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Probing $SO(10)$ symmetry breaking patterns through sfermion mass relations

B. Ananthanarayan
Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India

P. N. Pandita
Department of Physics, North-Eastern Hill University, Shillong 793 022, India

We consider supersymmetric $SO(10)$ grand unification where the unified gauge group can break to the Standard Model gauge group through different chains. The breaking of $SO(10)$ necessarily involves the reduction of the rank, and consequent generation of non-universal supersymmetry breaking scalar mass terms. We derive squark and slepton mass relations, taking into account these non-universal contributions to the sfermion masses, which can help distinguish between the different chains through which the $SO(10)$ gauge group breaks to the Standard Model gauge group. We then study some implications of these non-universal supersymmetry breaking scalar masses for the low energy phenomenology.

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I. INTRODUCTION

Despite its stupendous success, the gauge group $SU(3) \times SU(2) \times U(1)$ remains a completely unexplained feature of the Standard Model (SM) of electroweak and strong interactions. The idea of grand unification [1] is, therefore, one of the most compelling theoretical ideas that goes beyond the Standard Model. In grand unified theories (GUT's), the SM gauge group can be elegantly unified into a simple group. Moreover, the fermion content of the SM model can be accomodated in irreducible representations of the unified gauge group. Also, one can understand the smallness of neutrino masses via the seesaw mechanism [2] in some of the grand unified models like $SO(10)$ [3] . The renormalization group flow of the gauge couplings leads to their unification at a very large scale [4].

This picture of physics beyond the SM leads to the well-known hierarchy problem due to the widely separated scales, the weak scale $\sim M_Z$, and the large unification scale characterizing by the gauge coupling unification. It has been argued that in supersymmetric [5] extensions of the standard model [6], the hierarchy between the two scales can be made technically natural. This leads us to the idea of supersymmetric grand unification. In supersymmetric GUTS, supersymmetry raises the GUT prediction for $\sin^2\theta_W$ [7], which becomes very close to the current measurement. One of the most important predictions of grand unification is that, because of the presence of baryon number violating interactions, the proton must decay. Since supersymmetry raises the GUT scale, the proton lifetime can be long enough to be consistent with experiment [8].

Presently the hope is that most of the supersymmetric particle spectrum would be observed at the Large Hadron Collider (LHC). One could then have detailed information on the properties, especially the masses, production mechanisms and decays of the sparticles either at LHC or a future Linear Collider (LC). The question can then be posed as to what one can infer from this information about grand unification, and in particular, whether the masses of the sparticles, and their interrelationships, can provide us with a clue as to the nature of the grand unified gauge group and its spontaneous breaking to the SM gauge group. Indeed one may go as far as to ask whether the pattern of sparticle masses can rule out simple grand unification.

In this work we try to address this question in detail. We recall that the simplest grand unified theory into which the SM can be embedded is the $SU(5)$ grand unified theory [1]. The rank of $SU(5)$ is the same as that of the SM gauge group. On the other hand, SM can also be embedded into a larger gauge group like $SO(10)$. However, since the rank of $SO(10)$ is higher than the SM gauge group, the breaking of $SO(10)$ to the SM gauge group involves the reduction of the rank by one unit. Thus, in the case of $SO(10)$ unification there will be $D-$term contributions to the soft supersymmetry breaking scalar masses. In general $D-$term contributions to the SUSY breaking soft scalar masses arise whenever a gauge symmetry is spontaneously broken with a reduction of rank [9]. These $D-$term contributions have important phenomenological consequences at low energies as they allow one to reach certain regions of parameter space which are not otherwise accessible with universal boundary conditions [10, 11]. Non-universality, and in particular the $D-$term contributions, may have a dramatic impact on certain sum rules [12] satisfied by the squark and slepton masses. Such effects are likely to help distinguish between different scenarios for breaking of grand unified symmetry at high energies [13, 14, 15].

In a recent work [16] we addressed the question of D-term non-universality in the context of $SO(10)$ unified gauge group when it breaks to the SM gauge group via one of its maximal subgroups $SU(5)\times U(1)$. In the present paper we systematically consider the $D-$ term non-universality that is generated in $SO(10)$ unification when it breaks to the SM gauge group via any of its maximal subgroups. Since no such contributions are generated in $SU(5)$ unification, $SO(10)$ is one of the two (E_6 being the other) supersymmetric grand unified theories in four dimensions where such contributions can arise. In Section II, we discuss in detail the embedding of the Standard Model in $SO(10)$ grand unified gauge group, and study the different chains through which it can break to the SM gauge group. Here we discuss why the embedding of SM into $SO(10)$ can be done in more than one way. In Section III we consider the renormalization group equations and their solutions for $SO(10)$ breaking into the SM for different chains of breaking. Using these solutions, we derive characteristic relations between the sfermion masses which hold for different patterns of $SO(10)$ breaking into the SM gauge group. In Section IV we carry out a numerical analysis of the renormalization group evolution for the $SO(10)$ breaking into the SM gauge group, and the implications of this evolution for the low energy phenomenology. In this Section we also address the question of naturalness of the large values of $\tan\beta$ in the context of grand unified $SO(10)$ models with non-universal D-term contributions as well as certain other kinds of non-universality. We conclude the paper with a summary and some remarks.

II. EMBEDDING OF SM IN $SO(10)$

As pointed out in the Introduction, the SM can be embedded into a larger gauge group, where an entire SM generation can be fitted into a single irreducible representation of the underlying gauge group. Indeed, there is chain of group embeddings [17]

$$SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8. \tag{1}$$

However, in four-dimensional grand unified theories the gauge groups E_7 and E_8 do not support a chiral structure of the weak interactions, and hence cannot be used as grand unified gauge groups. This leaves out only the three groups, $SU(5)$, $SO(10)$, and E_6 as possible grand unified gauge groups in four dimensions. The gauge group $SO(10)$ appears at present to be the most attractive because it contains an entire SM generation in the fundamental representation. Furthermore, one has as bonus the right handed neutrino, necessary to generate neutrino masses, sitting in the same fundamental representation. Also, a complex **10** dimensional representation of $SO(10)$ can be employed to accomodate the two Higgs doublets of the low energy minimal supersymmetric standard model. However, since the rank of $SO(10)$ is one unit higher than the SM gauge group, it leads to D-term contributions to the soft scalar masses at the scale of symmetry breaking. These D-term contributions will depend on the manner in which the $SO(10)$ gauge group is broken to the SM gauge group. In order to study the implications of these D-term contributions for the phenomenology, we shall, in the following, discuss the breaking of $SO(10)$ in detail.

