Discrete Multitone Modulation with Least Squares Inverse Channel Filtering

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ABSTRACT

ADSL(Asymmetric Digital Subscriber Line) transmission technology meets internet access requirement quite well and offers telephone companies a technology for connecting virtually all internet users at megabit rates.The ANSI (American National Standara Institute) proposes discrete multitone (DMT) modulation with DFT (discrete Fourier tranasform) bases [1] as the carriers for the ADSL system. Recently, discrete wavelet multitone modulation has been studied with non-blind and blind equalization schemes [2,3,4]. In the present paper, we propose an entirely different approach which combines the multitone modulation and optimal orthonormal subband coding.

1. INTRODUCTION

Multicarrier modulation (MCM) schemes have been attracting a lot of attention lately. Theoretically, MCM with infinite number of subchannels is known to be optimum. In practice, by assigning different number of bits with different bit energy, depending on the channel frequency response, to different subchannels, near-optimum performance can be achieved at a reasonable computational cost with a finite number of subchannels [5].

MCM possesses several attractive properties. In MCM, the modulated signal can be processed in the receiver without the enhancement of the noise because of the orthogonality of the subchannel receivers. Also, as a consequence of its longer symbol time, MCM provides superior immunity to impulsive noise and fast fades compared to single carrier system.

Discrete multitone (DMT) modulation is a discrete-time version of the conventional MCM scheme. The ANSI proposed DMT modulation with DFT bases as the carriers. Since FFT (fast Fourier transform) is an efficient algorithm for computing the DFT, computational complexity of modulator and demodulator in the DFT based DMT is low. On the other hand, wavelet based DMT scheme is computationally more complex.

In this paper, we take an entirely different approach to the problem. We use suffix of zeros instead of cyclic prefix as is the case in the DFT based DMT, and apply certain preprocessing to the received signal. The objective of the preprocessing is to remove the channel effect. However, the noise at the output of the preprocessor becomes correlated. The problem then is to design an optimum demodulator. We formulate the design of the demodulator so as to achieve minimum average symbol error probability for a given average bit rate. We show that the desired demodulator is the orthonormal subband coder.

2. PROPOSED SCHEME

In the DMT with cyclic prefix, the modulator output block of length M is extended cyclically to a length of $M+\nu-1$ where ν is the effective channel impulse response length. For such case, DFT bases are the natural choice[1]. Instead of adding a cyclic prefix, we append the modulator output block with $\nu-1$ zero valued samples. That is, if $\mathbf{x}^k = \left[x_0^k, x_1^k, \cdots, x_{M-1}^k\right]^T$ is the k^{th} block of the modulator output, the transmit block becomes $\left[x_0^k, x_1^k, \cdots, x_{M-1}^k, 0, \cdots, 0\right]^T$ where the superscript T denotes vector transpose. When this transmitted block goes through the channel, the channel impulse response is convolved with each block to give non-overlapping blocks of $M+\nu-1$ samples. Let us denote the channel impulse response as $\mathbf{c} = [c_0, c_1, \cdots, c_{\nu-1}]^T$. Then, the received block can be expressed as

$$\mathbf{y}^k = \mathbf{C}\mathbf{x}^k + \mathbf{n}^k \tag{1}$$

where $\mathbf{n}^k = \left[n_0^k, n_1^k, \cdots, n_{M+\nu-1}^k\right]^T$ is a zero mean white Gaussian noise vector of power spectral density $\frac{N_0}{2}$ Watts/Hz and

[1]It can be shown that the DFT bases are optimum in the sense that the average symbol error probability, at the output of the demodulator, is minimum for a given average bit rate.

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$$\mathbf{C} = \begin{bmatrix} c_0 & 0 & \cdots & 0 & 0 \\ c_1 & c_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & c_0 & 0 \\ 0 & 0 & \cdots & c_1 & c_0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & c_{\nu-1} & c_{\nu-2} \\ 0 & 0 & \cdots & 0 & c_{\nu-1} \end{bmatrix} \quad (2)$$

At the receiver, symbol recovery is done in two steps. First is the pre-processor. In the pre-processor, we estimate the transmitted block. Since the transmitted blocks remain non-overlapping after convolution with the channel impulse response, all the $M + \nu - 1$ samples of each block can be used to estimate the corresponding transmitted block independent of the other blocks. In this subsection, we derive the maximum likelihood estimate of the transmitted block. It will be seen that the estimate will have two parts- the undistorted transmitted block and a noise vector whose components are in general correlated even though the noise introduced by the channel is uncorrelated. After the transmitted block is estimated, demodulation is done using a set of orthonormal demosulator bases. In subsection 2.2, we derive optimum demodulator bases to minimize the average probability of symbol error keeping probability of symbol error same in each subchannel.

