

New Net Models for Spectral Netlist Partitioning

P.S.Nagendra Rao,
Department of Electrical Engineering ,
Indian Institute of Science,
Bangalore -560 012.
email : nagendra@ee.iisc.ernet.in.

C.S.Jayathirtha,C.S.RaghavendraPrasad,
University Visvavaraya College of Engineering
Bangalore University,
Bangalore - 560 001.

Abstract

Spectral approaches for partitioning netlists that use the eigenvectors of a matrix derived from a weighted graph model of the netlist(hypergraph) have been attracting considerable attention. There are several known ways in which a weighted graph could be derived from the netlist. However, the effectiveness of these alternate net models for netlist partitioning has remained unexplored. In this paper we first evaluate the relative performance of these approaches and establish that the quality of the partition is sensitive to the choice of the model. We also propose and investigate a number of new approaches for deriving a weighted graph model for a netlist. We show through test results on benchmark partitioning problems that one of the new models proposed here, performs consistently better than all the other models.

1.0 INTRODUCTION :

The problem of partitioning the modules of a netlist is encountered in many situations. Some examples are, laying out circuits on chips and printed circuit boards [1-3], program segmentation [4,5], etc. In these cases the netlist can be represented by a **hypergraph** - a generalized graph. In such a hypergraph each module is **represented** as a node and each net is represented as a generalized edge which may connect more than two modules. In this paper we consider a netlist as a hypergraph H containing n nodes (modules), $V = \{ 1, 2, 3, \dots, n \}$, and $|E|$ generalized edges (nets). The general problem considered here is of partitioning these n nodes into k disjoint sets $V_1, V_2, V_3, \dots, V_k$ such that the number of nets connecting these k blocks is minimized. An additional requirement is that the sizes of the partitions must be nearly equal. We only consider the case where $k=2$.

The netlist partitioning problem can be considered as a generalized version of the well known graph partitioning

problem. Hence, several graph partitioning algorithms have been used with or without modifications for solving this problem. The graph partitioning problem is known to be **NP-complete** and several heuristic algorithms have been proposed in past [6-8]. These methods start with an initial partition and seek to iteratively improve the partition by interchanging modules between partitions. It is well known that the quality of the final partition depends on the initial partition and in all these schemes the initial partition is randomly generated. These methods can be readily used for netlist partitioning. In view of their simplicity and reasonably acceptable performance these methods are popular and are widely used for netlist **partitioning**.

Spectral methods are a new class of methods which are being widely investigated in the recent past for solving this problem. Spectral methods were first proposed for partitioning **graphs** [9-11]. In view of their promise efforts are being made to modify and extend them to netlist partitioning [12-19]. For applying these methods the hypergraph is first approximated as a weighted **graph**. The weights of the edges are used to form a matrix which captures the netlist connectivity in some sense. Eigenvectors of this matrix form the basis for determining the partitions. Some of the spectral methods [12-14] require k or more eigenvectors and significant additional computation for partitioning the netlist into k blocks. This is a major disadvantage of these methods as the evaluation of a number of eigenvectors of the large matrix derived from the netlist involves a large amount of computation.

From this point of view spectral methods similar to the one first proposed in [11], and later adopted in [15,16], are more attractive as they require the evaluation of only one eigenvector. This paper aims to investigate this type of spectral methods **further**. The first question that we attempt to answer here is whether the quality of partitions is sensitive to the method of obtaining the graph representation of the netlist. We believe that this question

is important because several methods of representing a net have been proposed in literature in different contexts. However, only one such model (a clique model) has been used [16] to obtain the spectral partitions using a single eigenvector. We first study the effectiveness of a number of other clique models that have been proposed in the literature [12,15-19] for getting a graph representation for the netlist for spectral partitioning. The study here comparing the effectiveness of the clique of models proposed in [12,15-19] shows that the quality of the partitions depends on the choice of the net model. Hence, we propose a number of alternate approaches for representing the netlist as a graph and compare the quality of the spectral partition obtained by the new approaches. In particular, we propose an alternate clique model and two different star models to represent a net of the netlist. We believe we are the first to explore star models for net representation in this context. We show that the quality of the partitions obtained using one of the new models proposed here is consistently superior to those obtained by using all the other approaches.