We start by recalling that when $SO(10)$ breaks via its maximal subgroup $SU(5)\times U(1)_Z$, with $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_X$, there are two possibilities for the hypercharge generator of the SM gauge group. In the "conventional" embedding via $SU(5)$, the hypercharge generator Y of the SM is identified with the generator X of $U(1)_X$. On the other hand, in the "flipped" embedding the hypercharge generator is identified with a linear combination of the generators X and Z.

Apart from the "natural" subgroup $SU(5)\times U(1)$, the group $SO(10)$ also has "natural" subgroup $SO(6)\times SO(4)$. Since $SO(6)$ is isomorphic to $SU(4)$, and $SO(4)$ is isomorphic to $SU(2)\times SU(2)$, $SO(10)$ contains [18] the group

$SU(4)\times SU(2)\times SU(2)$. We shall focuss on the signatures of the $SO(10)$ breaking via its two natural subgroups, and try to find distinguishing features of the sparticle spectrum in the two cases.

In $SO(10)$ grand unification, all the matter particles of one family of the Standard Model (SM) together with a right handed neutrino belong to the spinor representation **16**. Each such spinor representation **16** and each **10**-dimensional representation can be decomposed under the maximal subgroup $SO(10)\supset SU(5)\times U(1)_Z$ as

$$\mathbf{16} = \mathbf{5^*}_3 + \mathbf{10}_{-1} + \mathbf{1}_{-5}, \tag{2}$$

$$\mathbf{10} = \mathbf{5}_2 + 5^*{}_{-2}. \tag{3}$$

Furthermore under $SU(5)\supset SU(3)_C\times SU(2)_L\times U(1)_X$, we have the decomposition

$$\mathbf{5} = (\mathbf{3},\mathbf{1})_{-2} + (\mathbf{1},\mathbf{2})_3, \tag{4}$$

$$\mathbf{5^*} = (\mathbf{3^*},\mathbf{1})_2 + (\mathbf{1},\mathbf{2})_{-3}, \tag{5}$$

$$\mathbf{10} = (\mathbf{3},\mathbf{2})_1 + (\mathbf{3^*},\mathbf{1})_{-4} + (\mathbf{1},\mathbf{1})_6, \tag{6}$$

$$\mathbf{1} = (\mathbf{1},\mathbf{1})_0. \tag{7}$$

We note that $U(1)_X$, which is the subgroup of $SU(5)$, is not identical with the $U(1)_Y$ of the SM at this stage. Note also that each **16** includes two pairs of $(\mathbf{3^*},\mathbf{1})$ and $(\mathbf{1},\mathbf{1})$.

In order to identify the hypercharge group, we consider the decomposition $SO(10)\supset SU(5)\times U(1)_Z\supset SU(3)_C\times SU(2)_L\times U(1)_X\times U(1)_Z$. Therefore, the hypercharge $U(1)_Y$ must be a linear combination of $U(1)_X$ and $U(1)_Z$, i.e. $U(1)_Y\subset U(1)_X\times U(1)_Z$. Thus, there are two ways to define the hypercharge generator of the SM:

$$Y = X, \tag{8}$$

$$Y = -\frac{1}{5}(X+6Z), \tag{9}$$

upto an overall normalization factor. The first case corresponds to the the Georgi-Glashow model [1], whereas the second identification of hypercharge corresponds to the flipped case [19, 20].

In the first case the $U(1)$ generator of $SO(10)$ that is orthogonal to Y and the diagonal generators of $SU(3)_C$ and $SU(2)_L$ is

$$Y^\perp = -Z, \tag{10}$$

whereas in the flipped case we have for the orthogonal generator[16]

$$Y^\perp = \frac{-4X+Z}{5}, \tag{11}$$

After a suitable identification of the fields lying in the relevant representations of $SO(10)$, the effect of $SO(10)$ breaking at the unification scale leads to D-term non-universality, which is computed in terms of the eigenvalues of the operator $Y^\perp$ on the fields. We will discuss this in the next section.

We now come to the case of $SO(10)$ breaking via the Pati-Salam subgroup. The breaking pattern to the SM gauge group is

$$SO(10)\xrightarrow{M_U} SU(4)_{PS}\times SU(2)_L\times SU(2)_R \xrightarrow{M_{PS}} SU(3)_C\times SU(2)_L\times U(1)_Y. \tag{12}$$

The decomposition of **16** and **10** of $SO(10)$ under the maximal subgroup $SO(10)\supset SU(4)_{PS}\times SU(2)_L\times SU(2)_R$ is given by

$$\mathbf{16} = (\mathbf{4},\mathbf{2},\mathbf{1}) + (\mathbf{4^*},\mathbf{1},\mathbf{2}), \tag{13}$$

$$\mathbf{10} = (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}). \tag{14}$$

Furthermore, under $SU(4)_{PS} \times SU(2)_L \times SU(2)_R \supset SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_V$ we have the decomposition

$$(\mathbf{4},\mathbf{2},\mathbf{1}) = (\mathbf{3},\mathbf{2},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1}, \quad (15)$$
$$(\mathbf{4}^*,\mathbf{1},\mathbf{2}) = (\mathbf{3}^*,\mathbf{1},\mathbf{2})_{-1/3} + (\mathbf{1},\mathbf{1},\mathbf{2})_{1}, \quad (16)$$
$$(\mathbf{6},\mathbf{1},\mathbf{1}) = (\mathbf{3},\mathbf{1},\mathbf{1})_{-2/3} + (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{2/3}, \quad (17)$$
$$(\mathbf{1},\mathbf{2},\mathbf{2}) = (\mathbf{1},\mathbf{2},\mathbf{2})_{0}. \quad (18)$$

The decomposition (16) shows that $(\mathbf{3}^*,\mathbf{1})$ and $(\mathbf{1},\mathbf{1})$ are $SU(2)_R$ doublets. It is easily seen that the generators $U(1)_Z$ and $U(1)_X$, and the generator $U(1)_V$ are related through

$$Z = -4I_{3R} - 3V, \quad (19)$$
$$6X = -I_{3R} + \frac{1}{2}V. \quad (20)$$

Furthermore, the generator $U(1)_V$ can be identified with the $B-L$:

$$B - L = V = -\frac{1}{5}(Z - 24X), \quad (21)$$

which implies that $U(1)_{B-L}$ subgroup of $SO(10)$ is orthogonal to its $SU(2)_R$ subgroup.

