix 2 are tested under constant amplitude loading (CAL) with a sinusoidal waveform of 2 Hz frequency. The minimum and maximum load amplitude are 0.25 kN and 80% of static peak load respectively. The data of load, CMOD, load point displacement and time are acquired simultaneously through a data acquisition system.

Fig. 4: Acoustic emission system schematic diagram

2.3 Acoustic emission set-up

To understand the micro mechanisms, the mechanical tests are aided with acoustic emission (AE) set-up. The specimen under loaded conditions, generate micro and macro cracks which results in stress waves. These stress waves are captured through AE sensors and locate the position of the crack in the specimen. An AE signal received by a single AE sensor is termed as a *hit* and if the same signal is received by multiple sensors then it is counted as an *event*. For a three-dimensional location of an AE event, minimum of four AE sensors are required. In this work, four piezoelectric AE sensors for small size beam and six AE sensors for medium and large sized beams are mounted on the specimen to obtain a three-dimensional location of AE events. The AE data such as number of hits, events, counts, energy, spatial positions, amplitude and time are simultaneously acquired during the experiments using a data acquisition system. In order to filter the background noise, a threshold value of 35 dB is adopted. Piezoelectric AE sensors (R6D), which are resonant type differential sensor with an operating frequency varying from 35 to 100 kHz are used in the present work. To ensure perfect contact between the specimen surface and AE sensor, high vacuum silicon grease is used as a couplant. The

signals received from AE sensors are amplified with a gain of 40 dB using pre-amplifiers and sent to data acquisition system as shown schematically in Figure 4.

Fig. 5: Dimensions of small, medium and large sized beams and location of AE sensors on the beam surface . (All dimensions are in mm)

The location and distance between the AE sensors are decided based on the attenuation curve. Attenuation is defined as the loss of signal amplitude due to damping and geometry of the material. The location of AE sensors on the small, medium and large sized specimen are shown in Figure 5. The lead break test is performed before each experiment to calibrate the position of AE sensors.

2.4 Digital image correlation set-up

Digital image correlation is a non-contact and non-destructive optical measurement technique to measure the full-field surface displacement and strain of the deforming body. It requires the specimen surface to be artificially coated

by a random speckle pattern and consecutive digital images of the deforming specimen is captured. The surface displacements and strains are obtained by tracking the pixels of digital images of the specimen in un-deformed and deformed state respectively, through a correlation algorithm.

In this work, the beam specimens are painted white and a random speckle pattern is sprayed with black paint on the surface of the specimen as shown in Figure 2 (a). A camera with 12 Mega pixel resolution is used for taking digital images. The camera is mounted on a tripod stand and adjusted such that the axis of camera is oriented perpendicular to plane of the deforming specimen. For monotonic test, the images were captured at a regular interval of 10 s. For fatigue loading, video of the deforming specimen is taken which captures images at a rate of 25 frames per second. From the video, 25 images per cycle are taken for analysis at different interval of fatigue life of the specimen. For mix 1 specimens, DIC analysis is performed for monotonic load case and for mix 2 specimens DIC analysis is performed for both fatigue and monotonic load cases.

DIC analysis is performed using MATLAB image processing toolbox along with Ncorr which is an open source 2D program. The DIC algorithm used in this research is elaborated by Blaber et al. [5]. The region of interest (ROI) is manually selected in Ncorr. The dimension of ROI is approximately 90x75 mm, 100x150 mm and 100x300 mm for small, medium and large specimens respectively. The DIC parameters such as subset radius and subset spacing greatly affects the accuracy of the results obtained. Based on number of trials and comparing with experimentally obtained CMOD, the subset radius ≥ 30 pixels and subset spacing ≤ 8 pixels is chosen for the DIC analyses. Surface crack tip through DIC analysis is computed using recently proposed procedure

Fig. 6: Load-CMOD curves of mix 1 and mix 2 concrete beam specimens. (a) and (b) Monotonic loading, (c) Variable amplitude fatigue loading and (d) Constant amplitude fatigue loading

based on the horizontal displacement gradient by Bhowmik and Ray [4].

3 Comparison of monotonic and fatigue experimental results

The load-CMOD curves obtained from monotonic and fatigue tests are shown in Figures 6 (a)-(b) and (c)-(d) respectively. The specimens of mix 1 and 2 under monotonic loading show a typical post-peak softening behaviour. This is attributed to the formation of fracture process zone where various energy dissipative mechanisms occur. The average peak load along with standard deviation and fracture energy computed according to RILEM [1] procedure is shown in Table 2.

Table 2: Experimental results of monotonic loading

Beam specimen	Peak load (P_u)		Fracture energy (G_F)	
	Mean	σ_{std}	Mean	σ_{std}
	kN	kN	N/mm	N/mm
Small (Mix 1)	2.92	0.15	0.112	0.009
Medium (Mix 1)	4.89	0.59	0.115	0.002
Medium (Mix 2)	3.35	0.08	0.119	0.011
Large (Mix 1)	8.57	1.39	0.126	0.010

Table 3: Experimental results of fatigue loading

Specimen	Failure cycles	P_{max} at failure	Specimen	Failure cycles
(Mix 1)	N_f	kN	(Mix 2)	N_f
S1	2255	2.50 ($0.87P_u$)	M1	7223
S2	2562	2.75 ($0.94P_u$)	M2	5270
S3	2282	2.50 ($0.87P_u$)	M3	8089
M1	4482	4.50 ($0.92P_u$)	M4	356
M2	4007	4.25 ($0.87P_u$)	M5	42286
M3	4462	4.50 ($0.92P_u$)	-	-
L1	6071	6.25 ($0.73P_u$)	-	-
L2	7402	7.50 ($0.88P_u$)	-	-
L3	6270	6.50 ($0.76P_u$)	-	-

