Modulo-Variable Expansion Sensitive Scheduling

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Abstract

Modulo scheduling is an aggressive scheduling technique for loops that exploits instruction-level parallelism by overlapping successive iterations of the loop. Due to the nature of modulo scheduling, the lifetime of a variable can overlap with a subsequent definition of itself. To handle such overlapping lifetimes, modulo-variable expansion (MVE) is used, wherein the constructed schedule is unrolled a number of times.

In this paper we propose a technique to improve the constructed schedule while performing MVE. In our approach, we unroll the data dependence graph of the original loop and re-schedule it with a MVE-sensitive scheduler. Such an approach is expected to result in better initiation rates as compared to the traditional approach. We have implemented our approach and evaluated its performance on a large number of scientific benchmark kernels.

1. Introduction

Modulo scheduling or software pipelining is an aggressive instruction scheduling technique for exploiting instruction-level parallelism in loops [6, 11]. In this scheduling method, successive iterations of the loop are initiated at a constant interval, known as the initiation interval (II) of the schedule. The constructed schedule must obey the resource constraints of the machine as well as the dependency constraints of the loop. Several methods have been proposed to perform software pipelining [1, 2, 3, 5, 6, 9, 8, 11, 10, 13].

Due to the nature of modulo scheduling, the lifetime of a variable can overlap with a subsequent definition of itself. To handle such overlapping lifetimes, some form of register renaming has to be provided to ensure that

successive definitions of a variable use distinct registers. Some architectures provide hardware support in the form of rotating register files [14]. When no hardware support is available, the problem of overlapping lifetimes can be solved by using **Modulo-Variable Expansion (MVE)**, wherein the kernel is unrolled and multiple definitions of a variable are renamed at compile time [6]. In the traditional approach, when MVE is required, the constructed *schedule* is unrolled a number of times. The disadvantage of this is that the final schedule obtained by unrolling the schedule will have an II which is an exact multiple of the II of the original loop. Even if there exists a *valid* schedule at an II less than the exact multiple, the traditional method does not exploit this possibility. By a *valid* schedule we mean a schedule that obeys the resource and data dependency constraints, and *does not need further MVE*.

We propose a technique that attempts to find such a schedule by unrolling the data dependence graph (DDG) of the loop and re-scheduling it with a MVE-sensitive scheduler. By unrolling the DDG, we also overcome the limitation that the II of the original loop is restricted to be an integer. If the lower bound on the II computed before scheduling is not an integer, the performance degradation caused by rounding it up to an integer may be reduced by unrolling the loop. Our approach attempts to take advantage of this. The approach proposed in [7] by Lavery and Hwu also attempts to overcome the same limitation by unrolling the loop a number of times. However, they unroll the loop *before* modulo scheduling. Their final schedule may still require MVE, in which case they perform MVE using the traditional approach.

The proposed MVE-sensitive scheduler was implemented and tested on a large number of standard scientific kernels for three different machine models including the MIPS R8000 processor [4]. We used Huff's Lifetime-Sensitive Modulo Scheduling algorithm as the

base scheduling method [5]. Our experiments show that a large number of loops (27% to 50% of the loops) required MVE. Of these, our experimental analysis revealed potential for improvement in Π in about 35% of the loops. However the improvement in Π was observed only in 6% of the loops. This can be attributed to the possibly high resource requirement of the MVE-sensitive scheduler.

The rest of the paper is organized as follows. Section 2 gives the necessary background on modulo scheduling and modulo-variable expansion through a motivating example. Section 3 describes our scheduling technique in detail. Experimental results are reported in Section 4 and Section 5. Section 6 summarizes the conclusions.

2. Background and motivating example

In this section, first we present the necessary background on modulo scheduling and modulo-variable expansion through a motivating example. Using the example, subsequently, we motivate our approach to improve the constructed schedule while performing modulo-variable expansion.