From the decomposition (2) of the **16** of $SO(10)$ under the maximal subgroup $SO(10) \supset SU(5) \times U(1)_Z$ as well as the decomposition (13) under the maximal subgroup $SO(10) \supset SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ we find that the $SU(5)$ multiplets $\mathbf{5}^*$,**10** and **1** have the content

$$\mathbf{5}^*{}_3 = (\mathbf{3}^*,\mathbf{1},I_{3R} = -1/2)_{-1/3} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1}, \quad (22)$$
$$\mathbf{10}_{-1} = (\mathbf{3},\mathbf{2},\mathbf{1})_{1/3} + (\mathbf{3}^*,\mathbf{1},I_{3R} = 1/2)_{-1/3} + (\mathbf{1},\mathbf{1},I_{3R} = -1/2)_{1}, \quad (23)$$
$$\mathbf{1}_{-5} = (\mathbf{1},\mathbf{1},I_{3R} = 1/2)_{1}. \quad (24)$$

under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Similarly, from the decomposition (3) of the **10** of $SO(10)$, we find that the multiplets **5** and $\mathbf{5}^*$ have the content

$$\mathbf{5}_2 = (\mathbf{3},\mathbf{1},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{2},I_{3R} = -1/2)_{0}, \quad (25)$$
$$\mathbf{5}^*{}_{-2} = (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2},I_{3R} = 1/2)_{0}, \quad (26)$$

under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Eqs. (22) - (24) show that the embedding $SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L$ is not unique. As long as $U(1)_Y$ is not defined, there is freedom of $SU(2)_R$ rotation. The hypercharge of the SM is not identical with $U(1)_V$ and must be orthogonal to $SU(3)_C$ and $SU(2)_L$. This fact shows that the $U(1)_Y$ is not orthogonal to $SU(2)_R$. Once the assignment of hypercharge is made, the freedom of $SU(2)_R$ rotation is eliminated. The hypercharge assignments (8) and (9) can now be expressed in terms of the third component of $SU(2)_R$ and the quantum number of $U(1)_V$ as

$$Y = X = -I_{3R} + \frac{1}{2}V, \quad (27)$$

for the Georgi-Glashow model and

$$Y = -\frac{1}{5}(X + 6Z) = I_{3R} + \frac{1}{2}V, \quad (28)$$

for the case of flipped emebedding. We note that hese two assignments differ from each other in only the sign of the third component of $SU(2)_R$. This means that the $SU(5)$ group of flipped model is obtained from that of Georgi-Glashow model by the π rotation in $SU(2)_R$. In other words particle assignment of the flipped $SU(5)$ model is obtained from that of the Georgi-Glashow $SU(5)$ model by "flipping" of the $SU(2)_R$ doublets

$$u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c. \quad (29)$$

Although the way of embedding $SU(5)$ as $SO(10) \supset SU(5) \times U(1)_Z \supset SU(3)_C \times SU(2)_L \times U(1)_Y$ is not unique, the freedom of $SU(2)_R$ rotation is no longer available now . Thus, there are only two possibilities of embedding $SU(5)$ in $SO(10)$, i.e. the Georgi-Glashow $SU(5)$ or the "flipped" $SU(5)$.

III. $SO(10)$ BREAKING AND RENORMALIZATION GROUP EQUATIONS

We now come to the question of the implications of the different patterns of the breaking of supersymmetric $SO(10)$ to the minimal supersymemtric standard model, based on the SM gauge group, for the sparticle spectrum. This can be addressed by studying the renormalization group evolution to the electroweak scale. For the squarks and sleptons of the first and second family (the light generations), the renormalization group (RG) equations for the soft scalar masses are given by

$$
\begin{aligned}
16\pi^2 \frac{dm^2_{\tilde{Q}_L}}{dt} &= -\frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S, && (30)\\
16\pi^2 \frac{dm^2_{\tilde{u}_R}}{dt} &= -\frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S, && (31)\\
16\pi^2 \frac{dm^2_{\tilde{d}_R}}{dt} &= -\frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S, && (32)\\
16\pi^2 \frac{dm^2_{\tilde{L}_L}}{dt} &= -6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S, && (33)\\
16\pi^2 \frac{dm^2_{\tilde{e}_R}}{dt} &= -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S, && (34)
\end{aligned}
$$

where $t \equiv \ln(Q/Q_0)$, with Q_0 being some initial large scale; $M_{3,2,1}$ are the running gaugino masses, $g_{3,2,1}$ are the usual gauge couplings associated with the SM gauge group, with $\alpha_i \equiv g_i^2/4\pi$, and

$$
S \equiv \mathrm{Tr}(Ym^2) = m^2_{H_u} - m^2_{H_d} + \sum_{\text{families}} (m^2_{\tilde{Q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{L}_L} + m^2_{\tilde{e}_R})\,. \qquad (35)
$$

The $U(1)_Y$ gauge coupling g_1 (and α_1) is taken to be in a GUT normalization throughout this paper. The quantity S evolves according to

$$
\frac{dS}{dt} = \frac{66}{5} \frac{\alpha_1}{4\pi} S \qquad (36)
$$

which has the solution

$$
S(t) = S(t_G) \frac{\alpha_1(t)}{\alpha_1(t_G)}. \qquad (37)
$$

We note that if $S = 0$ at the initial scale, which would be the case if all the soft sfermion and Higgs masses are same, then the RG evolution will maintain it to be zero at all scales.