2.0 THE PARTITIONING APPROACH :

The proposed spectral partitioning schemes consist of the following main steps.

1. Deriving a matrix form a graph model of the netlist.
2. Computing the second eigenvector of the above matrix.
3. Constructing the partition using the eigenvector and some additional heuristics.

2.1 Netlist Matrix Model :

A netlist consists of a number of modules and nets. In order to derive a graph model each module is considered as a node. A net cannot be readily be represented as an edge as it could connect more than two modules and several alternatives are possible for representing a net connecting 'p' modules as a p node graph with 't' weighted edges between these nodes. The Using any of these alternative approaches (described in Section 3) a netlist containing n nodes and IEI nets can be represented as a graph with n nodes and 1 weighted edges where $l = \sum t_j$, $V_j \in IEI$. Using this graph we define the n x n matrix model D where

$$D_{ij} = -(\text{Sum of the weights of the edges between nodes } i \text{ and } j) \quad i \neq j \quad (1)$$

$$D_{ii} = - \sum_{j=1,2,\dots,n} D_{ij}, j = 1,2,\dots,n \text{ and } j \neq i \quad (2)$$

2.2 Computing the Eigenvector :

It is known that the matrix defined in (1) and (2) is a positive semidefinite symmetric matrix. Its smallest eigenvalue is zero. As the matrix is sparse the eigenvector corresponding to the second smallest eigenvalue can be

calculated very efficiently using the Lanczos method [10,15]. It is well known that this computation is not excessive and the effort involved is a function of the sparsity of the matrix D.

2.3 Constructing the Partition :

We consider four different heuristics for determining the partitions U and W from V using the second eigenvector x of D. Three of these are proposed in [16] and the fourth one is proposed here. The heuristics chosen from [16] are the sign (SGN), ratiocut (RCUT) and the median (MED) heuristics. The new heuristic being proposed here is a modification of the median (MODMED) heuristic.

2.3.1 Sign (SGN) . Partition the modules based on sign of components of the eigenvector x. i.e. $U = \{ \text{module } i : x_i > 0 \}$ $W = \{ \text{module } i : x_i < 0 \}$.

2.3.2 Ratiocut (RCUT) . Sort x to give a linear ordering of the modules and determine the splitting index r that yields the best ratiocut. This is done as follows. The n components of x of the eigenvector are sorted yielding an ordering $V = V_1, V_2, V_3, \dots, V_n$ of the modules. The splitting index r, $1 < r < n-1$ is then found which gives the best ratiocut cost when modules with index wazzu greater r than are placed in U and the rest are placed in W. The ratiocut cost is defined as

$$z = e(U, W) / (|U||W|) \quad (3)$$

where $e(U, W)$ is the number of nets between the partitions.

2.3.3 Median (MEDIAN) . Find the median x_m of x. Partition the modules around the median x_m value. The two halves constitute U, and W. i.e., $U = \{ \text{module } i : x_i \geq x_m \}$ and $W = \{ \text{module } i : x_i < x_m \}$.

2.3.4 Modified Median (MODMED) . Sort x and obtain a linear ordering $V = V_1, V_2, V_3, \dots, V_n$ of nodes and determine the number of nets cut by varying the splitting index i between $0.4n < i < 0.6n$ and find the value of i that results in the minimum number of nets cut. The motivation behind this new heuristic is that there is no need to keep the partition sizes strictly equal, if it is possible to achieve an improved partition. Hence, this heuristic attempts to minimize the number of nets cut by seeking natural partitions around the median by permitting a certain amount of inequality in the partition sizes. We have chosen a range of $\pm 10\%$ of n around the median. However, there is nothing special about this value and it could be modified according to the needs.