The load-CMOD curves of medium sized specimen of mix 1 subjected to variable amplitude loading and mix 2 specimen subjected to constant amplitude loading are shown in Figures 6 (c) and (d) respectively. These illustrate gradual decrease in the slope of hysteresis loop with increasing number of loading cycles, indicating degradation in the stiffness. The rate of stiffness degradation increases towards the failure due to coalescence of micro cracks into a macro crack. The fatigue failure cycles (N_f) along with the standard deviation (σ_{std}) obtained for nine specimens of mix 1, three of each size: small (S), medium (M) and large (L) and five medium specimens of mix 2 are shown in Table 3.

Fig. 7: Superposed monotonic and fatigue load-CMOD curves. (a), (b) and (c) Small, medium and large beam specimens of mix 1, respectively. (d), (e) and (f) Medium beam specimens of mix 2

In order to further investigate the fracture behaviour, the mechanical response under monotonic and fatigue loadings are compared.

The load-CMOD response of monotonic and fatigue loadings are superposed for specimens of mix 1 and mix 2 and are shown in Figures 7 (a)-(f). For mix 1 specimens, the maximum CMOD at fatigue failure is comparable with

the CMOD at peak load (P_u) of monotonic response. For mix 2, the monotonic load-CMOD response of three samples are superposed with fatigue load-CMOD response of one specimen as shown in Figures 7 (d)-(f). These figures indicate that CMOD at fatigue failure is slightly more than monotonic CMOD at peak load. However, the permanent deformation is observed close to monotonic peak load. To examine the repeatability of this trend, the CMOD values obtained at fatigue failure and monotonic peak load for all the mix 1 and mix 2 specimens are shown in Table 4. For all the specimens of mix 1, CMOD at fatigue failure and monotonic peak load falls in the same range. For mix 2 specimens, CMOD at P_{min} of second last fatigue failure cycle (N_{f-1}) falls between CMOD corresponding to monotonic peak load and 80% of peak load in the softening region. The difference in the trend observed between mix 1 and mix 2 specimens is attributed to difference in the type of fatigue loading (VAL and CAL), loading frequency, concrete strength and heterogeneity. These observations clearly indicate that fracture mechanisms are different under monotonic and fatigue loadings. However, from literature [14, 16] it is known that the failure criterion for concrete under fatigue is established based on the concept of static envelope curve. This criterion is briefly discussed here.

3.1 Fatigue failure criterion based on static envelope curve

Static envelope curve criterion states that monotonic or quasi-static response of the specimen provides a bound for deflection or strain values for the specimen subjected to fatigue loading. This implies that the failure strain or deflection under fatigue loading between two fixed stress levels depends only on maximum stress level (σ_{max}) as illustrated in Figure 8. Points O' and A correspond

Table 4: Comparison of CMOD values obtained from monotonic and fatigue tests

Mix	Specimen	CMOD values under monotonic			CMOD values under fatigue	
		P_u (mm)	$0.8P_u$ (mm)	At failure (mm)	At P_{min} of N_{f-1} (mm)	N_f (mm)
1	S1	0.038	0.054	1.1	0.036	0.037
	S2	0.020	0.024	0.5	0.041	0.045
	S3	0.036	0.039	0.89	0.030	0.039
	M1	0.035	0.048	0.87	0.049	0.054
	M2	0.032	0.032	0.83	0.031	0.034
	M3	-	-	-	0.044	0.047
	L1	0.055	0.07	1.07	0.062	0.085
	L2	-	-	-	0.056	0.063
	L3	-	-	-	0.049	0.065
2	M1	0.052	0.083	1.15	0.098	0.185
	M2	0.049	0.069	1.09	0.059	0.130
	M3	0.051	0.067	1.27	0.053	0.148

to the minimum and maximum load amplitude of fatigue loading respectively. Failure envelope criterion states that, fatigue failure will occur when ultimate deflection under fatigue reaches displacement δ_c at point C which corresponds to maximum fatigue load amplitude. In the experiments conducted, it is observed that for mix 1 specimens subjected to variable amplitude loading, the load-CMOD response is well within monotonic load-CMOD curve. In contrast, for mix 2 specimens subjected to constant amplitude loading, CMOD value at failure cycle corresponding to P_{max} falls outside the monotonic load-CMOD curve. This trend is clearly illustrated in Figures 7(a)-(f). Similar observations have been reported in the literature [13, 28, 29] for concrete subjected to different fatigue loading conditions.

