1. Dynamics of helical nanobots under rotating magnetic fields

A helical nanorobot actuated under rotating magnetic field shows different kind of dynamics depending upon the applied field, frequency and the fluid viscosity. This difference in dynamics is intrinsically related to the drag experienced by the nanorobot during motion. The different dynamics are distinguished by two cut off frequencies: $\Omega_1$ and $\Omega_2$ which represent a change in the dynamics of the nanorobot. The existence of these critical frequencies arise from the steady state configuration of the helix subjected to a rotating magnetic field of frequency $\Omega_B$. The magnetic torque on the helix due to the rotating magnetic field is denoted by $\tau$. The dynamics of the helix can be written as: $\tau = \gamma \omega$ where $\gamma$ is the rotational friction tensor and $\omega$ is the angular velocity vector. The torque is related to the magnetic moment $m$ and $B$ as: $\tau = m \times B$.

The magnetic field in the body frame ($x' y' z'$) of a helix is related to a magnetic field rotating in the lab frame by the following equation: 

$$
\begin{bmatrix}
B_{x'} \\
B_{y'} \\
B_{z'}
\end{bmatrix} = R \times \begin{bmatrix}
B \cos(\Omega_B t) \\
B \sin(\Omega_B t) \\
0
\end{bmatrix},
$$

where $R$ is the transformation matrix and $t$ is the time elapsed. The body frame magnetic field may be used to derive the body frame torque:

$$
\begin{bmatrix}
\tau_{x'} \\
\tau_{y'} \\
\tau_{z'}
\end{bmatrix} = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
m \cos \theta m & 0 & m \sin \theta m \\
B_{x'} & B_{y'} & B_{z'}
\end{bmatrix},
$$

where $m$ is the magnetic moment projected along the long axis and short axis.

Using standard notations to represent the Euler angles to describe the generalized orientation of a symmetric elongated object we can obtain the angular velocities in the body frame which are:

$$
\omega_{x'} = \dot{\phi} \sin \theta \sin \psi + \dot{\theta} \cos \psi, \omega_{y'} = \dot{\phi} \sin \theta \cos \psi - \dot{\theta} \sin \psi, \omega_{z'} = \dot{\phi} \cos \theta + \dot{\psi}.
$$
Equating the two expressions for the torque and solving for $\dot{\phi}, \dot{\psi}, \dot{\theta}$, we get the Euler equations with $\beta = \Omega_B t - \phi$:

$$\dot{\phi} = \frac{mB}{\gamma_s \sin \theta} (\sin \theta_m \cos \beta + \cos \theta_m \sin \beta \sin \theta \cos \phi)$$

$$\dot{\theta} = -\frac{mB}{\gamma_s} \sin \beta (\sin \theta_m \cos \theta + \cos \theta_m \sin \theta \sin \psi)$$

$$\dot{\psi} = \frac{mB \cos \theta_m (\sin \beta \cos \theta \cos \psi - \cos \beta \sin \psi)}{\gamma_l} - \dot{\phi} \cos \theta$$

The above equations can be solved for the steady state configurations where $\theta$ and $\psi$ remain constant in time. This leads to two different dynamical configurations for an object rotated by an external torque namely ‘tumbling’ and ‘precession’. Tumbling motion means a precession angle $\theta = 90^\circ$. This occurs for all frequencies below $\Omega_1$ denoted by $mB/\gamma_s$. At very low actuating frequencies ($\Omega_B < \Omega_1$), the magnetic moment of the nanorobot can follow the applied magnetic field with a constant phase difference and hence a phase locked tumbling motion of the nanorobot is observed, i.e., the nanorobot shows rotation about its geometric short axis. Above $\Omega_1$, the nanorobot starts to precess about the axis of rotating field. This happens because beyond this frequency the angle between $m$ and $B$ becomes more the $90^\circ$ and the moment can no longer follow the magnetic field, thus causing phase slip. Precessional motion ($\theta < 90^\circ$) is a solution to the Euler angles and the object can show precessional phase locked motion for $\Omega_B > \Omega_1$. Above the critical frequency $\Omega_2$, the magnetic moment of the helix starts to phase slip with the magnetic field. The frequency range ($\Omega_1$ to $\Omega_2$) can be adjusted as required using applied magnetic field.

2. Choice of ferromagnetic material
A proper choice of magnetic material is extremely important for our studies as a stable magnetic moment is expected for these measurements which may not be true for materials having low coercive field. In such cases, we see both phase slip tumbling and propelling dynamics at same field and frequencies. For example, the nanorobot which was coated with Nickel and showed tumbling and propelling motion at same field and frequency (45 G, 35 Hz) depending upon initial orientation as presented in Figure S1. For our experiments, we mostly coat the helices with Iron-Cobalt which has higher coercivity and saturation magnetization than the pure components.