3.0 GRAPH MODELS FOR NETS

The first step of the proposed partitioning approach requires that each net is represented as a weighted graph. Considering a net that connects p modules we need a p node graph with weighted edges between them. The existing models represent the net connecting p modules as a p -node clique. We discuss various clique models as also some new star models for representing the nets.

3.1 CLIQUE Models

Several authors [12,17-19] have proposed clique models for representing a net connecting p nodes as a p node clique. These models have been proposed in different contexts. A clique connecting p nodes consists of $p(p-1)/2$ edges. The models differ in the way weights are assigned to each of the edges of the clique. Various clique models that we have considered are summarized in Fig 1. Of these only the **CLIQ 1** (the standard clique model) and has used [16] for spectral partitioning in a way similar to the approach we are discussing here.

3.2 STAR Models

A major drawback of using a clique model for representing a net is that it reduces the sparsity of D . This in turn increases the computational effort while calculating the eigenvector. The net is actually a 'tree' type structure and one can conjecture that a model that captures this type of connectivity could help in obtaining better partitions. In addition, improved sparsity of D would contribute to the reduction in computational effort. There are several ways in which p nodes can be connected using $(p-1)$ edges in the form of a tree. In order to have a consistent strategy we use the functionality of the net to arrive at a unique tree structure. A net connecting p modules essentially carries a signal from one module (source) to the remaining $(p-1)$ modules (destinations). This functionality can be represented as a star graph having p nodes and $(p-1)$ edges; each of the edge connecting the source with one of the $(p-1)$ destinations. We consider two variants of this model. In the first model which we refer to as the **STAR** model, all the $(p-1)$ edges are assigned a weight of 1. The second model is referred to as the **Weighted Star (WTSTAR)** model and in this case the edges are assigned a weight of $1/(p-1)$,

4.0 NUMERICAL RESULTS

The performance of the various models considered here is compared by obtaining partitions for a number of benchmark examples. First we compare the performance of

the clique models. In all these cases we use the **MODMED** heuristic for finding the two partitions based on the second **eigenvector**. test results are summarized in Table 1, where the number of nets cut is given for a number of cases by the five clique models.

From Table 1 it is clear that the quality of partitions is sensitive to the choice of the model. Among the models considered the partitions obtained using the **CLIQ 1** and **CLIQ 4** models are consistently superior to those obtained by the others. It is interesting to note that the **CLIQ 2** model [18] which assigns the edge weight such that the weight of any cut in the clique underestimates any cut in the generalized edge, performs poorly as compared with the standard clique model **CLIQ1** even though it is rather arbitrary. It is also noteworthy that the new clique model proposed here (**CLIQ 3**), even though very simple, does not exhibit any significant improvement. The **CLIQ 5** model has been preferred over the **CLIQ 1** model in [12,13]. However, with the present approach, the performance of the **CLIQ 5** model is extremely poor. Based on the results in Table 1 we found it difficult to assess the relative capability of **CLIQ 1** and **CLIQ 4** models. Some more examples i.e. **S298, DECOD, COMP, COUNT** were considered. For these circuits the number of nets cut by the **CLIQ 1** model was 7, 18, 3, 4 where as that by **CLIQ 4** was 74, 19, 47, 35 respectively. Based on this we can infer that the **CLIQ 1** model appears to be the best among all the clique models considered here. In view of this, in the remaining part of the paper when we refer to clique models, we refer to only **CLIQ 1** model- as we consider it as the best among this class of methods.

The effectiveness of the new star models for partitioning netlists has also been assessed by finding partitions of a number of examples. We have compared the quality of the partitions obtained by the new models with those obtained by the best clique model (**CLIQ 1**). The results are summarized in Tables 2,3 and 4 corresponding to the **STAR**, **WTSTAR** and **CLIQ** models respectively. In these tables n_1 and n_2 indicate the partition sizes and **NC** the number of nets cut. While using each of this models the final partitions are obtained by all the four different heuristics discussed earlier; **SGN**, **RCUT**, **MEDIAN** and **MODMED**.

