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Resource allocation in a MAC with and without security via game theoretic learning

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Abstract

In this paper, we study a K-user fading multiple access channel (F-MAC), with and without an eavesdropper (Eve). In the system without Eve, we assume that each user knows only its own channel gain and is completely ignorant about the other users' channel state. The legitimate receiver sends a short acknowledgement message Acknowledge (ACK) if the message is correctly decoded and a No Acknowledge (NACK) if the message is not correctly decoded. Under these assumptions, we use game theoretic learning setup to make transmitters *learn* about the power allocation under each state. We use multiplicative weight *no-regret* algorithm to achieve an ϵ-coarse correlated equilibrium. We also consider the case where a user can receive other users' ACK/NACK messages. Now, we can maximize a weighted sum utility and achieve Pareto optimal points. We also obtain Nash bargaining solutions, which are Pareto points that are fairer to the transmitting users. Fairness among users is quantified using Jain's index. With Eve, we first assume each user knows only its own channel gain to the receiver as well as to Eve. The receiver decides whether to send an ACK or a NACK to the transmitting user based on the secrecy-rate condition. We use the above developed algorithms to get the equilibrium points. Next, we study the case where each user knows only the *distribution* of the channel state of Eve. Finally, we also consider the system where the users do not know even the distribution of the Eve's channel.

Keywords: Physical layer security, Power control, Fading channel, Coarse correlated equilibrium, Nash bargaining, Multiple access wiretap channel

1 Introduction

A multiple access channel (MAC) is a basic building block in wireless networks [1]. Also, it models the uplink in a wireless cellular system. Therefore, it has been studied extensively over the years ([2–4]). More recent, it has also received attention from information theoretic security point of view. In this paper, we study a MAC with and without an eavesdropper using game theoretic techniques. This allows operating in the capacity region which is fair to the users and also provides distributed algorithms with local information at the users. First, we provide a literature survey on this problem.

A general M-user fading MAC is considered in [5] where the receiver has perfect channel state knowledge and broadcasts channel state information of all the users to all the transmitters. The authors prove that the capacity region of a M-user MAC has a polymatroid structure, and they exploit this structural property to find the optimal power and rate control policy. Time-varying additive white Gaussian noise (AWGN) MAC is studied in [6] where it is assumed that only the receiver can track the channel and not the transmitters. In that case, the transmitters allocate fixed powers (which satisfy the average power constraint) and transmit data over the channel.

In [7], the authors propose a distributed power allocation scheme using *Game Theory*. The authors assume that each user knows the channel gain of other users also, in addition to knowing his own channel gain. The authors prove that the sum-rate point on the capacity region is a Nash equilibrium when the decoding strategy of the receiver is not known to the transmitters. The authors also prove the existence of a Stackelberg equilibrium in which the receiver acts as a leader and the transmitters play a low-level game. Using repeated games, the authors prove that each point on the capacity region of a fading

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MAC is achieved by some power control policy. In [8], the authors prove stronger results by assuming that each user knows only its own channel gain but knows the distribution of channel gains of the other users. Under these conditions, the authors prove the existence and uniqueness of a *Bayesian equilibrium*. In an orthogonal multiple access channel, the authors in [9] have used evolutionary game theory to obtain a power allocation scheme, while assuming that each user knows the channel gain of all users via feedback.

With security constraints, a multiple access wiretap channel (MAC-WT) has been well studied in literature. One of the early works is reported in [10] where only one user has confidential messages to be transmitted. The authors have obtained upper bounds on the secrecy-rate regions. In [11], the authors consider a more general setup wherein they consider a discrete memoryless multiple access channel where the transmitting users receive a noisy version of each others' conversation, and they do not trust each other. In this scenario, the authors have obtained an achievable secrecy-rate region and some outer bounds. In some special cases, this provides secrecy capacity region. A multiple access wiretap channel with feedback has been studied in [12]. An achievable region of a Gaussian multiple access wiretap channel (G-MAC-WT) was obtained in [13] (the secrecy capacity region is still an open problem).