In order to assess the static envelope curve as a fatigue failure criterion, it is

Fig. 8: Schematic representation of fatigue failure criterion based on static envelope curve

applied to medium specimen of mix 2 as shown in Figure 9. Point *B* corresponds to peak load of monotonic load-CMOD response. It is observed that CMOD corresponding to point *B* coincides with the second inflection point of fatigue life versus CMOD curve and as observed earlier point *B* falls close to the permanent deformation of fatigue loading. However, according to static envelop criterion, the failure should occur at deflection corresponding to point *C* but the actual failure takes place at point *D* which falls outside the static envelope curve. Based on these observations, deflection corresponding to peak load as the critical limit for fatigue failure may be more suitable as it coincides with second inflection point of fatigue curve. Furthermore, it is straightforward to obtain the CMOD or deflection values corresponding to static peak load compared to other points in the softening region. However, it is important to analyse the micro structural activities of the beam as it can exhibit a contradictory behaviour. For instance, scatter of acoustic emission (AE) events which indicate micro cracks are shown corresponding to points *B* and *D* (Figure 9) for monotonic and fatigue loadings. It is observed that at deflection corresponding to point *B* very few events have occurred under monotonic loading

Fig. 9: Application of fatigue failure criterion based on static envelope curve for medium specimen of mix 2

but they constitute high energy magnitude events indicated by bigger markers (red colour); whereas under fatigue loading comparatively more number of low energy AE events have occurred indicated by smaller (blue colour) markers. These contradicting micro mechanisms under static and fatigue loadings, indicate that the stiffness of concrete beams for the same deflection will be different. Thus, using static envelope curve as fatigue failure criterion which is based only on global mechanical response can be misleading. To further explore the underlying fracture mechanisms at micro and macro scale, digital image correlation (DIC) and acoustic emission (AE) techniques are used as elaborated in the following section.

4 Evolution of damage in concrete from acoustic emission and DIC

Initiation and propagation of micro cracks dissipate energy of different magnitude depending on their size and location. This dissipated energy is captured through AE sensors which give us information on internal micro mechanisms. The micro cracks formed in the specimen gradually coalesce to form a macro crack which propagates leading to failure. The propagation of macro crack is visualised through surface displacement plots obtained through DIC analysis. In this research, AE parameters such as absolute energy, event counts and localization maps are used to understand the intricate fracture mechanisms in concrete.

Fig. 10: Progression of AE events with different AE energy magnitude with time in concrete beam subjected to (a) monotonic loading and (b) fatigue loading.

4.1 AE events and absolute energy

AE absolute energy is recognized as an important parameter to characterise different levels of damage in the material [23, 26]. AE absolute energy is referred to as AE energy in the rest of this paper. The energy levels are divided

based on the percentage contribution of each energy level to the total cumulative AE energy. Four energy levels E_1, E_2, E_3 and E_4 which correspond to magnitude range $0 - 10^3 aJ$ (atto Joules), $10^3 - 10^4 aJ$, $10^4 - 10^5 aJ$ and $10^5 - 10^9 aJ$ respectively are defined. The evolution of micro cracks of varying energy magnitude for specimen subjected to monotonic loading is shown in Figure 10 (a). On the same figure, the variation of load with time is also shown. The size and colour of the markers are varied according to different energy levels defined. The AE events of all energy levels predominantly occur in the post peak region. Energy level (E_4) shown by bigger markers (red colour) are effectively generated in the post-peak region indicating the propagation of macro crack. The nucleation of micro cracks with varying energy magnitude, their coalescence to form macro crack and propagation of macro crack all occur simultaneously as illustrated in this figure. Under monotonic loading, close to failure, the density of the AE events decreases indicating that macro crack has propagated further towards the top surface of the beam and little volume of material is left for the formation of micro cracks.

Figure 10 (b) illustrates the variation of hysteresis energy and evolution of AE events with different energy magnitude with respect to time for beam specimen subjected to fatigue loading. The hysteresis energy dissipated in each load cycle is nearly constant for major portion of fatigue life and it increases precipitously just before the failure. This trend is very well complemented by the AE data. For a significant period of fatigue life, low energy AE events (E_1) are observed. After 75% of the fatigue life, increase in the density of micro cracks is observed but higher energy events (E_4) have still not occurred. About 90% of micro cracks are concentrated post 85-90% of the fatigue life. The high magnitude energy events (E_4), occurs only at final few cycles which may be

Fig. 11: Temporal variation of cumulative AE energy and relative crack length (a/D ; a =crack length, D =depth of beam) computed from DIC technique in concrete beam subjected to (a) monotonic loading and (b) fatigue loading attributed to unstable macro crack propagation close to failure.

4.2 Analysis of AE energy levels

Figure 11 (a) and (b) shows the evolution of cumulative AE energy of four levels as defined earlier, for monotonic and fatigue loadings respectively. For

monotonic loading, the variation of normalised load and relative crack length obtained from DIC is also shown in the same plot. Immediately after the specimen is loaded, a surge of AE energy of all the levels (E_1, E_2, E_3 and E_4) is observed. Relative crack length (a/D) computed from DIC has increased from its initial value 0.2 to 0.4 before the peak load. This immediate surge of AE energy and crack length is attributed to high stress concentration at the site of initial notch due to flaws and defects present in the pristine state of the material. In the post peak region, relative crack length, a/D has reached upto 0.7 corresponding to post-peak normalised load (P/P_u) of 0.8. Thereafter, both (a/D) and cumulative AE energy shows gradual increase until failure. All the four levels of AE energy show a similar trend. E_4 associated with higher energy closely correlates with the total cumulative AE energy curve. This indicates that maximum energy contribution is from energy level E_4 and it constitutes about 80% of total AE energy.

On the other hand, the evolution of AE energy shows different trend for fatigue loading. Initial surge of AE energy is observed even in the case of fatigue, but the magnitude is about 10^3 aJ for fatigue whereas for monotonic it is about 10^6 aJ. This indicates that the initial surge of AE energy under fatigue loading is only due to events with energy level E_1 . Further, it is observed that cumulative energies E_1 and E_2 closely follow the total energy curve while the contributions from energy levels E_3 and E_4 are observed only after 70% and 90% of fatigue life, respectively. This indicates that fracture mechanisms in fatigue are dominated by micro structural activities. Crack length computed from DIC is observed only after about 50% of fatigue life and thereafter it follows the trend of energy levels E_3 and E_4 . In fatigue, subcritical crack growth occurs as a result of progressive increase in the micro crack density.