Considering the partitions obtained by the **SGN** based heuristics in Tables 2,3, and 4, we see that this heuristic does not succeed in keeping the partition sizes nearly equal. This behavior is observed with all the three models **CLIQ**, **STAR** and **WTSTAR**. In cases where the partitions are reasonably balanced the quality of the partitions is inferior to that obtained by the **MODMED** heuristic. In situations where the nets cut are small as compared to the **MODMED** heuristic, the partitions are highly imbalanced.

The performance of the **RCUT** heuristic is also similar to the **SGN** heuristic across all three models. As a

matter of fact this is counter intuitive, since the **ratio-cut** heuristic is evolved with an explicit purpose of incorporating the size balance feature into the algorithm. However, it is very clear from the results in Tables 2, 3 and 4 that the **ratio-cut** heuristic cannot generate reasonably balanced good partitions. Even with this heuristic when the number of nets cut is small the sizes are not balanced and when the sizes are nearly **balancedKL**, the partitions obtained by the MODMED heuristic are invariably better.

The partitions obtained by the MEDIAN heuristic are generally better than the **RCUT** and **SGN** heuristics. These partitions are obviously the best from the point of view of the equality of sizes. The only limitation of this approach is that since the partition sizes are implicitly fixed, and if there are good partitions which are almost balanced but not exactly equal in size, this heuristic fails to recognize them. The new heuristic MODMED is intended to overcome this problem and it succeeds in achieving it as it is clear from Tables 2, 3 and 4.

Hence, it is clear that for all the three models the performance of the MODMED heuristic is clearly superior to the other heuristics and we limit our further comparison of the methods to only this heuristic. In order to facilitate the comparison of these three models, we summarize their performance in Table 5. For all the examples we have also obtained the partitions using the **Kernighan** and **Lin (KL)algorithm** [7] also. The number of nets cut given in Table 5 for the algorithm is the average of ten trials. From Table 5 it is clear that the partitions obtained by using the **WTSTAR** net model and MODMED heuristic is superior to those obtained by the other models. The only exception being the circuit **EXAMPLE2**, for which the **STAR** and **CLIQ** models give a slightly better partitions. However, comparing the number of nets cut in each case we see that the performance of the **WTSTAR** model is almost as good as the other models in this case also.

In order to assess the quality of the spectral partitions obtained by the various models, we have tried to determine whether the partitions could be improved further. We used the partitions obtained by the various models as the initial partition for the **KL** algorithm and recorded the improvement in the **partitions**, in each case. We have used only the MED heuristic for this experiment as only with this heuristic same size partitions are obtained in all the

cases. These results are given in Table 6. It can be seen that **KL** algorithm improves the partitions obtained by the **STAR** and **CLIQ** models to a considerable extent in all cases where as the improvement observed with **WTSTAR** model is very marginal. Even then, the final partitions obtained by the **WTSTAR** model and **KL** algorithm together are superior to the others in all the cases. This also shows that the quality of the partitions obtained by **WTSTAR** model are intrinsically very good.

5.0 CONCLUSION

In this paper we have considered the problem of partitioning a netlist using only one eigenvector of a matrix derived from the weighted graph model of the netlist. We have established that the quality of the partitions is dependent on the method used to obtain the weighted graph representation of the netlist. This has been done by considering a number of approaches that have been proposed by earlier works for generating the weighted graph of a netlist and using this matrix to obtain the partitions by spectral methods. In our effort to investigate whether better partitions can be obtained using alternate models of representing the netlist by a weighted graph, we propose three new models. We have also shown that one of the new approaches (**WTSTAR** model for net representation) succeeds in providing partitions that are comparable to or better than the partitions generated by all the other approaches. The fact that the partitions obtained using the **WTSTAR** model with MODMED heuristic could not be improved to any significant extent in any of the test cases by the **KL** algorithm is also an indicator of the very good quality of the partitions produced by this approach.