2.1. Modulo Scheduling

In a modulo scheduled loop the constant interval between the initiations of successive iterations is called **Initiation Interval** (Π) [10]. The Π of a software pipelined schedule is governed by both resource constraints and recurrences in the loop. The minimum initiation interval **MII** is the maximum of **RecMII**, due to inter-iteration dependences in the loop, and **ResMII**, due to the resource constraints of the architecture [10]. **RecMII** and **ResMII** are defined as

$$\mathbf{RecMII} = \max_{\text{All cycles}} \frac{\text{Length of recurrence cycle}}{D}$$

where *Length of recurrence cycle* is the sum of latencies of all instructions in the cycle and D represents the sum of dependence distances on the arcs in the cycle. *Dependence distance* or *loop-carried dependence* on an arc indicates how many iterations later the value produced by an instruction is consumed by its successor.

$$\mathbf{ResMII} = \max_{r \in \text{set of FU types}} \frac{N_r * d_r}{F_r}$$

where N_r represents the number of instructions that are executed in function unit (FU) type r , d_r represents the maximum number of cycles during which any stage

of FU type r is used by an instruction, and F_r is the number of FUs in type r . Lastly,

$$\Pi > \mathbf{MII} = \max(\lceil \mathbf{RecMII} \rceil, \lceil \mathbf{ResMII} \rceil).$$

Further details on **RecMII** and **ResMII** can be found in [5, 6, 10, 13]. Consider the data dependence graph

Func Units	Number of Units	Latency	Pipelined
Load/Store	2	2	yes
Add/Sub	1	1	yes
Multiply	1	1	yes

(b)

Figure 1. (a) Motivating Example (b) Machine Configuration

(DDG) shown in Figure 1(a), and a machine configuration as shown in Figure 1(b). Each node in the DDG represents an operation and an arc represents data dependence between operations. Dependence distance is represented by means of a token on the arc. Since there are no recurrence paths, the **MII** of the DDG is the **ResMII**, which is 2. The loop has a schedule at **MII** as shown in Figure 2(a).

Figure 2. (a) Schedule of Loop shown in Fig.1 (b) Kernel

2.2. Modulo-Variable Expansion

A variable defined by an operation needs to be stored and retained in a register until all its successors have used the value. The lifetime of a variable is defined as the duration between the first assignment and its last use. Due to the nature of modulo scheduling, the lifetime of a variable defined in one iteration can overlap with the lifetimes of variables defined in successive iterations. Furthermore, for a variable with a lifetime greater than Π , new values are generated before the previous one is used, i.e., the lifetime of a variable can overlap with a subsequent definition of itself. For

Figure 3. (a) Schedules of two consecutive iterations and (b) Lifetimes of variables

the motivating example, the lifetimes of all variables for two consecutive iterations of the loop are shown in

Figure 3(b). We see that lifetime of the virtual register V3 begins in cycle 2, i.e., the cycle in which the operation sub0 starts execution, and ends in cycle 5 which corresponds to the *last use*, i.e., use by the operation mul0 from the next iteration. Thus, the lifetime of V3 has a length of 3 cycles, which is greater than the Π of the loop. Hence this lifetime overlaps with the next definition of itself, which occurs in cycle 4 as shown in the figure.

To handle such overlapping lifetimes, some form of register renaming has to be provided to ensure that successive definitions of the variable use distinct registers. Some architectures provide hardware support in the form of rotating register files [14]. Rotating register files rename different instantiations of a variable during execution time.

When no hardware support is available, the problem of overlapping lifetimes can be solved by using **Modulo-Variable Expansion (MVE)**, wherein the kernel is unrolled and multiple definitions of a variable are renamed at compile time [6].

In the traditional approach, when MVE is required, the constructed *schedule* is unrolled a number of times. For any loop the required degree of unroll, henceforth referred to as *unroll_factor*, is determined by the longest lifetime among the variables [12]. Because iterations are initiated every Π cycles, *unroll_factor* can be calculated as :

$$\text{Unroll Factor} = \max_i \left\lceil \frac{\text{lifetime}_i}{\Pi} \right\rceil$$

The unrolled schedule has an Π which is equal to the Π of the original loop multiplied by the *unroll_factor*. In our example, the kernel has to be unrolled $\lceil \frac{3}{2} \rceil = 2$ times. In this case, the Π of the unrolled schedule is 4.