In the above work, weak secrecy criterion is used. A strong secrecy-based achievable rate region for a MAC-WT is reported in [14]. In [15], the authors find secure degrees of freedom for a MAC-WT. More recently in [16], the authors have studied a compound MAC-WT and have characterized inner and outer bounds on the secrecy capacity region. In [17], the authors have studied a fading MAC-WT with full Channel State Information (CSI) of Eve and also when each user knows the channel state of all the users to the receiver but is ignorant of the instantaneous value of channel state to the eavesdropper (only its distribution is known). But knowing other users' channel gains to the legitimate receiver may also not be practical: it needs a lot of signalling overhead and feedback information. Hence, in this paper, we present a game-theoretic solution to the resource allocation scheme under the hypothesis that each user only knows its own channel gain and is completely ignorant of other users' channels (not even the distributions).

In interference channel model [18], the authors use learning algorithms to study a stochastic game and learn optimal power allocation policies. The authors use no-regret algorithm to prove the existence of a *correlated equilibrium*. It is assumed that each user knows power allocation policy of other users, which is not always realistic. The same authors extend this work to the case where each user knows only his own channel gain and does not know the power levels used by other users. The authors prove the existence of a coarse correlated equilibrium using *multiplicative weights* no-regret algorithm [19].

In this paper, we first consider a fading MAC (F-MAC) without security constraint. We assume each user knows only its individual channel gain (unlike [8], we do not assume that it knows the distributions of channel gains of others). Since the receiver is receiving data from all the users, it is quite practical to assume that the receiver has channel state information of all the transmitting users. Once a user sends a codeword corresponding to a particular message, the receiver sends an ACK if it decodes it successfully, else it sends a NACK. Each user computes a utility based on the ACK/NACK. We use multiplicative weight no-regret algorithm to obtain a coarse correlated equilibrium (CCE). We also assume in the later part of the paper that each user can decode ACK/NACK of other users and hence knows their utility. Then we aim to maximize the sum utility and propose an algorithm to obtain a Pareto point (PP). We also find a Nash bargaining solution (NBS) which provides a Pareto point and ensures fairness among users. We also study the case where users can transmit at multiple rates rather than fixed rates.

Next, we consider a fading MAC-WT where we first assume that each user knows its channel gains to the receiver and Eve. In this case, we repeat all the algorithms which we used for a F-MAC (without security), i.e. multiplicative weight (MW), PP, and NBS, and also consider the multiple rates case. Since it is not practical to assume instantaneous channel gain of the eavesdropper to be known at the transmitter and the receiver, we next consider the case where the receiver only knows the distribution of the Eve's channel gains. The receiver calculates secrecy outage and sends an ACK/NACK based on that. We again obtain a CCE, PP and a NBS. To the best of our knowledge, this is the first paper which is using game theory on MAC-WT.

Finally, we compare the sum rates obtained via all these algorithms to the global CSI case and also with the sum rate obtained in [17].

The rest of the paper is organized as follows. In Section 2, we describe the channel model for F-MAC without the security constraint and formulate the problem. In Section 2.1, we use multiplicative weight algorithm to obtain a CCE. In Sections 2.2 and 2.3, we obtain Pareto optimal points and NBS. In Section 3, we consider a fading MAC-WT. First, we consider the case when CSI of Eve is available at the transmitters. In Section 3.1, we consider the case when CSI of Eve is not available at the transmitters (only its distribution is known) and obtain a CCE, a NBS and a PP for both the scenarios. Section 4 generalizes these results to multirate scenario. In Section 5, we compare the various schemes on an example. Finally, in Section 6, we conclude the paper.