2.1 Preprocessor

In the preprocessor, we estimate the corresponding demodulator output block from the received block. We want the maximum likelihood (ML) estimate $\hat{\mathbf{x}}^k$ of $\mathbf{x}^k$ for given $\mathbf{y}^k$. That is, we seek

$$\begin{aligned} \hat{\mathbf{x}}^k &= arg \max_{\mathbf{x}^k} \{p\{\mathbf{y}^k|\mathbf{x}^k\}\} \\ &= arg \max_{\mathbf{x}^k} \{p\{\mathbf{n}^k = \mathbf{y}^k - \mathbf{C}\mathbf{x}^k\}\} \\ &= arg \max_{\mathbf{x}^k} \left\{\exp\left(-\frac{||\mathbf{y}^k - \mathbf{C}\mathbf{x}^k||^2}{N_0}\right)\right\} \\ &= arg \min_{\mathbf{x}^k} \{||\mathbf{y}^k - \mathbf{C}\mathbf{x}^k||^2\} \end{aligned} \quad (3)$$

Thus, the ML estimate $\hat{\mathbf{x}}^k$ is the least squares solution of $\mathbf{y}^k = \mathbf{C}\mathbf{x}^k$. If $\mathbf{C}^\# = \left(\mathbf{C}^T\mathbf{C}\right)^{-1}\mathbf{C}^T$ denotes the pseudo inverse of $\mathbf{C}$, then

$$\hat{\mathbf{x}}^k = \mathbf{C}^\#\mathbf{y}^k = \mathbf{x}^k + \mathbf{C}^\#\mathbf{n}^k = \mathbf{x}^k + \mathbf{w}^k \quad (4)$$

where $\mathbf{w}^k = \mathbf{C}^\#\mathbf{n}^k$. So the maximum likelihood estimate of the modulator output block can be obtained by transforming the received block by $\mathbf{C}^\#$.

2.2 Demodulator

We demodulate the estimated transmit block using an orthonormal demodulator bases (with the modulator bases as the mirror image of those of the demodulator). Though the signal part in $\hat{\mathbf{x}}^k$ is in undistorted form, the noise samples are now correlated with each other. Since the demodulator bases are orthonormal, the demodulator output will be

$$\hat{\mathbf{X}}^k = \mathbf{X}^k + \mathbf{v}^k \quad (5)$$

where $\mathbf{X}^k$ is the vector of symbols (at the input of the modulator) in k^{th} block and $\mathbf{v}^k = \left[v_0^k, v_1^k, \cdots, v_{M-1}^k\right]$ is a vector of noise components. Here v_i^k represents the noise component in i^{th} subchannel. We want to design the orthonormal demodulator bases such that the average symbol error probability $\frac{1}{M}\sum_{i=0}^{M-1} P_i$, where P_i denotes the symbol error probability at the output of i^{th} subchannel, is minimized for a given average transmitted bit rate, keeping the symbol error probability same in all the subchannels. To derive such bases, we proceed as follows.

Let us assume for simplicity that we transmit same average energy in each subchannel and we use PAM (pulse amplitude modulation) symbols in each subchannel. Then, the symbol error probability in i^{th} subchannel is given by

$$\mathbf{P}_i = \frac{2\left(2^{b_i} - 1\right)}{2^{b_i}} Q\left(\frac{\sqrt{3\mathcal{E}_{av}}}{\sigma_i\sqrt{2^{2b_i} - 1}}\right) \approx 2Q\left(\frac{\sqrt{3\mathcal{E}_{av}}}{2^{b_i}\sigma_i}\right) \quad (6)$$

for $2^{b_i} \gg 1$ where b_i is the number of bits transmitted in i^{th} subchannel, $\mathcal{E}_{av}$ is average symbol energy in each subchannel, and σ_i^2 is the variance of v_i. To maintain the same symbol error probability in all the subchannels, we need