Fig. 12: Comparison of percentage of AE events and AE energy in concrete beam subjected to monotonic and fatigue loadings.

The percentage of AE events in each energy level under monotonic and fatigue loadings are shown in the form of a bar plot in Figure 12 (a) and (b), respectively. In case of monotonic loading, the percentage of AE events with energy level E_1 is about 80% which is the highest when compared to other energy levels but it has least contribution of about 1% to the total AE energy. Energy level E_4 constitutes about 80% of total energy but it constitutes only 2% of the total number of events. This indicates that macro cracking is attributed to energy level E_4 which occurs as a consequence of nucleation and coalescence of micro cracks from lower energy levels. All the specimens tested under monotonic showed a similar behaviour. Under fatigue loading, AE events decrease with increasing energy levels similar to the case of monotonic. However, the percentage contribution of each energy level to total AE energy falls within a smaller range between 20 and 35%. This implies that all the energy levels, effectively contribute to the total fatigue damage. Energy levels E_1 and E_2 occur for dominant period of fatigue life indicating progressive damage.

Fig. 13: Temporal variation of normalised load (P/P_u), relative crack length (a/D) and AE event rate of different energy levels in concrete beam subjected to monotonic loading. (a) Energy level E_1 , (b) Energy level E_2 , (c) Energy level E_3 and (d) Energy level E_4

4.2.1 AE event rate of different energy levels

Figures 13 and 14 show the rate of change of AE events of different energy levels under monotonic and fatigue loadings, respectively. For monotonic case, all the energy levels show acceleration of AE events up to a 10% of normalised load in the post-peak region. Thereafter, near the tail portion of softening curve AE event rate decreases. This means that a lot of damage causing activity is continuously taking place which include microcracking in mortar and along the transition zone between aggregates and mortar until the load drops to about 10% of the peak value. At this point, the relative crack length has

Fig. 14: Temporal variation of relative crack length (a/D) and AE event rate of different energy levels in concrete beam subjected to fatigue loading. (a) Energy level E_1 , (b) Energy level E_2 , (c) Energy level E_3 and (d) Energy level E_4

almost reached 90% of the beam depth resulting in hardly any material volume for the FPZ to extend. Thereafter, the applied driving force is dissipated to dissociate cohesive and frictional interactions that may exist between the already cracked surface. Therefore, deceleration of AE events is observed in the tail end of the softening region.

Under fatigue loading, as seen in Figure 14 the AE event rate of energy levels, E_1 and E_2 are nearly constant till around half the fatigue life. Thereafter, rate gradually increases and becomes infinite close to failure. The event rate of energy level E_3 is zero for major part of fatigue life and it spikes at a relative crack length (a/D) of 0.4. This indicates that due to progressive micro

damage under fatigue the material can produce macro cracks with lower energy dissipation compared to that of monotonic loading. The event rate due to high energy level E_4 occurs only close to failure. The relative crack length (a/D) computed from DIC shows stable increase up to 60%, thereafter unstable crack propagation occurs. Unstable macro crack propagation produces higher energy events because 40% of beam depth is still intact and breaking of intact portion produces higher acoustic energy.

All the results presented on AE energy and event counts clearly indicate that the evolution of micro and macro processes are completely different for concrete subjected to monotonic and fatigue loadings. This implies that the fracture process zone formation (FPZ) will also be different in both the load cases. To further understand the FPZ formation, AE event localisation maps are analysed in the following section.

4.3 *AE events localisation*

In order to visualise the formation of FPZ, the scatter of micro cracks and macro crack propagation are depicted on the specimen dimension at various stages of fatigue and monotonic loadings for medium sized specimen. The scatter of AE events on the beam dimension indicate micro cracks and propagation of macro crack is observed from horizontal surface displacement contours on region of interest (ROI) of beam obtained from DIC analysis. Figure 15 shows the evolution of micro cracks and macro crack corresponding to different points (A,B,C,D,E) of the monotonic load-CMOD response. The following observations are made from this figure:

Fig. 15: Evolution of micro and macro cracks in medium sized concrete beam (mix 2) under monotonic loading. (a) Monotonic load-CMOD response; (b), (g)-(i) AE events scatter plots; (c)-(f) Corresponding surface displacement contours on region of interest (ROI) from DIC analysis

- From the initial point when load increases from zero to point B corresponding to peak load (P_u), only 0.6% of the total AE events have occurred. The surface displacement plot up to point B shows small increase in the crack

length near the notch tip. Due to heterogeneity of concrete, the crack tip in the interior of the specimen is expected to be less than the surface crack tip computed from DIC.

- Between points B and C on the load-CMOD plot, where C corresponds to 40% of peak load in the post peak region, the number of micro cracks have increased. This is complemented by visible increase in the macro crack on the surface displacement plot. This indicates that a macro crack has formed after the peak load through the coalescence of microcracks.
- From points C to D , where D corresponds to 14% of peak load, the density of micro cracks has substantially increased and macro crack has propagated up to 75% of the beam depth. This shows that the microcracking and their coalescence are taking place in the fracture process zone (FPZ) continuously.
- The width of FPZ increases from point D to E . High energy AE events are observed in the centre of the narrow band formed (denoted by bigger markers in red colour) indicating the propagation of a dominant macro crack. This region is also called as damage core zone.
- Under monotonic loading the micro cracking and toughening mechanisms leads to gradual increase in CMOD consequently propagating the macro crack. The fracture process zone (FPZ) consisting of toughening mechanisms, more predominantly the aggregate interlock mechanism is formed under monotonic loading. This causes redistribution of stresses by alleviating the effect of high stress at the vicinity of major crack and driving it to propagate further into the FPZ formed along the depth of the specimen.