2 Fading MAC: without security constraint

A time-slotted F-MAC channel is considered with K-users who have messages to be transmitted to a receiver. Let $\{\widetilde{H}_i(t)\}$ be the channel gain process from user i to the receiver at time t. It includes path loss, fading and shadowing. User i transmits $\widetilde{X}_i(t)$ and the receiver receives

$$\widetilde{Y}(t) = \sum_{i=1}^{K} \widetilde{H}_i(t)\widetilde{X}_i(t) + \widetilde{\eta}_b(t), \tag{1}$$

at time t, where $\widetilde{\eta}_b(t)$ is white Gaussian noise with mean zero and variance σ_b^2, denoted by $\mathcal{N}\left(0, \sigma_b^2\right)$, an d independent of $\{\widetilde{X}_i(t)\}$ and $\{\widetilde{H}_i(t)\}$. We transform the input-output relationship of F-MAC in (1) by appropriate scaling:

$$Y(t) = \sum_{i=1}^{K} \widehat{H}_i(t)X_i(t) + \eta_b(t), \tag{2}$$

where

- $X(t) = \frac{1}{\sigma_b}\widetilde{X}(t)$
- $\widehat{H}(t) = \frac{1}{\sigma_b}\widetilde{H}(t)$
- $\eta_b = \frac{1}{\sigma_b}\widetilde{\eta}(t)$, where now $\eta(t) \sim \mathcal{N}(0, 1)$

Let $H_i(t) \triangleq \left|\widehat{H}_i(t)\right|^2$. The channel gains are assumed discrete valued, in the sets $\mathcal{H}_i \triangleq \{h_i^{(1)}, \ldots, h_i^{(M)}\}$, where M is the number of possible states. Also, $\{H_i(t), t \geq 0\}$ are independent and identically distributed (*iid*) sequences with distributions $\left\{\alpha_i^{(1)}, \ldots, \alpha_i^{(M)}\right\}$. To transmit any codeword, user i can choose any power level from the set $\mathcal{P}_i \triangleq \left\{P_i^{(1)}, \ldots, P_i^{(M)}\right\}$. Also, user i has average power constraint $\overline{P}_i$.

Assumption of discrete channel states Before we describe our game model, we first clarify our assumptions of discrete channel states. The assumption that the channel states take values from discrete sets is based on two observations: (1) In practice, the channel gains are estimated by transmitting known pilots from the transmitter to receiver (a base station in case of uplink scenario), and then the receiver feeds back the estimated value of channel gains up to some precision level. Hence, if n bits are used to represent the value of channel gains (as in digital communication, the information is represented by a finite number of bits), the total possible number of channel states is 2^n [8]. (2) The continuous state of a channel (e.g. Rayleigh distribution) can be quantized to obtain any level of accuracy by a finite set. Also, algorithms in game theory require a finite set. This finite CSI is a commonly made assumption in literature [8, 20, 21].

User i transmits at a fixed rate r_i (to be generalized later) via a usual point to point channel encoder. If the receiver successfully decodes a message, it sends an ACK to that particular user. Otherwise, it sends a NACK. We assume that the NACK and ACK are transmitted at low rates so that these can be received with negligible error at the intended transmitter. The goal of each user is to maximize its probability of successful transmission.

Each user i is assumed to know its own channel gain $H_i(t)$ at time t. Since the receiver can estimate the channel gain of all the users (either by receiving known pilots or by using initial data received), the receiver can use successive cancellation decoding strategy to decode all the users.