Even though no effort has been made here to quantify the computational advantage of this approach it is easy to see that one can expect a significant saving in computational effort while computing the eigenvector using the **WTSTAR** model instead of any of the **CLIQ** models in view of the increased sparsity of the matrix **D**. Our proposed approach can be readily extend to partition the netlists into more than two partitions and even in this case the new approach performs better than the other approaches. This aspect, however, is discussed elsewhere.

NAME	PROPOSED BY	EDGE WEIGHTS
CLIQ 1	Lengaur [17]	$1/(p-1)$
CLIQ 2	Hadley et. al [18]	$1/(\lfloor p/2 \rfloor * \lfloor p/2 \rfloor)$
CLIQ 3	present authors	1
CLIQ 4	Frankle et. al[19]	$(2/p)^{3/2}$
CLIQ 5	Alpert and Yao[12]	$(1-2/2^p) * 4/(p*(p-1))$

Fig 1. Summary of CLIQUE models.

CIRCUIT	CLIQ 1			CLIQ 2			CLIQ 3			CLIQ4			CLIQ5		
	n1	n2	NC	n1	n2	NC	n1	n2	NC	n1	n2	NC	n1	n2	NC
MUX	53	59	18	57	55	24	57	55	24	67	45	6	45	67	37
LDD	77	54	13	65	66	23	66	65	23	66	65	22	52	79	48
DECOD	28	18	18	19	27	18	19	27	19	18	28	18	18	28	18
FRG1	92	47	15	59	80	24	56	83	23	56	83	20	65	74	41
UNREG	87	78	42	63	102	50	57	108	42	72	93	7	69	96	54

n1,n2 - Sizes of the partitions, NC-Nets cut. Table 1. Performance of CLIQUE models (MODMED heuristic).

CIRCUIT	Number of Nodes	SGN			RCUT			MEDIAN			MODMED		
		n1	n2	NC	n1	n2	NC	n1	n2	NC	n1	n2	NC
MUX	112	29	83	10	4	108	1	56	56	24	44	68	20
LDD	131	66	65	15	59	72	14	65	66	22	58	73	14
PCLER8	147	88	59	11	104	43	8	73	74	15	87	60	10
CHT	315	115	200	2	116	199	2	157	158	28	130	185	9
FRG1	139	109	30	1	109	30	1	69	70	30	82	57	22
UNREG	165	139	26	6	131	34	6	82	83	50	87	78	42
EXAMPLE2	517	275	242	19	81	436	3	257	258	24	309	208	7
ALU2	376	75	301	47	34	342	11	188	188	87	122	254	71
X4	608	364	244	14	361	247	14	304	304	73	361	247	14
DECOD	46	6	40	2	8	38	2	23	23	22	18	28	18
S298	142	58	84	14	9	133	2	71	71	12	69	73	9
COMP	211	109	102	4	49	162	2	105	106	4	110	101	3
COUNT	195	99	96	5	100	95	4	97	98	5	100	95	4
B9	186	136	50	54	1	185	1	93	93	92	110	76	69
CC	100	89	11	3	95	5	1	50	50	17	41		14

n1,n2- Sizes of the two partitions, NC-netscut.

Table 2. Partitions obtained using STAR model.

CIRCUIT	Number of Nodes	SGN			RCUT			MEDIAN			MODMED		
		n1	n2	NC	n1	n2	NC	n1	n2	NC	n1	n2	NC
MUX	112	85	27	6	90	22	4	56	56	14	53	59	14
LDD	131	65	66	13	49	82	11	65	66	13	64	67	13
PCLER8	147	75	72	5	74	73	4	73	74	4	74	73	4
CHT	315	199	116	2	199	116	2	157	158	23	185	130	7
FRG1	139	109	30	1	109	30	1	69	70	27	90	49	15
UNREG	165	95	73	6	82	83	6	82	83	7	82	83	6
EXAMPLE2	517	211	306	10	176	341	4	257	258	26	206	311	10
ALU2	376	211	165	62	30	346	11	188	188	56	223	153	53
X4	608	29	579	1	29	579	1	304	304	16	280	328	10
DECOD	46	38	8	2	38	8	2	23	23	9	22	24	7
S298	142	52	90	11	70	72	6	71	71	7	70	72	6
COMP	211	107	104	4	49	162	2	105	106	4	110	101	3
COUNT	195	99	96	5	100	95	4	97	98	15	100	95	4
B9	186	143	43	3	156	31	1	93	93	17	110	76	11
CC	100	8	92	1	8	92	1	50	50	12	52	48	9