Figure 16 shows the evolution of micro cracks and macro crack for every quarter of fatigue. The following observations are made from this figure:

- Until 50% of fatigue life only 4% of total AE events have occurred. The

Fig. 16: Evolution of micro and macro cracks at different stages of fatigue life (N_f) in medium sized beam under fatigue loading. (a)-(c), (g)-(i) Surface displacement contours from DIC analysis; (d)-(i), (j) AE events scatter plots

micro cracks are isolated and randomly scattered in the specimen.

- Between 50% and 75% of fatigue life small increase in the AE events is observed, but these events fall within the energy level E_1 and E_2 . Surface displacement plot shows only minimal changes near the notch tip.
- Between 75% of fatigue life and failure, substantial increase in the density of micro cracks is observed. Three plots of surface displacement from DIC analysis are shown corresponding to $0.98N_f$, N_{f-1} and N_f . Distinct macro crack propagation is observed at $0.98N_f$. Surface crack length is more prominent at second last failure cycle N_{f-1} and at failure cycle N_f . This indicates that the macro crack effectively propagates only after 95% of fatigue life. Therefore, only subcritical crack growth occurs for a dominant period of fatigue life.
- Even though the sudden failure under fatigue is associated with macro crack propagation, hardly any high energy events are observed in fatigue. As the fatigue failure occurs suddenly, the medium of material contact for AE signal to pass across the AE sensors located on either side of the propagating crack is instantly lost. Consequently high energy events are not captured.
- The magnitude of CMOD at fatigue failure is in the same range as the CMOD at peak load under monotonic (Figure 9). But the density of AE events (or internal micro cracks) at fatigue failure is much higher compared to the ones observed for the same monotonic CMOD. This indicates that CMOD under fatigue is not directly correlated with the formation of micro cracks.
- For the specimen loaded under fatigue, the FPZ predominantly consists of isolated and randomly distributed micro cracks formed over the depth of the specimen for major part of the fatigue life. The stresses are released at the location where the micro cracks are formed, but the coalescence of micro

cracks into a macro crack does not take place until close to failure cycle. This behaviour is attributed to the presence of randomly oriented micro cracks ahead of major crack tip which blunts the effect of high stresses, which in turn increases the energy required to cause failure.

These observations emphasise that the fundamental mechanisms of fracture are different for concrete subjected to monotonic and fatigue loadings. Under monotonic loading the FPZ is formed and its size increases along with the increase in crack length whereas under fatigue, the higher energy events which are responsible for macro crack propagation are not formed. Furthermore, only subcritical crack growth occurs which eventually propagates as a macro crack only after 95% of fatigue life. Therefore, numerical characterization of FPZ is very important to understand the various fracture processes in concrete and is discussed in the following section.

5 Characterization of FPZ size

Several approaches based on AE technique have been proposed in the literature to characterize the size of FPZ. Lertsrisakulrat et al. [18] and Watanabe et al. [32] proposed different simplified empirical relations to compute localized compressive fracture length. They considered the length of FPZ to be equal to the zone with local energy greater than 15% of total energy of the specimen or the region where the distribution of peak amplitude or when AE energy is greater than 30% of the total value. The concepts of probability and statistics [12] were also used to characterize the size of FPZ, wherein the damage zone is divided into two areas: the zone of confidence with high damage and a weak damage zone based on the distribution of either amplitude or number of

AE events. Saliba et al. [26] estimated the length of FPZ based on AE event density along the ligament length. Based on their experimental observation, the length of FPZ was determined as length of the segment from notch tip to the intersection of the histogram with the horizontal line located at 20% of maximum number of AE events. Furthermore, they obtained the width of FPZ and fracture core zone (FCZ) according to the method proposed by Otsuka and Date [23] wherein they considered AE events density along the length of the specimen (X-direction) and determined width of FPZ and FCZ as the horizontal line at 20% and 50% of maximum number of AE events (N_{max}) respectively. In this paper, the method followed by Otsuka and Date [23] and Saliba et al. [26] is used to quantify the size of FPZ.

Fig. 17: Length and width of FPZ from AE analysis of medium sized concrete beam (mix 2) subjected to monotonic loading

Fig. 18: FPZ formation in small and large sized concrete beam subjected to fatigue loading from AE analysis

Figure 17 shows the length and width of FPZ and distribution of microcracks along the thickness of the specimen for concrete subjected to monotonic loading. AE events with higher energy level E_1 fall within the width of fracture core zone (FCZ) and all the major and minor AE events which contribute to the fracture of concrete surface fall within the width and length of FPZ. The higher energy AE events are randomly scattered along the thickness of the specimen indicating undulations in the fracture surface along the thickness of the specimen. Similar fracture characteristics is observed in all the specimens subjected to monotonic loading. The length of FPZ (L_{FPZ}), width of FCZ (W_{FCZ}) and FPZ (W_{FPZ}) computed for all the specimens subjected to monotonic loading are shown in Table 5. It is observed that the length and width of FPZ increases with the size of specimen. Furthermore, the length of FPZ for different sizes of specimen is comparable with its corresponding ligament length. Another important observation is that when the length of FPZ is normalised with respect to depth (D) of the specimen, a constant value of 0.78 is obtained for all the specimens irrespective of different sizes and mixes. Hence, the parameter $\frac{L_{FPZ}}{D}$ can be taken as the material parameter.