The solution for the renormalization group equations (30)–(34) can then be written as

$$
\begin{aligned}
m^2_{\tilde{u}_L}(t) &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36} C_1 + (\frac{1}{2} - \frac{2}{3} \sin^2\theta_W) M_Z^2 \cos(2\beta) - \frac{1}{5} K, && (38)\\
m^2_{\tilde{d}_L}(t) &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36} C_1 + (-\frac{1}{2} + \frac{1}{3} \sin^2\theta_W) M_Z^2 \cos(2\beta) - \frac{1}{5} K, && (39)
\end{aligned}
$$

$$m^2_{\tilde{u}_R}(t) \;=\; m^2_{\tilde{u}_R}(t_G) + C_3 + \frac{4}{9}C_1 + \frac{2}{3}\sin^2\theta_W M_Z^2\cos(2\beta) + \frac{4}{5}K, \tag{40}$$

$$m^2_{\tilde{d}_R}(t) \;=\; m^2_{\tilde{d}_R}(t_G) + C_3 + \frac{1}{9}C_1 - \frac{1}{3}\sin^2\theta_W M_Z^2\cos(2\beta) - \frac{2}{5}K, \tag{41}$$

$$m^2_{\tilde{e}_L}(t) \;=\; m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + (-\frac{1}{2} + \sin^2\theta_W) M_Z^2\cos(2\beta) + \frac{3}{5}K, \tag{42}$$

$$m^2_{\tilde{\nu}_L}(t) \;=\; m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + \frac{1}{2}M_Z^2\cos(2\beta) + \frac{3}{5}K, \tag{43}$$

$$m^2_{\tilde{e}_R}(t) \;=\; m^2_{\tilde{e}_R}(t_G) + C_1 - \sin^2\theta_W M_Z^2\cos(2\beta) - \frac{6}{5}K, \tag{44}$$

where C_1, C_2 and C_3 are given by

$$C_i(t) = \frac{a_i}{2\pi^2}\int_t^{t_G} dt\; g_i(t)^2\; M_i(t)^2,\; i = 1,2,3 \tag{45}$$

$$a_1 = \frac{3}{5}, a_2 = \frac{3}{4}, a_3 = \frac{4}{3}, \tag{46}$$

and

$$K = \frac{1}{16\pi^2}\int_t^{t_G} g_1^2(t)\; S(t)\; dt = \frac{1}{2b_1}S(t)\left[1 - \frac{\alpha_1(t_G)}{\alpha_1(t)}\right], \tag{47}$$

is the contribution of the non-universality parameter S to the sfermion masses, and $b_1 = -33/5$.

The solutions of the RG equations for the soft scalar masses given above involve the values of these masses at the initial scale (GUT scale). These initial values will be determined by the pattern of the breaking of the grand unified group to the SM gauge group. For the case of direct breaking of $SO(10)$ to the Standard Model gauge group, these initial values are given by

$$m^2_{\tilde{Q}_L}(t_G) \;=\; m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{16} + g^2_{10}D, \tag{48}$$

$$m^2_{\tilde{L}_L}(t_G) \;=\; m^2_{\tilde{d}_R}(t_G) = m^2_{16} - 3g^2_{10}D, \tag{49}$$

$$m^2_{H_u}(t_G) \;=\; m^2_{10} - 2g^2_{10}D, \tag{50}$$

$$m^2_{H_d}(t_G) \;=\; m^2_{10} + 2g^2_{10}D, \tag{51}$$

at the $SO(10)$ breaking scale M_G, where the normalization and sign of D is arbitrary. Here m_{16} and m_{10} are the common soft scalar masses, corresponding to the **16** and **10** dimensional representations, respectively of $SO(10)$, at the unification scale. We note here that in the breaking of $SO(10)$ the rank is reduced by one, and hence the D-term contribution to the soft masses is expressed by a single parameter D. these intial values in the solutions (38) - (44) of the renormalization equations, and eliminating the quantities m^2_{16}, g^2_{10}, and D, we obtain the following two sum rules for the sfermion masses:

$$2m^2_{\tilde{Q}} - m^2_{\tilde{u}_R} - m^2_{\tilde{e}_R} \;=\; (C_3 + 2C_2 - \frac{25}{18}C_1), \tag{52}$$

$$m^2_{\tilde{Q}} + m^2_{\tilde{d}_R} - m^2_{\tilde{e}_R} - m^2_{\tilde{L}} \;=\; (2C_3 - \frac{10}{9}C_1), \tag{53}$$

where we have used the notation

$$m^2_{\tilde{Q}} = \frac{1}{2}(m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L}),\;\; m^2_{\tilde{L}} = \frac{1}{2}(m^2_{\tilde{e}_L} + m^2_{\tilde{\nu}_L}).$$

We note that g_{10}^2 and D enter in the combination $g_{10}^2 D$ in the initial conditions (48) - (51), and therefore constitute only one parameter. We note from the above that $S(t_G) = -4g_{10}^2 D$. The solution for K is obtained by eliminating C_1, C_2, C_3, m_{16}^2, and m_{10}^2 from the sfermion mass equations. For the case of direct breaking of $SO(10)$ to the SM gauge group, we have

$$K = -\frac{1}{4}(m_{\tilde{Q}}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 + m_{\tilde{e}_R}^2 - m_{\tilde{L}}^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta). \tag{54}$$