2.3. Our Approach

In our approach, instead of unrolling the schedule, we unroll the DDG itself and schedule it. The unrolled DDG is shown in Figure 4. Again, since there are no recurrence paths, the $\text{Res}\Pi$ of the unrolled DDG is the $\text{Res}\Pi$, which is 3. When the unrolled DDG is scheduled, we see that it gets scheduled at an Π of 3, as opposed to an Π of 4 achieved by the traditional approach. The schedule and lifetimes of the unrolled DDG are shown in Figure 5(a) and Figure 5(b) respectively. While scheduling the unrolled DDG, we have to ensure that the schedule obtained from the unrolled DDG will not require further modulo-variable expansion, i.e., the scheduler which schedules the unrolled DDG should be *sensitive* to modulo-variable expansion. In other words, we should ensure that the lifetime of

Figure 4. Unrolled DDG

Figure 5. (a) Schedule of unrolled loop (b) Kernel (c) Lifetimes

a virtual register in the final schedule for the unrolled DDG does not overlap with a subsequent definition of itself. In order to ensure this, we derive the necessary conditions in Section 3.3. The following section discusses the scheduling method used in our work.

3. Scheduling Technique

In this section we describe our scheduling technique which is sensitive to modulo-variable expansion. The method is based on Huff's Lifetime-Sensitive Modulo Scheduling algorithm [5]. In Section 3.1 basic definitions of a few terms used in Huff's algorithm and in

our approach are given. For the sake of completeness we briefly describe Huff's algorithm in Section 3.2. Our approach is described in Section 3.3. Readers familiar with Huff's algorithm could skip Sections 3.1 and 3.2.

3.1. Definitions

This subsection defines certain basic terminology .

Definition 3.1 (Mindist) $\text{mindist}(i,j)$ is the minimum number of cycles by which operation i must precede operation j in any feasible schedule.

Intuitively, $\text{mindist}(i,j)$ corresponds to the maximum distance in terms of execution latencies of operations along any path from i to j .

Definition 3.2 (Estart) Estart is the earliest time step at which an operation can be scheduled such that all its predecessors have completed their execution.

Definition 3.3 (Lstart) Lstart is the latest time step at which an operation can be scheduled such that it completes its execution before its successors can start executing.

In Section 3.2 we explain how Estart and Lstart of an operation can be computed.

Definition 3.4 (Slack) Slack of an operation i is defined as the difference between its Lstart and Estart values. i.e.,

$$\text{Slack}(i) = \text{Lstart}(i) - \text{Estart}(i)$$

3.2. Huff's Algorithm

Huff's Lifetime-Sensitive Modulo Scheduling algorithm schedules a loop based on the scheduling freedom or *slack* of an operation. The method attempts to schedule a loop for minimal register pressure as well as minimal II. To aid in the scheduling process, two pseudo-operations are added to the loop body: Start and Stop. Start is the predecessor of each operation and Stop is the successor of every operation. Using these nodes and the mindist relation, the scheduler initializes the Estart and Lstart bounds for each operation to:

$$\text{Estart}(i) = \text{mindist}(\text{Start}, i)$$

and

$$\text{Lstart}(i) = \text{Lstart}(\text{Stop}) - \text{mindist}(i, \text{Stop})$$

Once the initial bounds are set up, the scheduler is ready to place the operations one by one into the partial

schedule. The algorithm tries to place operations into the partial schedule until either a feasible schedule is found or the heuristics give up after a large number of tries. The algorithm consists of the following basic steps:

- Choose an operation to place in the partial schedule based on a priority function. Huff's algorithm prioritizes operations based on their slacks.
- Search for a valid issue slot within the $Estart$ and $Lstart$ bounds of the operation. An issue slot is valid if placing the operation in that slot does not create any resource conflict in the partial schedule.
- If no conflict-free slot is found, then the operation is forced into the time-slot

$$\max (Estart, (PrevScheduleTime + 1))$$

where $PrevScheduleTime$ is the time at which the operation was scheduled previously. After forcing the operation, all operations in the partial schedule whose resource requirements conflict with the current operation are ejected. Further, if the operation is placed beyond its $Lstart$, all its successors are also ejected.