Table 5: Length and width of FPZ for concrete under monotonic loading

Specimen	Width of FPZ W_{FPZ} mm	Width of FCZ W_{FCZ} mm	Length of FPZ L_{FPZ} mm	Ligament $l_d = D - a_0$ mm	$\frac{L_{FPZ}}{depth(D)}$
S1 (Mix1)	34.0	5.0	58.0	60	0.77
S2 (Mix1)	37.3	17.0	59.4	60	0.79
S3 (Mix1)	42.5	31.0	59.0	60	0.78
Average	37.9	17.7	58.8	60	0.78
M1 (Mix1)	46.0	25.0	117.5	120	0.78
M2 (Mix1)	73.7	22.0	117.8	120	0.79
M3 (Mix1)	92.0	38.0	117.3	120	0.78
Average	70.6	28.3	117.5	120	0.78
L1 (Mix1)	160.5	68.0	232.5	240	0.78
L2 (Mix1)	174.0	98.5	215.4	240	0.72
L3 (Mix1)	166.0	102.2	220.6	240	0.74
Average	166.8	89.6	222.8	240	0.75
M1 (Mix2)	90.0	51.0	115.5	120	0.77
M2 (Mix2)	92.0	38.5	114.4	120	0.76
M3 (Mix2)	111.0	43.0	117.0	120	0.78
Average	98.0	44.2	115.6	120	0.77

For concrete under fatigue loading, the distribution of microcracks on the specimen geometry is shown in Figure 18 for small and large beam specimen. AE events formed are more scattered and AE events with higher energy which defines FCZ has not occurred in the specimen. These observations illustrate that the method used for characterizing the size of FPZ through AE results for monotonic loading cannot be applied for concrete under fatigue loading.

All the results shown so far is for one specimen each under monotonic and fatigue loadings. A similar trend is observed for all the specimens tested and distribution of AE events on the specimen geometry for mixes 1 and 2 subjected to monotonic and fatigue loadings are shown in Figures 19 and 20

Fig. 19: Scatter of AE events on concrete beam specimens of mix 1. (a), (c) and (e) respectively correspond to small, medium and large specimens subjected to monotonic loading. (b), (d) and (f) respectively correspond to small, medium and large specimens subjected to fatigue loading

respectively.

Fig. 20: Scatter of AE events on medium sized beam specimens of mix 2. (a), (c) and (e) corresponds to three different beam specimens subjected to monotonic loading. (b), (d) and (f) corresponds to three different beam specimens subjected to fatigue loading

5.1 Does FPZ exist in concrete under fatigue?

One of the primary motives of this experimental investigation is to explore the fundamental question of how FPZ formed in concrete under fatigue differs from that of monotonic loading. The conventional definition of FPZ which

states that it is a cohesive zone formed ahead of crack tip wherein redistribution of stresses occur due to toughening mechanisms which characterizes the quasi brittle nature of concrete. Our experiments have provided reliable evidences that FPZ within the conventional definition is not formed in concrete under fatigue. Figures 19 and 20 show the AE events formed until failure for specimens of mix 1 and mix 2 respectively. It is clearly observed that a FPZ is formed in all the specimens subjected to monotonic loading irrespective of type of mix. According to the findings of Otsuka [23], the entire damage process of concrete, consisting of micro and macro cracks occurs within a total energy zone (TEZ). FPZ and fracture core zone (FCZ) constitutes almost 95% and 70% of total energy zone, respectively. The same characteristics are observed in the specimens subjected to monotonic loading in this study. A narrow band of micro cracks is formed along the central notch of the specimen and the FCZ indicated by bigger markers (red colour) constituting high energy events are clearly illustrated in these figures. On the contrary, for all specimens under fatigue loading, the micro cracks are isolated and scattered randomly along the depth of specimen. The fracture core zone (FCZ) along which the macro crack propagates is not developed under fatigue. Hence, using the same terminology as FPZ without explicitly mentioning the above observations for fatigue damage can be misleading. Under fatigue, the formation of isolated and random scatter of micro cracks gradually reduces the effective stiffness of the material. Hence, a FPZ in its traditional definition, is not formed in concrete under fatigue loading under load control and FPZ is developed only in concrete subjected to monotonic loading tested under displacement control.

Alternatively, the fracture process in concrete can be understood using AE data through b -value analysis. This provides useful information on damage

evolution process that can strengthen and supplement the current arguments on concrete fracture behaviour under monotonic and fatigue loadings as discussed in the following section.

6 AE based b-value analysis

A close analogy exists between AE data produced from the fracture processes occurring in the brittle material and the seismic waves caused during the earthquakes. In seismology, seismic signals emitted due to earthquakes are related to the cumulative number of seismic events through the Gutenberg-Richter (GR) relationship which is expressed as [11]:

$$\log N_c = a - bM_L \quad (1)$$

where N_c is the number of seismic events, M_L is the Richter magnitude of events, ' a ' and ' b ' are empirical constants. The empirical constant ' b ' known as b -value, is defined as the log-linear slope of the frequency-magnitude distribution of seismic events. In other words, b -value represents the relative proportion of large to small events which occur during earthquakes. The concept of b -value is applied in AE technique to study the scaling of the amplitude distribution of the acoustic emission waves generated during the cracking process in engineering structures. The Gutenberg-Richter (GR) formula is modified for the use of b -value in AE technique as

$$\log_{10}(N_{AE}) = a - b(A_{dB}/20) \quad (2)$$

where, N_{AE} is the number of AE hits or events with amplitude greater than the threshold amplitude A_t , a is an empirical constant, b is the b -value and A_{dB} is

the peak amplitude of the AE event in decibels. The factor $1/20$ is multiplied in the Equation 2, so that the b -value obtained through this equation is equivalent to the one used in seismology since the AE peak amplitude is measured in decibels (dB) and the Richter magnitude of earthquake is defined in terms of the logarithm of maximum amplitude.