Let $\pi(i)$ be the user which has the ith highest channel gain (in case of a tie, we arbitrarily order them). The decoder first decodes the user $\pi(1)$ with the best channel gain first, taking the transmissions from the other users as noise. Then it removes it from the received signal $Y(t)$ and then decodes the next best user, taking the other users as noise and so on. Let

$$C_b\left(P_{\pi(i)}, P_{-\pi(i)}, H_{\pi(i)}\right) \triangleq \frac{1}{2}\log\left(1 + \frac{H_{\pi(i)}P_{\pi(i)}(H_{\pi(i)})}{1 + \sum_{j=i+1}^{K} H_{\pi(j)}P_{\pi(j)}(H_{\pi(j)})}\right). \tag{3}$$

Then the receiver will send an ACK to the transmitting user $\pi(i)$ if

$$r_{\pi(i)} \leq C_b\left(P_{\pi(i)}, H_{\pi(i)}\right). \tag{4}$$

The above constraint follows from the successive cancellation decoding scheme chosen. Each user i takes action (allocating power) $P_i^{(j)}$ when its channel gain is $H_i^{(j)}$ to transmit at its rate. We define feasible action space for user i as

$$\mathcal{P}_i = \left\{\mathbf{P}_i = \left(P_i^{(1)}, \ldots, P_i^{(M)}\right) : P_i^{(k)} \in \left\{p_i^{(1)}, \ldots, p_i^{(M)}\right\},\right.$$

We define $|\mathcal{P}_i| \triangleq M_i$ (where $|A|$ denotes the cardinality of set A) and index the elements of set $\mathcal{P}_i$ as $\{1, \ldots, M_i\}$. Let a_i denotes a feasible power policy of user i, i.e. a_i takes a value from $\mathcal{P}_i$, and $a_i(h)$ is the power level used by user i when its channel gain is $h \in \mathcal{H}$ under policy a_i. The action space of K-users is denoted as

$$\mathcal{P} = \prod_{i=1}^{K} \mathcal{P}_i, \tag{5}$$

and the action space of users, other than user i, is

$$\mathcal{P}_{-i} = \prod_{j=1, j\neq i}^{K} \mathcal{P}_j, \tag{6}$$

where $\prod_{i=1}^{K} A_i = A_1 \times A_2 \ldots \times A_K$. The action profile of all the users is denoted as $a = (a_1, \ldots, a_K)$. A probability distribution $\psi(i)$ on $\mathcal{P}_i$ is called a strategy of user i. When a certain action is chosen with probability one, it is called a *pure strategy*. The objective of each transmitter is to maximize its probability of successful transmission. Since the actions chosen by one user may influence the outcome for the other users in terms of probability of successful transmission, this can be formulated as a stochastic game. Let, for user i, the channel gain in time slot t be $H_i(t)$ and the action profile chosen is $a_i^{(t)}$. Also, let $a^{(t)} = \left(a_1^{(t)}, \dots, a_K^{(t)}\right)$ and $H(t) = (H_1(t), \dots, H_K(t))$. We define the reward of user i,

$$\omega_i^{(t)}\left(a^{(t)}, H(t)\right) = \begin{cases} 1, & \text{if user } i \text{ receives an ACK,} \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

We are interested in the time average of the reward process, for $a = \left(a^{(1)}, a^{(2)}, \dots\right)$,

$$\nu_i(a) \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \omega_i^{(t)}\left(a^{(t)}, H(t)\right). \quad (8)$$

We will restrict ourselves to Markov stationary policies, i.e. action of user i at time t depends only on its current state $H_i(t)$. Then $\{\omega_i(a^{(t)}, H(t))\}$ are *iid* across time t. Hence, by strong law of large numbers, the average reward $\nu_i(a) = \mathrm{E}\left[\omega_i^{(t)}\left(a^{(t)}, H(t)\right)\right]$ is the same as the probability of successful transmission. In terms of a mixed strategy $\psi = (\psi_i, \psi_{-i})$, the average reward is

$$\nu_i(\psi_i, \psi_{-i}) = \sum_{a\in\mathcal{P}} \left[\prod_{j=1}^{K} \psi_j(a_j)\right] \nu_i(a_i, a_{-i}). \quad (9)$$

Hence, this stochastic game can be modelled as a one-shot game in which player i maximizes its utility (9). In the rest of the paper, we develop algorithms to compute equilibrium points for this game.