n1,n2- Sizes of the two partitions, NC-netscut.

Table 3. Partitions obtained using WTSTAR model.

CIRCUIT	Number of Nodes	SGN			RCUT			MEDIAN			MODMED		
		n1	n2	NC	n1	n2	NC	n1	n2	NC	n1	n2	NC
MUX	112	84	18	8	108	4	1	56	56	22	53	59	18
LDD	131	74	57	13	78	53	12	65	66	21	74	57	13
PCLER8	147	90	57	10	88	59	10	73	74	14	88	59	10
CHT	315	199	116	2	199	116	2	157	158	27	184	131	11
FRG1	139	109	30	1	109	30	1	69	70	33	92	47	15
UNREG	165	151	14	6	160	5	2	82	83	70	87	78	42
EXAMPLE2	517	239	278	27	81	436	3	257	258	23	309	208	6
ALU2	376	299	77	49	341	35	11	188	188	80	255	121	72
X4	608	255	353	19	241	367	13	304	304	68	243	365	14
DECOD	46	8	38	2	8	38	2	23	23	22	28	18	18
S298	142	53	89	18	22	120	3	71	71	7	68	74	7
COMP	211	103	108	4	162	49	2	105	106	4	110	101	3
COUNT	195	80	115	5	88	107	4	97	98	31	88	107	4
B9	186	31	155	2	30	156	1	93	93	30	74	112	23
CC	100	89	11	3	95	5	1	50	50	20	56	44	13

n1,n2- Sizes of the two partitions, NC-nets cut.

Table 4 Partitions obtained using CLIQ 1 model.

CIRCUIT	Number of nets cut				Percentage Improvement over		
	KL	STAR	CLIQ	WTSTAR	KL	STAR	CLIQ
MUX	21	20	18	14	50	43	29
LDD	24	14	13	13	85	8	0
PCLER8	14	10	10	4	250	150	150
CHT	15	9	11	7	114	29	57
FRG1	22	22	15	15	43	43	0
UNREG	23	42	42	6	283	600	600
EXAMPL2	52	7	6	10	220	-30	-40
ALU2	71	71	72	53	34	34	36
X4	66	14	14	10	560	40	40
DECOD	12	18	18	7	70	154	154
S298	20	9	7	6	233	50	17
COMP	10	3	3	3	233	0	0
COUNT	16	4	4	4	300	0	0
B9	16	69	23	11	45	528	109
CC	15	14	13	9	67	56	44

Table 5 Comparison of WSTAR model with other models (MODMED heuristic).

CIRCUIT	STAR		CLIQ		WTSTAR	
	INITIAL	FINAL	INITIAL	FINAL	INITIAL	FINAL
MUX	24	14	22	12	14	8
LDD	22	22	21	21	13	13
CHT	28	22	27	23	23	19
FRG1	30	22	33	26	27	24
UNREG	50	29	70	33	7	5
EXAMPLE2	24	20	23	20	26	19
ALU2	87	69	80	79	56	56
X4	73	79	68	68	16	15
DECOD	22	21	22	9	9	9
S298	12	12	7	6	7	6
COMP	4	4	4	4	4	4
COUNT	5	3	3	5	5	3
B9	92	19	30	15	17	15
CC	17	14	20	14	12	12

Table 6. Improvement of Spectral partitions by KL algorithm (nets cut).

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