There are two ways in which b -value can be computed: (i) instantaneous b -value and (ii) long term b -value. In the former case, the b -value is computed for every time window or AE event window, i.e. the length of the window is fixed. For example, if a window of 100 AE events is considered, for every 100 AE events the b -value is computed. In the latter case, b -value at i^{th} window is calculated by considering $100i$ number of AE events, where i is the number of windows. In this paper, long term b -value is calculated in order to capture the cumulative effect. For monotonic load case, window of a 200 AE events is considered. In the case of fatigue, since the number of events generated are sparse in the dominant period of fatigue life, the AE events from initial point to 25%, 50%, 75%, 90% and 100% of fatigue life are considered.

In general, during a fracture process, it is observed that the AE events of large magnitude occur less frequently than the AE events of small magnitude. As b -value is the log-linear slope of frequency magnitude distribution, it gives significant information on the damage evolution process. b -value for concrete has been reported to range between 0.5 and 1.5. A higher b -value arises due to a large number of small magnitude AE events representing large number of micro crack formation and slow macro crack growth, whereas lower b -value indicates faster or unstable macro crack growth accompanied by relatively high amplitude of AE events in large numbers.

Fig. 21: Temporal variation of b -value for medium size concrete beam specimen

As our main focus is to differentiate between the fracture process of concrete under fatigue and monotonic loadings, the evolution of b -value for both the load cases are shown in Figure 21. Figure 21 (a) shows the temporal variation of b -value and load for concrete subjected to monotonic loading. b -value is higher than unity in the initial stage of softening. Higher b -value indicates nucleation of micro cracks. Reduction in b -value is observed oscillating between 0.90 and 0.75 which indicates that nucleation, coalescence and propagation of macro crack have all occurred simultaneously forming a damage localized zone and close to failure the b -value is nearly constant at 0.88. Carpinteri et al. [6] have demonstrated that b -values are always greater than unity ranging between 1.5 and 2.0 at beginning stages of loading due to randomly distributed micro cracks either in the whole or part of a structure. Close to final collapse, b -value approaches unity as the damage process localizes forming a narrow band near the final fracture surface. Further, they substantiated that decreasing b -value indicates both increase in damage as well as the increase in crack size. This argument is well complemented by the b -values computed from monotonic tests. On the other hand, different trend is observed in the b -value calculated for fatigue loading as shown in Figure 21 (b). The figure shows the

evolution of b -value up to every quarter and 90% of fatigue life along with the maximum CMOD in each cycle with normalised time. The b -value increases from 1.40 to 1.82 until 90% of fatigue life and at final failure it drops to 1.68 due to a few high magnitude AE events. As the entire range of b -value under fatigue is between 1.4 and 1.9, it confirms that the damage localization or FPZ does not occur in the specimen and micro cracks are isolated and randomly distributed which ultimately leads to failure in the final cycle of loading. A newly defined damage index is proposed based on the AE data and is discussed in the following section.

7 Damage index based on AE data

Acoustic emission provides high sensitivity real-time health monitoring and is widely employed in concrete bridges for condition assessment and damage characterization. Under fatigue loading conditions, concrete structures are prone to cracking at early stages of service life which cannot be characterized from the global response. In such situations, the ability of AE technique in early detection of cracks is useful for condition assessment of structures and prevent possible catastrophic failure in future.

There have been several investigations to quantify damage in concrete by correlating AE features with fracture mechanics theory. Dunegan et al. [9] assumed that the total AE count is proportional to the volume of plastic zone at the crack tip. Based on this assumption, they proposed a theoretical model relating total number of AE counts (N_{AE}) from a cracked specimen with the fourth power of stress intensity factor (K), given as

$$N_{AE} \propto K^4. \quad (3)$$

The effect of loading variables on acoustic emission from fatigue crack propagation was studied experimentally by Morton et al. [20]. They related total cumulative AE count and stress intensity factor range (ΔK) through an analogous Paris law type equation as

$$\frac{dN_{AE}}{dN} = A(\Delta K)^n \quad (4)$$

where, A and n are constants determined from experiments. While several researchers [22, 27] have used similar models validated with laboratory experiments, Zarate et al. [34] argued that the stress intensity factor range (ΔK) which is a function crack length, structural geometry and loading conditions, is difficult compute for operational bridges as most of them are composed of complex geometries. As a solution to this problem Zarate et al. [34] proposed a methodology that does not require knowledge of initial crack length or load amplitude. Their methodology was developed using Bayesian inference to account for uncertainties in AE features and fracture mechanics parameters.