2.1 Multiplicative weight algorithm for learning CCE

In this section, we use MW algorithm [22] to compute coarse correlated equilibrium (CCE). The cost of each user can be defined as $\mathfrak{C}_i((a_i, a_{-i}) \triangleq -\nu_i(a_i, a_{-i})$. A coarse correlated equilibrium is defined as follows:

Definition 2.1 *A joint distribution ψ on $\mathcal{P}$ is called a CCE if*

$$\mathsf{E}_{a\sim\psi}\left[\mathfrak{C}_i(a)\right] \le \mathsf{E}_{a\sim\psi}\left[\mathfrak{C}_i\left(\hat{a}_i, a_{-i}\right)\right], \quad (10)$$

for all $\hat{a}_i$ and for all users i, when the expectation on the RHS is with respect to the joint distribution with $a_i = \hat{a}_i$. A joint distribution ψ on $\mathcal{P}$ is called an ϵ-CCE if

$$\mathsf{E}_{a\sim\psi}\left[\mathfrak{C}_i(a)\right] \le \mathsf{E}_{a\sim\psi}\left[\mathfrak{C}_i\left(\hat{a}_i, a_{-i}\right)\right] + \epsilon, \quad (11)$$

for all $\hat{a}_i$ and for all users i.

A coarse correlated equilibrium exists for any finite game unlike a Nash equilibrium. A CCE is a generalization of a mixed strategy. A Nash equilibrium and every mixed strategy Nash equilibrium is also a CCE. We now define external regret that plays a key role in the MW algorithm.

Definition 2.2 Arora et al. [22] *The external regret of user i given the action sequence $a_i^{(1)}, a_i^{(2)}, \dots, a_i^{(T)}$ with respect to an action a_i is*

$$\mathcal{R}(a_i) \triangleq \frac{1}{T}\sum_{t=1}^{T} \mathsf{E}_{a_{-i}\sim\psi_{-i}}\left[\mathfrak{C}_i^{(t)}\left(a_i^{(t)}, a_{-i}\right) - \mathfrak{C}_i^{(t)}(a_i, a_{-i})\right]. \quad (12)$$

The MW algorithm is a no-regret algorithm, meaning that the users update their strategies based on the received cost such that the external regret converges to zero ([22, 23]). The MW algorithm is presented in Algorithm 1.

2.1.1 Explanation of Algorithm 1

We provide a brief explanation of Algorithm 1. In each iteration t, a_i is the action chosen by user i with probability $\Phi_i^t(a_i) = \frac{w_i^{(t)}(a_i)}{\sum_{a_i} w_i^{(t)}(a_i)}$, and then it pays an average cost $c_i^{(t)}(a_i)$. Based on the cost incurred, the users update their weight as $w_i^{(t+1)}(a_i) = w_i^{(t)}(a_i)[1-\epsilon']^{c_i^{(t)}(a_i)}$. Number of iterations needed to get an ϵ-approximate point is $T > 2^{(\ln(n)/\epsilon^2)-1}$, as given by Theorem 2.1 below.

Algorithm 1 Multiplicative Weights Algorithm

For each i, a_i, initialize weights, $w_i^{(1)} = 1, \forall\, a_i,\ i$
Fix $\epsilon' > 0$, $\epsilon > 0$ (to compute ϵ-CCE), initialize $t = 1$
do
    Choose number of iterations $T > 2^{\frac{\ln n}{\epsilon^2}-1}$, where n is the maximum number of action of a user which depends on average power constraint.
    For each i, choose $a_i \sim \Phi_i^{(t)}$, where $\Phi_i^{(t)}(a_i) = \frac{w_i^{(t)}(a_i)}{\sum_{a_i} w_i^{(t)}(a_i)}$
    Compute average cost of user i upto time t, $c_i^{(t)}(a_i) = -\frac{1}{t}\sum_{k=1}^{t} \omega_i^{(k)}\left(a_i^{(k)}\right) \mathbb{1}_{\left\{a_i^{(k)} = a_i\right\}}\ \forall\, a_i$
    Update the weight $w_i^{(t+1)}(a_i) = w_i^{(t)}(a_i)\left[1-\epsilon'\right]^{c_i^{(t)}(a_i)}$
    $t \leftarrow t+1$
while $t \le T$