The right hand side of the sum rules (52) and (53) involve the functions $C_i(t)$. These functions can be written in terms of quantities whose values can be inferred from experiment. In terms of the gluino mass $M_{\tilde{g}} = M_3(t_{\tilde{g}})$, we can write $C_3(t)$ from (45) as

$$C_3(t) = \frac{8}{9}\frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})}[\alpha_3^2(t) - \alpha_3^2(t_G)], \tag{55}$$

where we have used the fact that gaugino masses run as

$$\frac{M_i(t)}{\alpha_i(t)} = \frac{M_i(t_G)}{\alpha_i(t_G)}. \tag{56}$$

In an underlying grand unified theory, we can require that all three gaugino masses be same at the high mass scale M_G so that $M_i(t_G) \equiv m_{1/2}$. We then have

$$\frac{M_1(t)}{\alpha_1(t)} = \frac{M_2(t)}{\alpha_2(t)} = \frac{M_3(t)}{\alpha_3(t)} = \frac{m_{1/2}}{\alpha_G}, \tag{57}$$

where $\alpha_1(t_G) = \alpha_2(t_G) = \alpha_3(t_G) \equiv \alpha_G$ is the grand unified gauge coupling. We note that the gaugino masses always satisfy the relation (57) irrespective of the breaking pattern [13] to the Standard Model gauge group if the underlying gauge group is unified into a simple group at a high mass scale M_G. We further note that (57) is a result of one-loop renormalization group equations, and does not hold at the two loop level [21]. However, the two-loop effect is numerically small [22]. From (56) it follows that

$$M_i(t) = \alpha_i(t)\frac{M_{\tilde{g}}}{\alpha_3(\tilde{g})}. \tag{58}$$

Using the above, we can now express the functions C_1 and C_2 in terms of the gluino mass and the corresponding gauge couplings. We have[12]

$$C_1(t) = \frac{2}{11}\frac{M_{\tilde{g}}^2}{\alpha_3^2(\tilde{g})}[\alpha_1^2(t_G) - \alpha_1^2(t)], \tag{59}$$

$$C_2(t) = \frac{3}{2}\frac{M_{\tilde{g}}^2}{\alpha_3^2(\tilde{g})}[\alpha_2^2(t_G) - \alpha_2^2(t)]. \tag{60}$$

We note here that the gluino mass in (55), (59) and (60) is the one-loop gluino mass and not the pole mass, although these are related. Using these results for C_i, we can write the sum rules (52) and (53) as follows:

$$2m_{\tilde{Q}}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 = \frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})}[\frac{8}{9}\alpha_3^2(t) - 3\alpha_2^2(t) + \frac{25}{99}\alpha_1^2(t) + \frac{184}{99}\alpha_G^2], \tag{61}$$

$$m_{\tilde{Q}}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{e}_R}^2 - m_{\tilde{L}}^2 = \frac{M_{\tilde{g}}^2}{\alpha_3^2(t_{\tilde{g}})}[\frac{16}{9}\alpha_3^2(t) + \frac{20}{99}\alpha_1^2(t) - \frac{196}{99}\alpha_G^2]. \tag{62}$$

Using a supersymmetric threshold of 1 TeV, and the values $M_G = 1.9 \times 10^{16}$ GeV, $\alpha_G = 0.04$, $\alpha_1(1\ TeV) = 0.0173, \alpha_2(1\ TeV) = 0.0328, \alpha_3(1\ TeV) = 0.091$, we can finally write our sum rules in terms of experimentally measurable masses as (at a scale of 1 TeV)

$$2m^2_{\tilde{Q}} - m^2_{\tilde{u}_R} - m^2_{\tilde{e}_R} \ = \ 0.85 M^2_{\tilde{g}}, \tag{63}$$

$$m^2_{\tilde{Q}} + m^2_{\tilde{d}_R} - m^2_{\tilde{e}_R} - m^2_{\tilde{L}} \ = \ 1.42 M^2_{\tilde{g}}. \tag{64}$$

We now come to the case of $SO(10)$ breaking via its other maximal subgroup $SO(10) \supset SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ As in the case of breaking via the $SU(5)$ subgroup, there appear D-term contributions to the soft scalar masses when the rank of the gauge group reduces from 5 to 4 at the intermediate Pati-Salam symmetry breaking scale M_{PS}. As discussed in Section II matter multiplets belong either to $L = (\mathbf{4}, \mathbf{2}, \mathbf{1})$ or $R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ representations, with masses m^2_L and m^2_R respectively above M_{PS}. When the $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ group breaks to G_{SM}, we obtain the following masses

$$m^2_{\tilde{Q}_L}(M_{PS}) \ = \ m^2_L + g^2_4 D, \tag{65}$$

$$m^2_{\tilde{u}_R}(M_{PS}) \ = \ m^2_R - (g^2_4 - 2g^2_{2R})D, \tag{66}$$

$$m^2_{\tilde{e}_R}(M_{PS}) \ = \ m^2_R + (3g^2_4 - 2g^2_{2R})D, \tag{67}$$

$$m^2_{\tilde{L}_L}(M_{PS}) \ = \ m^2_L - 3g^2_4 D, \tag{68}$$

$$m^2_{\tilde{d}_R}(M_{PS}) \ = \ m^2_R - (g^2_4 + 2g^2_{2R})D, \tag{69}$$

at the Pati-Salam breaking scale. Here D represents the D-term contributions whose normalization is arbitrary. We note that these expressions do not depend on a particular choice of the Higgs representation which breaks the Pati-Salam group, and is fixed only by the symmetry breaking pattern. We further note that the gauge coupling g^2_4, g^2_{2R} can be determined from the low-energy gauge coupling $\alpha_i(m_Z)$ $(i = 1, 2, 3)$ as a function of M_{PS} alone. The solutions (38) - (44) of the renormalization group equations (30) - (34), together with the boundary conditions (65) - (69) lead to the sum rule

$$\begin{aligned} m^2_{\tilde{Q}} + m^2_{\tilde{d}_R} - m^2_{\tilde{e}_R} - m^2_{\tilde{L}} \ &= \ (2C_3 - \frac{10}{9}C_1) \\ &= \ \frac{M^2_{\tilde{g}}}{\alpha^2_3(t_{\tilde{g}})}[\frac{16}{9}\alpha^2_3(t) + \frac{20}{99}\alpha^2_1(t) - \frac{196}{99}\alpha^2_G], \end{aligned} \tag{70}$$

which is the only sum rule valid in this case. The notation is same as in the case of direct breaking of $SO(10)$ to the SM gauge group. As in the case of direct breaking of $SO(10)$, this can be written as

$$m^2_{\tilde{Q}} + m^2_{\tilde{d}_R} - m^2_{\tilde{e}_R} - m^2_{\tilde{L}} = 1.42 M^2_{\tilde{g}}. \tag{71}$$

