Perturb and Observe with Momentum Term applied to Current Referenced Boost Converter for PV Interface

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Abstract—The Photovoltaic (PV) current follows the same direction as the perturbed duty cycle for a DC-DC converter interfaced PV system. Hence current referenced control is well suited for Maximum Power Point Tracking (MPPT). A small signal model for a Boost converter with PV is developed, which gives the input current to duty cycle control signal transfer function in order to design the controller. In addition to the inner current control loop, an MPPT control loop is added to track the Maximum Power Point (MPP). The standard Perturb and Observe (P&O) Algorithm has an inherent trade-off between dynamic performance during rapidly changing irradiation conditions and oscillations about the MPP. To overcome this problem a Momentum term is introduced in the perturbation of the reference control signal in order to accelerate tracking and to reduce oscillations. The Momentum term which is a scaled version of the previous perturbation can be easily implemented in the existing P&O algorithms, with only slight modification and almost no change in computational effort.

Index Terms—Boost Converter, Photovoltaics (PV), Maximum Power Point Tracking (MPPT), Perturb and Observe (P&O), Small Signal Analysis, Momentum Term, Current Controller

I. INTRODUCTION

Standard PV modules of around 300W with the same size have a maximum power point voltage of around 35 – 60V based on the technology of the solar cell. It is required to boost this voltage to above 650V in three phase grid connected PV systems. A simple calculation can show that a minimum of 15 modules are required in series, (i.e. an array of 5kW) to get to the desired voltage level which can be an expensive proposition. Hence, a Boost Converter is necessary in such cases. Another advantage of having an intermediate Boost converter is that it decouples the MPPT control with the grid control by providing a fixed DC voltage at the DC side of the grid tied inverter [1]. Even in standalone systems where PV is to be interfaced with energy storage the Boost Converter is widely used. Hence, this topology of the DC-DC converter family has been looked into, in this study.

II. SMALL SIGNAL MODEL

The small signal model requires the characteristics of the PV source to be included [2], which is given by the dynamic resistance of the I-V characteristics as shown in (1).

$$r_{pv} = \frac{dv}{di} \approx \frac{\tilde{v}_{pv}}{\tilde{i}_{pv}} \quad (1)$$

This dynamic resistance is negative in value as can be found from the I-V characteristics of the PV source shown in Fig. 1. The I-V characteristics can be broken down into four portions as in [3]: current source region, power source region 1, power source region 2 and voltage source region based on the dynamic resistance values in these regions.

Fig. 1. Linear Fitting of Photovoltaic I-V Characteristics

The state space equation is given by (2), where V_{bat} is the battery voltage at the output of the Boost Converter, V_F is the forward voltage of the diode, R_L is the series resistance with the inductor (L) and C is the input side capacitance.

$$\begin{bmatrix} \frac{d\hat{i}_L}{dt} \\ \frac{d\hat{v}_{pv}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_L}{L} & \frac{1}{L} \\ \frac{1}{r_{pv}C} & \frac{1}{C} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_{pv} \end{bmatrix} + \begin{bmatrix} \frac{V_F + V_{bat}}{L} \\ 0 \end{bmatrix} \quad (2)$$

The input current to duty cycle transfer function was found to be as shown in (3):

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$$\frac{\tilde{i}_L}{\tilde{d}} = \frac{\left(\frac{V_{bat}+V_F}{R_L-r_{pv}}\right) \left(1 + \frac{s}{\omega_z}\right) \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \quad (3)$$

$$\omega_i = \sqrt{\frac{-r_{pv} + R_L}{-r_{pv}LC}} \quad (4)$$

$$\zeta_i = \frac{r_{pv}R_L C - L}{2\omega_i r_{pv}LC} \quad (5)$$

$$\omega_z = \frac{1}{-r_{pv}C} \quad (6)$$

Table 1 shows the values of the system components and specifications. The open loop parameters of the small signal model are shown in Table 2. The parameters vary in the different zones which have been defined.

TABLE I
SPECIFICATIONS

System Specifications	
PV panel	250 W _p
	V _{oc} = 43.53V
	I _{sc} = 7.78A
	FF=0.738
C _{pv}	500μF
L	1mH
R _{L}	0.12 Ω
V _{bat}	72V
V _{F}	0.6V

III. PI CONTROLLER

A standard PI controller is utilized in order to have zero steady state error to step input in current and the required phase margin. The circuit implementation of the system with the PI controller has been shown in Fig. 2. The PI controller is designed using operational amplifiers and control output is given as duty cycle to the boost converter. The continuous conduction mode averaged switch model is used to improve speed of simulation.

The transfer function of the PI controller is shown in (7). The step response of the close loop system is shown in Fig. 3.

$$C(s) = 0.0806 + \frac{500}{s} \quad (7)$$

The current source region sees very slow rise to the steady state value because current can't rise above the short circuit current in this region. Other regions see a settling time of less than 5ms. This sets the perturbation period for the Perturb and observe algorithm [4].