Theorem 2.1 *(Remark 2.3 of chapter 17 from* [24]*) After T iterations, where $T > 2^{\frac{\ln n}{\epsilon^2}-1}$, of no-regret dynamics (MW algorithm), every player of a cost minimization game has regret* (12) *at most ϵ for each of its strategies.*

Let $\psi^{(t)} = \prod_{i=1}^{K} \Phi_i^{(t)}$ denote the outcome distribution at time t and $\psi = \frac{1}{T}\sum_{t=1}^{T} \psi^{(t)}$, the time averaged history of these distributions. Then ψ is an ϵ-coarse correlated equilibrium.

2.2 Pareto optimal points

In a wireless environment, it is realistic to assume that the ACK/NACK bits sent to a particular user can be successfully decoded by all the other users also (because these are sent at a low rate using robust codes). In that case, all users can learn about the utility of each other at time t. We show in this section that this information can be used to get a socially optimal Pareto point which generally provides a better performance than a CCE.

Definition 2.3 *An action profile $a \in \mathcal{P}$ is a Pareto point if there does not exist another profile $\tilde{a}$ such that $\nu_i(\tilde{a}) \geq \nu_i(a), \forall\, i \in \mathcal{K}$ and $\nu_j(\tilde{a}) > \nu_i(a)$ for some $j \neq i$.*

Define

$$\Omega(a) = \sum_{i=1}^{K} \gamma_i \nu_i(a), \tag{13}$$

for fixed $\gamma_i \geq 0,\ i = 1, \ldots, K$. Then a solution to the optimization problem

$$\max_a \Omega(a),\ \text{subject to} \qquad a \in \mathcal{P},$$

is a Pareto point [25].

In Algorithm 2 below, we provide a distributed algorithm in which the users update their strategies in a sequential fashion so as to improve $\Omega(a)$. This distributed algorithm is the variation of a heuristic stochastic local search algorithm. In this algorithm, each user chooses a random action and uses it for a fixed number of time slots (say T). Then each user finds weighted sum of the utilities (since each user receives ACK/NACK of other users, it can calculate this quantity). After T slots, a user experiments randomly with probability say, ρ, and then with some probability updates the action profile according to its channel state. Now, one user uses this action for next T slots and the other users use the previous action. Based on the weighted sum of utilities, the particular user defines a benchmark. The details of algorithm in the scenario of interference channel can be found in [18].

Algorithm 2 Distributed Algorithm to obtain Pareto Points

User i: choose $a_i \in \mathcal{P}_i$ uniformly.
Use a_i for T time slots.
procedure WEIGHT UPDATE
  Update weight of each user i
  $\hat{\Omega}(a) \leftarrow \sum_{i=1}^{K} \gamma_i \left(\frac{1}{T} \sum_{t=1}^{T} \omega_i^{(t)}(a_i, H_i(t), G_i(t)) \right)$
  After T slots: $w.p\ \rho_i$ user i experiments
  procedure ACTION UPDATE
    $w.p\ \epsilon$ choose $a_i' \neq a_i$, $a_i' \in \mathcal{P}_i$
    $w.p.\ 1 - \epsilon$
      choose $a_i' \neq a_i$ s.t. h_i with high α_i gets higher power level
    If α_i same for all h_i, then higher value of channel state gets higher power level.
  end procedure
  Call new action $\hat{a}_i$
  User i: use $\hat{a}_i$ for T time slots.
  $\hat{a}_j = a_j$ if user j is not experimenting.
  User i: find $\hat{\Omega}(\hat{a}_i, a_{-i})$

  if $\hat{\Omega}(\hat{a}_i, \hat{a}_{-i}) > \hat{\Omega}(a_i, \hat{a}_{-i})$ **then** $a_i \leftarrow \hat{a}_i$
    $P_{benchmark} = \hat{\Omega}(\hat{a}_i, \hat{a}_{-i})$
  else
    Randomly select another action
  end if
end procedure