Thus, this sum rule serves as a crucial distinguishing feature of $SO(10)$ breaking via the Pati-Salam subgroup. If both the sum rules (63) and (64) are seen to hold experimentally, then in the context of $SO(10)$ unification, the breaking of $SO(10)$ takes place directly to the SM gauge group. On the other hand, if only the sum rule (71) is seen to hold experimentally, then the breaking of $SO(10)$ must take place via the Pati-Salam subgroup. We recall the relation (57) that has been used in the derivation of (71) is valid irrespective of the breaking pattern [13] to the Standard Model gauge group if the underlying gauge group is unified into a simple group at a high mass scale M_G. Furthermore, we note that the parameter K cannot be determined in the case of breaking via the Pati-Salam subgroup.

It is important to distinguish between the situation where there is a grand unification of the SM gauge group into a simple group like $SO((10)$, and the situation where there is no such unification into a simple group. A typical example of the latter case is the flipped $SU(5) \times U(1)$ model which is not grand-unified into a simple group. In this case we have two independent gauge couplings $g_{SU(5)}$ and $g_{U(1)}$ at the GUT scale. On the other hand there are three soft scalar masses ${m_{10}}^2$, ${m_{\bar{5}}}^2$, and ${m_1}^2$ at the GUT scale. In addition there is the unknown D-term. The intial values of the soft scalar masses are given by

$$
\begin{aligned}
m^2_{\tilde{Q}_L}(t_G) &= {m_{10}}^2 + \left(\frac{1}{10}{g_{SU(5)}}^2 + \frac{1}{40}{g_{U(1)}}^2\right) D, && (72)\\
m^2_{\tilde{u}_R}(t_G) &= {m_{\bar{5}}}^2 + \left(\frac{1}{5}{g_{SU(5)}}^2 - \frac{3}{40}{g_{U(1)}}^2\right) D, && (73)\\
m^2_{\tilde{e}_R}(t_G) &= {m_1}^2 + \frac{1}{8}{g_{U(1)}}^2 D, && (74)\\
m^2_{\tilde{L}_L}(t_G) &= {m_{\bar{5}}}^2 - \left(\frac{3}{10}{g_{SU(5)}}^2 + \frac{3}{40}{g_{U(1)}}^2\right) D, && (75)\\
m^2_{\tilde{d}_R}(t_G) &= {m_{10}}^2 - \left(\frac{2}{5}{g_{SU(5)}}^2 - \frac{1}{40}{g_{U(1)}}^2\right) D. && (76)
\end{aligned}
$$

In this case, eliminating the unknown soft mass parameters ${m_{10}}^2$, ${m_{\bar{5}}}^2$, and ${m_1}^2$, the gauge couplings $g_{SU(5)}$ and $g_{U(1)}$, and the parameter D from the solutions (38) - (44) of the renormalization group equations (30) - (34), together with the boundary conditions (72) - (76), we get the sum rule

$$
m^2_{\tilde{Q}} - m^2_{\tilde{u}_R} - m^2_{\tilde{d}_R} + m^2_{\tilde{L}} = -(C_3 - 2C_2 + \frac{5}{18}C_Y), \qquad (77)
$$

where, to avoid confusion, we have denoted the function C corresponding to the $U(1)_Y$ subgroup of the Standard Model as C_Y (with the usual GUT normalization). That this can be done is a consequence of the general argument for elimination of D-terms.

We can now try to write the right hand side of (77) in terms of measurable quantities, just as we did in the case of the unified gauge group $SO(10)$. To do this, we note that the value of $g_{SU(5)}$ at the scale M_G is the same as in the case of a grand unified supersymmetric gauge theory like $SO(10)$, i.e. $g^2_{SU(5)}(M_G)/(4\pi) = \alpha_G$, since it is determined by the evolution of $SU(3)$ and $SU(2)$ gauge couplings, which is unaltered. This implies

$$
\frac{M_2(t)}{\alpha_2(t)} = \frac{M_3(t)}{\alpha_3(t)} = \frac{m_{1/2}}{\alpha_G}. \qquad (78)
$$

Furthermore, we can again identify the gluino mass as $M_{\tilde{g}} = M_3(t_{\tilde{g}})$. We then have

$$
\begin{aligned}
C_3(t) &= \frac{8}{9}\frac{M^2_{\tilde{g}}}{\alpha^2_3(t_{\tilde{g}})}[\alpha^2_3(t) - \alpha^2_3(t_G)], && (79)\\
C_2(t) &= \frac{3}{2}\frac{M^2_{\tilde{g}}}{\alpha^2_3(\tilde{g})}[\alpha^2_2(t_G) - \alpha^2_2(t)]. && (80)
\end{aligned}
$$

On the other hand, we can write the function C_Y as

$$
C_Y(t) = \frac{2}{11}\frac{M^2_Y(t)}{\alpha^2_Y(t)}[\alpha^2_Y(t_G) - \alpha^2_Y(t)], \qquad (81)
$$

where we have denoted the soft gaugino mass corresponding to the $U(1)_Y$ gauge group as M_Y. Using (79), (80) and (81) we can write the sum rule (77) as

$$
m^2_{\tilde{Q}} - m^2_{\tilde{u}_R} - m^2_{\tilde{d}_R} + m^2_{\tilde{L}} = -\frac{M^2_{\tilde{g}}}{\alpha^2_3(t_{\tilde{g}})}[\frac{8}{9}\alpha^2_3(t) + 3\alpha^2_2(t) - \frac{35}{9}\alpha^2_G] - \frac{5}{99}\frac{M^2_Y(t)}{\alpha^2_Y(t)}[\alpha^2_Y(t_G) - \alpha^2_Y(t)]. \qquad (82)
$$

We note that the gaugino mass parameter M_Y can be extracted from the experimental measurements in the neutalino sector [23], so that the sum rule (82) can be tested. Thus, this sum rule could serve to distinguish the flipped ununified $SU(5) \times U(1)$ model from the unified $SO(10)$ model.