$$\begin{aligned} \frac{\sqrt{3\mathcal{E}_{av}}}{\sqrt{2^{2b_i}\sigma_i^2}} &= \frac{\sqrt{3\mathcal{E}_{av}}}{\sqrt{2^{2b_j}\sigma_j^2}} \\ \Rightarrow 2^{2b_i}\sigma_i^2 &= 2^{2b_j}\sigma_j^2 \end{aligned} \quad (7)$$

for $0 \le i, j \le M - 1$. Minimization of the average symbol error probability subject to the condition (7) is equivalent to minimizing $\left(\prod_{i=0}^{M-1} 2^{2b_i}\sigma_i^2\right)^{\frac{1}{M}} = 2^{2\bar{b}/M}\left(\prod_{i=0}^{M-1}\sigma_i^2\right)^{\frac{1}{M}}$, where $\bar{b} = \sum_{i=0}^{M-1} b_i$. Thus, for a given average transmitted bit rate, the demodulator bases should be chosen so as to minimize $\left(\prod_{i=0}^{M-1}\sigma_i^2\right)^{\frac{1}{M}}$. This requirement is same as the requirement for optimality of a subband coder for a given input, which is the noise $\mathbf{w}^k$ in the present case.

First, let us consider the demodulator filter length N to be restricted to M. Then, the optimum subband coder is KLT (Karhunen - Loeve Transform), i.e., the demodulator filters are mirror images of the eigenvectors of the noise correlation matrix

$$\begin{aligned} \mathbf{R}_{ww} &= E\left\{\mathbf{w}^k\mathbf{w}^{k^T}\right\} = E\left\{\mathbf{C}^\#\mathbf{n}^k\mathbf{n}^{k^T}\mathbf{C}^{\#^T}\right\} \\ &= \mathbf{C}^\# E\left\{\mathbf{n}^k\mathbf{n}^{k^T}\right\}\mathbf{C}^{\#^T} = \frac{N_0}{2}\mathbf{C}^\#\mathbf{C}^{\#^T} \end{aligned} \quad (8)$$

Suppose, the singular value decomposition(SVD) of $\mathbf{C}$ is $\mathbf{C} = \mathbf{A}\begin{bmatrix}\Gamma \\ \cdots \\ 0\end{bmatrix}\mathbf{B}^T$, where $\Gamma = diag(\gamma_0, \gamma_1, \cdots, \gamma_{M-1})$. Then the SVD of $\mathbf{C}^\#$ is $\mathbf{C}^\# = \mathbf{B}\left[\Gamma^{-1} \vdots 0\right]\mathbf{A}^T$ giving the columns of $\mathbf{B}$ as the eigenvectors of $\mathbf{C}^\#\mathbf{C}^{\#^T}$. Thus, the optimum demodulator is equivalent to transformation by the matrix $\mathbf{B}^T$, and the corresponding modulator is the transformation by the matrix $\mathbf{B}$. The demodulated symbol vector is given by

$$\hat{\mathbf{X}}^k = \mathbf{B}^T\hat{\mathbf{x}}^k = \Gamma^{-1}\mathbf{A}_1^T\mathbf{y}^k = \mathbf{X} + \Gamma^{-1}\mathbf{A}_1^T\mathbf{n}^k \qquad (9)$$

where $\mathbf{A}_1$ is the $(M + \nu - 1) \times M$ matrix composed of the first M columns of $\mathbf{A}$. This suggests a computationally more efficient implementation of the demodulator as shown in Figure 1.

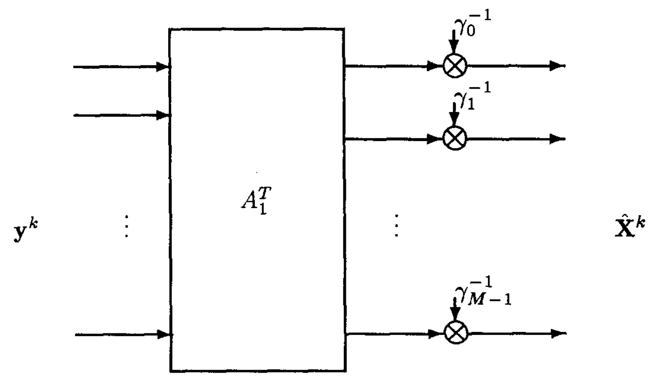

Figure 1: Demodulator structure

Now, KLT decorrelates the noise blockwise. Since the demodulator does not cause any inter-block interference, noise components in the demodulated symbols are uncorrelated across the blocks as well. That is,