Fig. 2. Schematic of Boost Converter with PI Current Controller

Fig. 3. Close Loop Step Response with PI controller

Fig. 4. Tracking of Reference Current and Input Current towards MPP for PI controller

TABLE II
OPEN LOOP PARAMETERS

Operating Point	$r_{pv}(\Omega)$	DC Gain (dB)	$\omega_i(rad/sec)$	ζ_i	$\omega_z(rad/sec)$
Current Source Region	-1737	-27.53	1.414×10^3	0.0428	1.151
Power Source Region 1	-12.65	15.07	1.421×10^3	0.0979	158.1
Power Source Region 2	-2.487	28.85	1.448×10^3	0.3191	804.2
Voltage Source Region	-0.841	37.50	1.511×10^3	0.8262	2.378×10^3

IV. 2-DOF CONTROL

Since the plant parameters are varying, 2 Degree of Freedom (2-DOF) control is utilized in order to minimize the effect of variations in plant parameters. In 2-DOF control one possible approach to minimize the variation of the closed-loop pole due to plant variation is to place an open-loop zero close to the desired closed-loop pole location and increase the open-loop gain to as high a value as possible. A pre-filter is then added to cancel the effect of the close loop zero which appears due to the addition of the zero in the controller. It is required to place dominant close loop poles at $-1000 \pm j1000$. The controller transfer function $C(s)$ is found out to be as

$$C(s) = \frac{4.156 \times 10^6 (s^2 + 2002.64s + 1998065)}{(s + 25000)^2} \quad (8)$$

The pre-filter $F(s)$ is designed as

$$F(s) = \frac{1998065}{(s^2 + 2002.64s + 1998065)} \quad (9)$$

The close loop response to variations in plant parameters is shown in Fig. 5. It can be seen that the step response in all the regions is almost invariant to the plant parameters.

Fig. 5. Close Loop Step Response with 2-DOF Controller

V. PERTURB AND OBSERVE WITH MOMENTUM TERM

In addition to the current referenced control loop, an outer MPPT loop has been designed to track the maximum power point. A wide number of MPPT techniques are available such as fractional open circuit voltage [5], fractional short circuit

Fig. 6. Tracking of Reference Current and Input Current towards MPP for 2-DOF controller

current [5], perturb and observe (P&O) [4], incremental conductance [6], hill climbing [7]. In this study the P&O algorithm is considered because according to [8] it is a true MPPT which is not panel dependant, implementation complexity is low, periodic tuning is not required and can be implemented in both analog and digital domains.

In P&O algorithm the voltage or current is perturbed and the corresponding change in power due to the perturbation decides in what direction the next step in voltage or current should be in. There are various modifications proposed to the P&O algorithm to prevent loss of tracking and drift during rapidly changing irradiance conditions and to solve the trade-off between tracking response and oscillations.

Most of the Modified P&O algorithms include a dynamic perturbation length which is based on the rate of change of power with voltage or current [9] [10]. Others include Artificial Neural Network and Genetic Algorithm based approaches [11] [12]. In our study, we consider a Momentum term that can be introduced in the perturbation similar to that used in gradient descent learning based back propagation algorithms for updating the weights of neural networks [13].

The idea for using this momentum term was introduced in [14] and it was shown that the addition of this term accelerates tracking when far away from the MPP and reduces the step size when MPP is reached. Also, it involved adding just a single term to the existing algorithm requiring only one extra memory location and still maintains the integrity and simplicity of the P&O Algorithm. In the present study a model of the system in terms of difference equations is determined

and a closed form solution to the system with momentum term is obtained for the first time. The system dynamics can then be understood in a better way.

Let $\Delta u(k)$ be the current perturbation for instant 'k' shown in (10), which is given in terms of the previous perturbation $\Delta u(k-1)$ scaled by a factor α and the step size of the P&O method given by $f(k)$.

$$\Delta u(k) = \alpha \Delta u(k-1) + f(k) \quad (10)$$

This factor $\alpha \Delta u(k-1)$ is called as the momentum term. The step size of P&O $f(k)$ is given by the following equation (11). If change in power in previous cycle is positive then the perturbation $f(k)$ will have same sign as $f(k-1)$ else sign is reversed as processed by the signum function in (11). The signum function makes the system non linear.

$$f(k) = f(k-1) \times \text{sign}[P(k-1) - P(k-2)] \quad (11)$$

VI. TIME DOMAIN ANALYSIS

To understand the system better it is considered to obtain a closed form solution to the response for the system with momentum term. The zero state and zero input response is found out to determine the complete response. Initially it is assumed that the system is linear by considering that the input of the system i.e. the perturbation step $f(k)$ is independent of the output $u(k)$.

Now we have,

$$\Delta u(k) = u(k) - u(k-1) \quad (12)$$

Thus from (10) and (12), a linear second order difference equation for the system is found.

$$u(k) = u(k-1) + \alpha [u(k-1) - u(k-2)] + f(k) \quad (13)$$

The equation (13) can be solved for the initial conditions and the input $f(k)$ assuming that the input is independent of the output. Two initial conditions are required to start the system as it is of order two. The initial conditions are taken as $u(0) = S$ and $u(1) = S + c$ where S is the starting value of current and c is the perturbation.