To sum up this Section, we have shown that there are characteristic sum rules obeyed by the sfermion masses when $SO(10)$ breaks to the SM gauge group via different breaking chains. We have also shown that in the case the SM does not unify into a simple gauge group, there is a sum rule which can help distinguish such a situation from the one where the SM unfies into a simple group at the grand unified scale.

IV. NUMERICAL RESULTS

In this Section we consider the phenomenological implications of the D-term contributions that arise in the breaking of $SO(10)$ to the SM gauge group, as well as typical non-universality associated with the Pati-Salam subgroup for purposes of illustration. Recalling here that one of the most attractive pictures of unification is the one where there is gauge coupling unification at a single scale, and in which Yukawa couplings of the heaviest generation have a common value. This also fixes the value of the hitherto unknown parameter $\tan\beta \equiv < H_u^0/H_d^0 >$ (the ratio of the vacuum expectation values of the two Higgs doublets of the minimal supersymmetric standard model (MSSM)) at the theoretically attractive value of $\sim m_t/m_b$. Furthermore, this framework also provides a candidate for the cold dark matter [24] of the universe in the form of a bino-like lightest supersymmetric particle [25]. However, this simple picture is not realised when threshold corrections, that depend crucially on the details of the spectrum, are taken into account [26]. Furthermore, it is well known that the model has problems of naturalness when the tree level Higgs potential is considered, although arguments have been presented to show that one-loop corrections might alleviate the problem [27]. In other words, one may have to give up exact unification, but still have large values of $\tan\beta$ and approximate unification, or alternatively constrain the parameter space significantly by demanding exact unification. Our approach here will be to take the simplest possible set of assumptions and study the implications of these for the phenomenology. This could, then, become a basis for further studies, and could become important when the supersymmetric particles are discovered experimentally.

In order to implement the above picture, we carry out numerical integration of the renormalization group equations for the gauge couplings, Yukawa couplings, the gaugino masses, the supersymmetry breaking soft scalar mass squared parameters and the soft trilinear couplings of the MSSM with $SO(10)$ breaking boundary conditions. For definiteness, we shall consider the case of $SO(10)$ breaking via the Pati-Salam subgroup, since this case has more parameter freedom. We wish to retain those aspects of unification that are approximately valid. Motivated by $SO(10)$ unification, these include a unified gauge coupling at M_X, and a unified Yukawa coupling for the heaviest generation. For all parameters except the mass squared parameters, which we study case by case, we assume universal boundary conditions. Starting with values of the common gaugino mass ($M_{1/2}$), trilinear couplings (A), with the third generation Yukawa couplings having a common value h at the unification scale, and the unified gauge couplingα_G, we integrate the set of coupled renormalization group equations from the $SO(10)$ breaking scale down to the effective supersymmetry breaking scale of ~ 1 TeV. At this scale, the parameters in the Higgs potential, after being evolved from the GUT breaking scale, must be such that the electroweak symmetry is broken. As is well known, one of the conditions for this to happen is

$$\frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} = \frac{m_Z^2}{2}, \tag{83}$$

where $\mu_1^2 = m_{H_d}^2 + \mu^2$ and $\mu_2^2 = m_{H_u}^2 + \mu^2$, with $m_{H_u}^2$ and $m_{H_d}^2$ being soft supersymmetry breaking Higgs mass squared parameters, and μ the supersymmetry conserving Higgs(ino) mass parameter. Proceeding in the by now well-known fashion [25] of determining $\tan\beta$ from the accurately known value of the τ-lepton mass, inserting it into the above equation, and using the values of the Higgs mass squared parameters determined from the RG evolution, yields the parameter μ. We chose the sign of D to be positive. This alleviates the problem of fine-tuning, inherent in $SO(10)$ unification, by allowing $m_{H_u}^2$ to evolve to values that are negative and larger in magnitude, compared to the situation when the D-term is absent.

We note that for the universal mass squared case, sufficiently large values of the common gaugino mass $M_{1/2}$ are required to ensure that the gluino is sufficiently heavy, and also fairly large values of the common soft scalar mass m_0 ($< M_{1/2}$) are required to ensure that the neutralino (and not the lightest slepton) is the LSP. The near degeneracy of the two slepton states in the absence of electroweak symmetry breaking makes the mixing between them, once $SU(2) \times U(1)$ is broken, significant. Indeed, it is important to observe the variation of the mass of the lighest slepton, since it has the tendency to become lighter than the lightest neutralino and to emerge as a candidate for the LSP, which is not acceptable. An upper bound on m_0 ensues when we require a sufficiently large $m_A \equiv \mu_1^2 + \mu_2^2 (\gtrsim M_Z)$. Keeping these features in mind, we study the effects of the $D-$term on the spectrum.

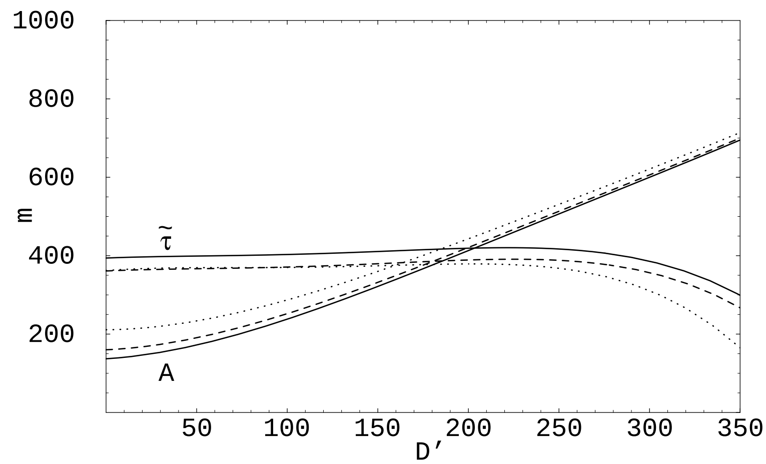

FIG. 1: Values of m_A and lighter stau ($\tilde{\tau}$) mass plotted as a function of D' with $M_{1/2} = 800, A = 0$ and $\delta m_0^2 = -(200)^2$ for the case of $SO(10)$ breaking via Pati-Salam subgroup. The solid line corresponds to $m_0 = 700, h_t = h_b = h_\tau = 2$, dashed line corresponds to $m_0 = 700, h_t = h_b = h_\tau = 3$, and dotted line to $m_0 = 600, h_t = h_b = h_\tau = 2$. All masses are in GeV.