In this paper, damage index is defined for concrete subjected to fatigue loading by considering only AE absolute energy. As loading and geometrical properties are difficult to estimate in the operational bridges, the damage index proposed in this paper can be used as preliminary indicator for condition assessment of concrete structures. Damage at any time, $D_m(t)$ is defined as the ratio of cumulative AE energy dissipated up to time t and total cumulative energy (E_{tot}) up to failure given by

Fig. 22: Temporal variation of hysteresis energy and different AE energy levels under fatigue loading. (a) Energy level E_1 , (b) Energy level E_2 , (c) Energy level E_3 , (d) Energy level E_4 , (e) Energy levels ($E_3 + E_4$) and (f) Total energy E_{tot}

$$D_m(t) = \frac{E(t)}{E_{tot}}. \quad (5)$$

As the damage variable is based on AE energy, the variation of different energy levels (E_1 , E_2 , E_3 and E_4) along with hysteresis energy is shown in Figures 22 (a)-(d). Low energy levels E_1 and E_2 show similar trend. Up to about half

the fatigue life (2000 s), E_1, E_2 and hysteresis energy curve show negligible variation, thereafter E_1 and E_2 curves increase at an accelerated rate but hysteresis energy curve shows substantial increase only close to the failure. Variation of E_3 and E_4 closely follow the trend of hysteresis energy. Hysteresis energy dissipation is a global mechanical response of the material influenced by macro damage. As E_1 and E_2 indicate progression of micro damage they do not follow the global trend. In Figure 22 (e), the cumulative energy of events in E_3 and E_4 are added and its variation with time is shown. A close correlation is observed with hysteresis energy and this indicates that E_3 and E_4 represent damage due to macro cracks. Based on these observations, damage index is divided into micro damage and macro damage as

$$D_{micro} = \frac{E_{micro}}{E_{tot}} \quad (6)$$

$$D_{macro} = \frac{E_{macro}}{E_{tot}} \quad (7)$$

where E_{micro} is the energy contribution from E_1 and E_2 and E_{macro} is the energy contribution from E_3 and E_4 . All the damage indices: total, micro and macro are shown in Figure 23 along with the hysteresis energy. It is observed that at failure, $D_{total} = 1$, $D_{micro} = 0.45$ and $D_{macro} = 0.55$.

Based on these observations it could be perceived that a close analogy exists between population growth model and the rate of micro-cracking under fatigue loading. According to population growth model, population (P) grows at a rate proportional to its size at any given time (t) and is given by

$$\frac{dP}{dt} = kP, \quad (8)$$

where k is a proportionality constant. The solution to the above equation is

Fig. 23: Evolution of hysteresis energy, micro, macro and total damage indices with time for concrete beam subjected to fatigue loading

given by

$$P(t) = ce^{kt} \quad (9)$$

where, $c = P(t = 0)$. This implies that the unconstrained natural population growth follows an exponential curve. Similar to population growth model, the micro cracks under fatigue loading nucleate exponentially until they coalesce to form macro crack which propagate in an unstable manner leading to failure. Damage (D_m) incurred by concrete at time (t) due to micro cracking is expressed by as

$$D_m(t) = ce^{kt}. \quad (10)$$

Utilizing this analogy, an exponential curve is fitted to the micro damage index D_{micro} and same fitting parameters are used to fit total damage index D_{total} as shown in Figure 24. It is observed that exponential curve fit is well suited to model progressive micro damage under fatigue loading. For the total damage index, the exponential fit correlates well until damage index $D = 0.3$. Therefore, the total damage index can be used for safety assessment of structures with a critical value of 0.3 beyond which unstable crack propagation occurs. This model has potential application in structural health monitoring

of infrastructures.

Fig. 24: Micro damage index and total damage index along with exponential curve fit for concrete beam subjected to fatigue loading

8 Summary and Conclusions

In this research, fracture and failure mechanisms of concrete under monotonic and fatigue loadings are experimentally investigated. Concrete beam specimens are tested under three point bending with the aid of digital image correlation (DIC) and acoustic emission (AE) techniques. The important conclusions drawn from this research are:

- (1) The mechanical, AE and DIC results have shown that the fundamental mechanics of fracture is different for concrete subjected to monotonic and fatigue loadings. The fatigue load-CMOD curve either exceeds or falls within the monotonic load-CMOD curve. This inconsistency is due to the combined effect of loading amplitude, strength, loading frequency and heterogeneity of concrete. From these observations it may be concluded that fatigue failure criterion based on static envelope curve is not suitable

for concrete.

- (2) Under monotonic loading, a gradual propagation of macro crack is observed in the post peak region. A narrow band of micro cracks is formed indicating the formation of fracture process zone (FPZ) and fracture core zone (FCZ). The length of FPZ when normalised with respect to specimen depth results in a constant value of 0.78 for all the specimens irrespective of different sizes and mixes.
- (3) Under fatigue loading, fracture process zone within its conventional definition is not formed. The micro cracks formed under fatigue are isolated and randomly scattered along the specimen dimension and consequently redistribution of stresses does not occur. The micro cracks formed in the specimen over its fatigue life continuously degrades the stiffness of the material until a critical stiffness is reached beyond which failure occurs suddenly. Surface displacements from DIC analysis indicate that macro-crack under fatigue effectively propagates only beyond 95% of fatigue life.
- (4) The evolution of b -value over entire fatigue life of concrete specimen ranges between 1.5 and 2.0 which confirms that under fatigue only distributed micro cracking occurs without the formation of a traditional FPZ for a majority of fatigue life.
- (5) A damage index based on AE energy is proposed for concrete under fatigue loading. The damage index is divided into micro and macro damage indices based on the magnitude of AE energy. It is shown that micro damage follows an exponential growth. A critical value for total damage index of 0.3 is proposed beyond which unstable crack growth occurs.

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