The closed form solution of zero input response is,

$$u_{ZI}(k) = S + \left(\frac{\alpha^k - 1}{\alpha - 1} \right) \times c \quad (14)$$

Impulse response is given by,

$$h(k) = \frac{\alpha^{k+1} - 1}{\alpha - 1} \quad (15)$$

Zero state response is given as the convolution sum of the impulse response and the input

$$u_{ZS}(k) = \sum_{m=0}^k f(m)h(k-m) \quad (16)$$

$$u_{ZS}(k) = \sum_{m=0}^k f(m) \frac{\alpha^{k-m+1} - 1}{\alpha - 1} \quad (17)$$

The complete response can finally be given as sum of zero input response and zero state response

$$u(k) = S + \left(\frac{\alpha^k - 1}{\alpha - 1} \right) c + \sum_{m=0}^k f(m) \frac{\alpha^{k-m+1} - 1}{\alpha - 1} \quad (18)$$

The solution for $u(k)$ can now be found by iteratively calculating given $f(k)$ for two cases

- $f(k) = c$ for tracking toward MPP
- $f(k) = +c, +c, -c, -c, \dots$ for around MPP

Fig 7. shows output for tracking toward MPP. It can be clearly seen that for current to rise from 5A to 8A the normal P&O algorithm takes around 30 cycles whereas for algorithm with momentum term it takes almost 10 to 15 cycles. This shows that with momentum term faster response can be achieved. Also it can be seen that as α increases the response gets faster.

Fig. 7. Rise of P&O Algorithm for different Momentum Terms

Fig. 8 shows output for oscillations around MPP. It can be seen that while oscillation size is 0.2A for normal P&O for algorithm with momentum term it is reduced to almost 0.13A. This shows that with momentum term ideally oscillation size can be reduced. It should be noted though that as α increases it takes longer time to settle toward the oscillation. Thus α should not be increased to too high a value. Also as α is also the characteristic root of the difference equation, α must be less than 1, since the root should be inside the unit circle for stability.

Although we have considered input to be independent of the output in our analysis, it is not the case as can be seen from (11). Also the signum function makes the system non-linear. From (11) and (13) the complete difference equation is given by:

$$u(k) = u(k-1) + \alpha [u(k-1) - u(k-2)] + f(k-1) \times \text{sign}[P(k-1) - P(k-2)] \quad (19)$$

This can be further evaluated considering $P(k)$ to be dependent on current $u(k)$ as in (20). Thus it can be seen clearly that the system is non-linear and depends on the IV characteristics of the PV panel. Finding a closed form solution

Fig. 8. Oscillations of P&O Algorithm for different Momentum Terms

to this equation is non trivial and hence the equation is solved numerically by modelling the system in Simulink environment.

$$u(k) = u(k-1) + \alpha [u(k-1) - u(k-2)] + f(k-1) \times \text{sign} \left\{ [u(k-1) - u(k-2)] \left[\frac{V}{I} + r_{pv} \right] \right\} \quad (20)$$

VII. SYSTEM DYNAMICS

To understand the dynamics of the system a model of the system was created in Simulink Version 8.8 (R2016b). The 2-DOF controller was used since it gives better response. The Averaged switch model is used for the Boost Converter. The algorithm for MPPT is written in a MATLAB function block. Initial set-point is at $S = 3.248A$, the perturbation length is taken as $c = 0.05A$, the sampling time is $0.01s$ and $\alpha = 0.7$. The tracking of output power for perturb and observe with and without momentum term is shown in Fig. 9. It is clear that the system responds faster with the momentum term.

Fig. 9. Output Power for Perturb and Observe with and without Momentum Term

The tracking of power in the case of ramp down and ramp up of irradiance signal for both the cases is shown in Fig. 10. The irradiance falls from $1000W/m^2$ to $500W/m^2$ in one second. It can be seen that with momentum term the tracking

is smooth, while in case of normal P&O since ramp down in irradiance is fast, it leads to loss in tracking for some time.

Fig. 10. Output Power for ramp-down and ramp-up of Irradiation

The variation in tracking due to changes in the momentum factor α is shown in Fig 11. The parameter α is also the characteristic root of the discrete time system. Hence α must be less than one for stability considerations. Also as α comes closer to one the speed, increases but natural oscillations becomes prominent as can be seen in Fig. 11. Hence proper trade-off must be considered between faster response and stability for the parameter α .

Fig. 11. Variation in tracking by change in Momentum factor α

VIII. CONCLUSIONS

The current controller for Boost Converter was designed using PI and 2-DOF techniques. The 2-DOF controller gives better response to the changing parameters due to closed loop pole placement, but its complexity in implementation is increased. The system was analysed for P&O with Momentum Term and a discrete time model for the system was arrived at. The solution was analysed for special cases where input is independent of output. It was seen that for a complete solution of the system a non-linear difference equation has

to be solved which was done in the Simulink Environment, as finding a closed form solution for it is a non-trivial task. The simulations showed that the speed of tracking is much faster in case of momentum term. Also the conditions for choosing the momentum factor were discussed.

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