$$E\left\{v_i^k v_j^m\right\} = \sigma_i^2 \delta(i-j)\,\delta(k-m) \qquad (10)$$

for $i, j = 0, 1, \cdots, M-1$ and $k, m = \cdots, -2, -1, 0, 1, 2, \cdots$. Thus the KLT satisfies the decorrelation property which is a necessary condition for optimality of a subband coder. Since $E\left\{v_i^k v_i^m\right\} = 0$ for $k \neq m$ and for $i = 0, 1, \cdots, M-1$, the noise component of the demodulator output in each subchannel is white with variance varying from subchannel to subchannel in general. The whiteness of the noise in each subchannel is sufficient for ensuring the majorization condition to be satisfied. Thus, the KLT demodulator satisfies the decorrelation and majorization conditions implying that the demodulator is the optimum orthonormal subband coder for the underlying correlated noise [6]. Note that the demodulator filters can be determined from the channel response. We may point out here that the demodulator of Fig. 1 is similar to that of DFT based DMT system. However, the proposed scheme requires computation of SVD of $\mathbf{C}$ to obtain $\mathbf{A}_1$, Γ and $\mathbf{B}$.

3. SIMULATION RESULTS

To see how the proposed method performs in comparison to the DMT with cyclic prefix, we considered two different channels of impulse response lengths, 14 and 5 and an SNR of 30 dB. In the DMT with cyclic prefix, we used PAM constellations for the real bases and rectangular QAM constellations for the complex bases. In the proposed method, we used PAM constellations for all the subchannels. In both the schemes, we used non-uniform bit-loading across the subchannels with $\bar{b} = 64$.

Table 1. Average bit error probability in the DMT with cyclic prefix and in the proposed scheme ($M = 16$ and $\bar{b} = 64$) with exact channel knowledge

	DMT	Proposed scheme
Channel-1	0.0117	0.0044
Channel-2	0.0707	0.0468

Table 2. Average bit error probability in the DMT with cyclic prefix and in the proposed scheme ($M = 16$ and $\bar{b} = 64$) with estimated channel

	DMT	Proposed scheme
Channel-1	0.0125	0.0047
Channel-2	0.0709	0.0471

Table 1 gives the average bit error probability when perfect knowledge of the channel is assumed. The results show that the proposed scheme yields lower bit error probability, more significantly for channel-1. This is because, with $\nu = 14$ and $M = 16$, (1) forms an over determined set of equations and this helps in reducing the effects of noise perturbations.

Table 2 gives the average bit error probability when channel impulse resonse is not known exactly but is estimated by transmiting a stream of training samples in the presence of noise keeping the SNR at 30 dB. We transmitted 500 samples and used Weiner formulation to estimate the channel impulse response. The simulation results show that the proposed scheme continues to perform better than the DMT with cyclic prefix even with inaccurate estimate of the channel.

REFERENCES

[1] K. Sistanizadeh, P. S. Chow, J. M. Cioffi. *Multi-tone Transmission for Asymmetric Digital Subscriber Lines (ADSL)* ICC93, pp. 756-760.

[2] S. D. Sandberg and M. A. Tzannes, "Overlap Discrete Multitone Modulation for High Speed Copper Wire Communications', *IEEE J. Select. Areas Commun.*, vol. 13, no. 9, pp. 1571-1585, Dec. 1995.

[3] S. Gracias and V. U. Reddy, "An Equalization Algorithm for Wavelet Packet Based Modulation Schemes", *to appear in Nov. 1998 issue of IEEE Trans. Signal Processing.*

[4] T. K. Adhikary and V. U. Reddy, "Complex Wavelet Packets for Multicarrier Modulation", *Proc. IEEE ICASSP* , Seattle, Washington, May 1998.

[5] P. P. Vaidyanathan, "Theory of Optimum Orthonormal Subband Coders", *IEEE Trans. Signal Processing.* vol. 46, no. 6, pp. 1528-1543, June 1998.