- Once the operation has been placed, the modulo reservation tables are updated to indicate the resource usage of the placed operation.
- The $Estart$ and $Lstart$ bounds of all unscheduled operations are updated to reflect the placement of the new operation. Let a new operation i be scheduled in time-step ti , then, for each operation j not in the partial schedule the bounds are updated to :

$$Estart(j) = \max (Estart(j), ti + mindist(i, j))$$

and

$$Lstart(j) = \min (Lstart(j), ti - mindist(j, i))$$

- If the number of operations ejected is beyond some threshold value, then all operations are removed from the partial schedule, II is incremented and all the above steps are repeated.

3.3. Our Scheduling approach

Once a valid schedule of the loop is found using Huff's algorithm, the lifetimes of all variables are computed. If any of the variables has a lifetime greater than II , then MVE has to be performed on the loop.

In the traditional approach, when MVE is required, the constructed *schedule* is unrolled. In our approach we unroll the DDG of the original loop and re-schedule it with a modulo scheduler sensitive to MVE.

In subsequent sections we will use the following notations to differentiate the II and MII of the original loop and the unrolled loop: Let,

II_{org} denote the II of the original loop.

MII_{org} denote the MII of the original loop.

II_{unroll} denote the II of the unrolled loop.

MII_{unroll} denote the MII of the unrolled loop.

The scheduler which schedules the unrolled loop must adhere to the following conditions :

Condition A *The schedule obtained for the unrolled DDG should not require further modulo-variable expansion.* Henceforth we refer to this condition as the MVE constraint.

Condition B *The schedule obtained for the unrolled DDG should have an II less than $II_{org} * unroll_factor$.* This constraint is mainly to ensure that scheduling the unrolled DDG is at least as efficient as unrolling the schedule.

In order to satisfy the Condition A the scheduler must ensure that the lifetime of a variable can never exceed $Hunroii$ and hence will not need MVE. To achieve this, the relations for $Estart$ and $Lstart$ bounds used in Huff's algorithm have to be refined. To see how $Estart$ and $Lstart$ bounds have to be modified, consider the two operations shown in Figure 6(a). Let the opO be scheduled in time-step t_0 and let $op1$ be a direct unscheduled successor of it. The $Lstart$ of $op1$ is first found using Huff's $Lstart$ bounds to satisfy the mindist relation. It is computed depending on the successors of $op1$ (not shown in the figure). Then, in order to ensure that the lifetime of the variable defined by opO does not exceed II_{unroll} , $op1$ should be scheduled before $t_0 + Hunroii$ cycles. Therefore the $Lstart$ of $op1$ is defined as:

$$Lstart(op1) = \min (Lstart(op1), t_0 + II_{unroll})$$

Now, consider the operations shown in Figure 6(b). In this case the dependence distance from opO to $op1$ is greater than zero (1 in case of Figure 6(b)). Assume again that opO is scheduled in time-step t_0 . Here, opO produces a value which is used by $op1$ in the next iteration. In other words, $op1$ uses the value defined by opO in cycle $t_1 + II_{unroll}$, where t_1 is the time-step in which $op1$ is scheduled. Hence, the lifetime of the

Figure 6. Lstart Calculation

variable is $t_1 + II_{unroll} - t_0$. By the MVE constraint this should be less than II_{unroll} , i.e.,

$$t_1 + II_{unroll} - t_0 < II_{unroll}$$

We see that in this case op1 should be scheduled before time-step t_0 .

In general, for any dependence distance d , we have to ensure that

$$t_1 + d * II_{unroll} - t_0 < II_{unroll}$$

Therefore, if an operation i is scheduled, for each unscheduled operation j which is a *direct* successor of operation i , the Lstart relation can be written as :

$$Lstart(j) = mm(Lstart(j), t_i + (1 - d) * II_{unroll})$$

where t_i is the time-slot in which operation i is placed.