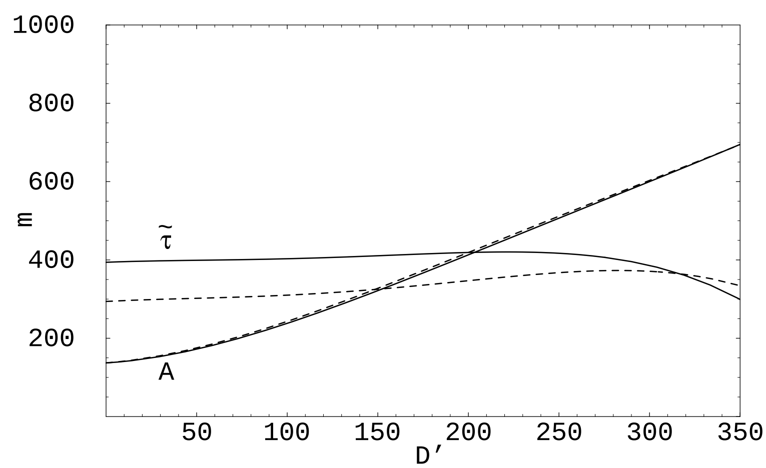

FIG. 2: Values of m_A and lighter stau ($\tilde{\tau}$) mass plotted as a function of D' for $M_{1/2} = 800, A = 0$ and $m_0 = 700, h_t = h_b = h_\tau = 2$ for the case of $SO(10)$ breaking via Pati-Salam subgroup. Here the solid line corresponds to $\delta m_0^2 = -(200)^2$ and dashed line corresponds to $\delta m_0^2 = 200^2$.

We start by determining the low-energy sparticle spectrum characteristics for a set of typical boundary conditions appropriate to the breaking of $SO(10)$ via the Pati-Salam subgroup. For this case, we have imposed the condition $g_4 = g_{2R} = g_{2L} (= g_{10})$ at the GUT scale. We consider the following cases: $m_h^2 = m_0^2$, $m_R^2 = m_0^2 - \delta m_0^2$, $m_L^2 = m_0^2 + \delta m_0^2$. With these boundary conditions, we expect significant changes in the masses of the squarks compared to the situation when there are universal boundary conditions for the squark masses. What is relevant here, however, is the influence on the masses of the stau's, because of the mixing between the left and right states after electroweak symmetry breaking. Normally part of the allowed parameter space is ruled out since the lighter stau tends to become lighter than the lightest neutralino, which is unacceptable on phenomenological grounds. We illustrate these features through our numerical results.

In Fig. 1, we illustrate the variation of the low energy observables as the parameter $D' \equiv \sqrt{g_{10}^2 D}$ is varied in the range of 0 to 350 GeV, where δm_0^2 is taken to be $-(200\mathrm{GeV})^2$, for a typical parameter choice of $M_{1/2} = 800$, $m_0 = 700$, $A_0 = 0$ in units of GeV, with the common Yukawa coupling taking the value $h_t = h_b = h_\tau = 2.0$. This is shown as a solid line in Fig. 1 We have also considered the case when all parameters take the above quoted values, except that the unified Yukawa coupling is taken as 3, and also for the case when only m_0 is changed to a value of 600 GeV. We recall that one of the stringent constraints is the requirement that the mass of $\tilde{\tau}$ exceed the lightest neutralino mass, which in the present case is almost entirely bino-like with a mass ~ 350 GeV. This implies that the larger value of m_0 is preferred. Thus a window of parameters is allowed when D-term non-universality contributes significantly to the boundary conditions. Note that we have not imposed any constraints on bino-purity which typically constrains larger values of the unified Yukawa coupling, since the possibility of Higgsino like dark matter is not excluded [28].

In Fig. 2 we have illustrate the numerical results for a typical choice of parameters with $\delta m_0^2 = \pm(200\mathrm{GeV})^2$. It may be inferred from here that the negative sign is preferred for lower values of D-term non-universality before the onset of strong mixing between the scalar leptons of the heaviest generation, while the converse is true for higher values of D-term non-universality. We conclude that with the non-universality coming from the D-term contribution via $SO(10)$ breaking, the naturalness problems are alleviated by the presence of additional parameters. The main reason for this is that such a $D-$ term non-universality succeeds in splitting the masses of the Higgs doublets at the electroweak scale in an efficient manner.

V. SUMMARY AND CONCLUSIONS

In this paper we have considered the breaking of $SO(10)$ grand unfied gauge group to the SM gauge group in a supersymmetric grand unified theory. Such a breaking of $SO(10)$ generates non-universal contributions to the soft scalar masses. We have studied the implications of such non-universal contributions for the sfermion masses. In particular, we have derived sum rules which hold among the sfermion masses when $SO(10)$ breaks to the SM model via different breaking chains. These sum rules may help in distinguishing between the different breaking patterns. We have also shown that these sum rules are different from the case when the SM is not unified into a simple group. We have studied the implications of the non-universal contributions to the low energy phenomenology. In particular, we have shown that it is possible to increase the unified Yukawa coupling to as large a value as 3, once D-term non-universality as well as additional non-universality is introduced. Typically, in the absence of such a non-universality it is not possible to achieve Yukawa unification and radiative electroweak symmetry breaking, and to satisfy all the phenomenological constraints for typical values of parameters. With the non-universality coming from the D-term contributions and additional non-universality, the naturalness problems associated with large values of $\tan\beta$ are alleviated by the presence of additional parameters.

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