We see that in order to satisfy the MVE constraint, the Lstart of an operation depends not only on its successors as in Huff's original algorithm, but also depends on its direct predecessors.

Similarly, the Estart relation can be derived as :

$$Estart(j) = \max(Estart(j), t_i + (d - 1) * II_{unroll})$$

Thus, in order to satisfy Condition A the MVE-sensitive scheduler uses the above modified equations for Estart and Lstart calculations.

In order to satisfy Condition B, our scheduler searches for a valid schedule only between MII_{unroll} and $II_{org} * unroll_factor$. If a valid schedule is not found in this range, the scheduler outputs the unrolled schedule as in the traditional approach.

Our MVE-sensitive scheduler incorporates the above mentioned modifications in Huff's algorithm and attempts to find a final schedule for the unrolled DDG.

The overall algorithm consists of the following steps:

Step 1 Schedule the original loop using Huff's algorithm.

Step 2 Find the lifetimes of all variables.

Step 3 Find the *unroll_factor* using relation given in Section 2.2.

Step 4 Unroll the DDG the required number of times.

Step 5 Schedule the unrolled DDG using the MVE-sensitive scheduler to obtain a valid schedule that does not require further MVE.

4. Experimental Setup

Our modulo scheduling approach, sensitive to modulo-variable expansion, is based on Huff's slack scheduling algorithm [5]. We used Huff's method because it is a fast and efficient method. However the approach presented in this paper can be applied to any modulo scheduling algorithm. The approach was tested for its performance on a set of 1450 benchmarks. These are basic blocks, extracted from a variety of scientific benchmark programs like Linpack, Livermore, NAS, and SPEC. We assumed three machine configurations with complex structural hazards. The first configuration closely resembles the R8000 machine model (Model 1). The other two are hypothetical models.

Model 1 This configuration resembles the R8000 machine model. It consists of eighteen functional units, details of which can be found in [4].

Model 2 This is a simple hypothetical model consisting of two integer add units with unit latency, one integer load unit with latency 3, one integer store unit with latency 4, one floating point add unit with latency 4, one floating point multiply unit with latency 3, and one divide unit with latency 18.

Model 3 This model has the same number of functional units as Model 2 but with more complex structural hazards and higher latencies. The integer add, integer load and floating point add units have a latency of 5, integer store unit has a latency of 4, floating point multiply unit has a latency of 7, and the floating point divide has a latency of 22.

The performance results for all the three machine configurations are presented in the following section.

5. Results

The performance results for the three machine models are tabulated in Tables 1 and 2.

Machine Model	Loops which required MVE	Loops in which $MII_{unroll} = II_{org} * unroll_factor$	Loops in which $MII_{unroll} < II_{org} * unroll_factor$	Loops scheduled with an improved II_{unroll}
Model 1	388	226	162	27
Model 2	502	352	150	29
Model 3	718	470	248	42

Table 1. Statistics on Improvement

A total of 1450 loops were run on the scheduler. Out of the 1450 loops about 49.4% of the loops required MVE in case of Model 3. The number of loops requiring MVE was less (27% and 35%) in case of Models 1 and 2. This is possibly because both Models 1 and 2 have simpler resource configurations compared to Model 3. From here on we discuss the results for Model 3 in greater detail while the results for Models 1 and 2 are tabulated.

In case of Model 3, out of the 718 loops that needed MVE, 470 loops (65%) of the loops had an MII_{unroll} equal to $II_{org} * unroll_factor$. This is because these loops got scheduled at their MII and the absolute lower bound on their MII was an integer, hence when unrolled, the unrolled DDG had an MII which was $MII_{org} * unroll_factor$. In these loops our MVE-sensitive scheduler would not have been able to give any performance gains. Therefore, our scheduler follows the traditional approach and outputs the unrolled schedule. In the remaining 248 (35%) loops there was potential for improvement in II . However, we could get schedules at an improved II only in about 42 (6%) of the loops. For the loops in which we did get an improvement in II , the improvement was significant (refer to Table 2). The performance improvement was calculated as $\frac{II_{org} * unroll_factor - II_{unroll}}{II_{unroll}}$, where II_{unroll} was the II obtained by the MVE-sensitive scheduler.

In Model 3, the improvement in II was as high as 37.5%. The average improvement in II was 12%. The performance improvements in Model 1 and 2 were higher. In Model 1 we observed a performance improvement of about 14% and in Model 2 the average performance improvement was about 23%.

Machine Model	Maximum %	Average %
Model 1	33.3	13.8
Model 2	60.0	23.1
Model 3	37.5	12.0

Table 2. Improvement in II

The loops which could not be scheduled at an $II < II_{org} * unroll_factor$ even though there was potential for improvement in them, is due to fact that the MVE constraint requires certain nodes to be placed in certain time slots which aggravates the resource requirements considerably. To empirically establish this we ran all the loops on an infinite resource model, i.e., with no resource constraints. The unrolled loops were scheduled using the MVE-sensitive scheduler. It was observed that all the loops got scheduled at the MII_{unroll} of the loops. In this experiment, we used the value of the MII_{unroll} for each loop as computed for the target machine model, i.e., with resource constraints. We did not attempt to schedule the loops for $II = MII_{unroll}$ of the infinite resource model. This is because our interest here is to find out whether the MVE constraint does really increase the resource requirement, and that it prevents the loops to be scheduled at an II less than $II_{org} * unroll_factor$.

In [7], various unrolling-based optimizations have been proposed to reduce the resource requirements and the height of the critical path of the unrolled loop body. Some of these optimizations include removing the loop exit branches from the unrolled loop body, induction variable elimination etc. Currently none of the optimizations have been included in our implementation. It is important to note that our approach can further benefit from these optimizations, whereas the traditional approach (of unrolling the schedule) cannot.

In our approach, we first compute the schedule for the original loop using Huff's scheduling method, and subsequently we schedule the unrolled loop, if MVE is needed, using the MVE-sensitive scheduler. This requires scheduling to be performed twice. Alternatively, it is possible to apply the MVE-sensitive scheduler on the original loop and obtain a schedule that does not require MVE. However, there are some loops which may inherently require MVE; e.g., this happens if the DDG of the loop consists of forward edges with dependence distance greater than one [8]. In these loops it is not possible to get a schedule that does not require MVE without unrolling it. Hence, we first estimate the min-

imum *unroll_factor* using the mindist relation as:

$$\text{Estimated unroll_factor} = \max_{\text{all edges}} \left\lceil \frac{\text{Min Lifetime}}{\text{MII}} \right\rceil$$

where minimum lifetime is computed using the mindist relation. The loop is then unrolled by the Estimated unroll_factor and the MVE-sensitive scheduler is applied to it. Hence, this approach eliminates the preliminary scheduling step.

The performance of this approach was measured for machine Model 3 for the 718 loops which needed MVE. The results showed that using this approach only 576 of the 718 loops could be scheduled and out of the 576 loops only in 20 of the loops the **II** for the original loop was better than the **II** obtained by using Huff's original approach. In all the remaining loops (556 loops), Huff's method produced schedules with a lower **II**.

6. Conclusions

Modulo-variable expansion is an essential, compile-time renaming technique required for certain modulo scheduled loops. In this paper we propose a technique to improve the constructed schedule while performing modulo-variable expansion. We improve, in terms of **II**, the schedule by unrolling the data dependence graph (DDG) of the loop and re-scheduling it rather than simply unrolling the *schedule* as in the traditional approach. We have derived the conditions that the scheduler which schedules the unrolled DDG must adhere to. These conditions ensure that our MVE-sensitive scheduler which schedules the unrolled DDG is at least as efficient as the traditional approach.

We have tested the approach for 1450 standard benchmarks and for three machine configurations. We have shown that the MVE-sensitive scheduler finds schedules at a lower **II** for about 6% of the loops. In those loops, there is an average improvement in **II** from 12% to 23%.

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