2.3 Nash bargaining solution

The Pareto points obtained in Section 4 are socially optimal but may not be fair to all users: some users may get much more rates than others. To obtain fair Pareto points, we use the concept of Nash Bargaining Solution (NBS) [26].

In NBS, we need to specify a *disagreement* strategy Δ and the corresponding outcome $\delta = (\delta_1, \ldots, \delta_K)$ that specifies the utility of each user that it receives by playing the disagreement strategy whenever there is no improvement over this utility in playing the bargaining outcome. Thus, each user i is guaranteed with δ_i if it is feasible. We define the set of all possible utilities as

$$\mathcal{V} = \{(\nu_1(a), \ldots, \nu_K(a)) : a \in \mathcal{P}\}. \tag{14}$$

This bargaining problem is denoted by $(\mathcal{V}, \delta)$.

The aim of the bargaining problem is to find a bargaining solution which is Pareto optimal and satisfies the axioms of symmetry, invariance and independence of irrelevant alternatives [27].

Theorem 2.2 Nash[26] *There exists a unique bargaining solution (provided the feasible region is non-empty) and it is given by the solution of the optimization problem:*

$$\max \prod_{i=1}^{K} (\nu_i - \delta_i)$$
$$\text{subject to } \nu_i \geq \delta_i, i = 1, \ldots, K,\ (\nu_1, \ldots, \nu_K) \in \mathcal{V}. \square \tag{15}$$

A crucial part of a Nash bargaining problem is to choose the disagreement outcome. It is common to consider an equilibrium point as a disagreement outcome. In our problem, we can consider the utility vector at a correlated equilibrium as the disagreement outcome. We can also choose $\delta_i = 0$ for each i. If we choose the disagreement outcome to be a correlated equilibrium, each user needs to evaluate a correlated equilibrium first before running the algorithm to find a solution of (15), which requires more computations. Rather, we obtain the disagreement outcome for our problem by the following procedure

- ⋆ Each user chooses an action that gives higher power level to the channel state that has higher probability of occurrence. In other words, among the set of feasible actions, choose a subset of pure strategies that gives the highest power level to the channel state with highest probability of occurrence. We shrink the subset by considering the actions that give higher power level to the second frequently occurring channel state, and we repeat this process until we get a single strategy.
- ⋆ If all the channel states occur with equal probability, we follow the above procedure by considering the value of the channel gain instead of the probabilities of occurrence of the channel gains.

Let a_i denote the pure strategy chosen by the ith user, and let T_δ be the number of time slots over which this strategy is used. Then the disagreement value for user i is

$$\delta_i = \frac{1}{T_\delta} \sum_{t=1}^{T_\delta} \omega_i^{(t)} \left(a_i, H_i(t)\right). \tag{16}$$

We modify Algorithm 2 to obtain a distributed solution of (15), with the objective function defined as

$$\Omega(a) = \prod_{i=1}^{K} \left(\nu_i(a) - \delta_i\right). \tag{17}$$

Hence, the algorithm for NBS is the following Algorithm 3.

From [26], if the set of utilities $\mathcal{V}$ is convex, then a Nash bargaining solution is also *proportionally fair*. In our problem, $\mathcal{V}$ is convex, and hence, the solution is proportionally fair also.

2.3.1 Fairness comparison via Jain's index

We use Jain's index to quantify the fairness in the allocation of rates to multiple users [28]. Let $\mathbf{r} \triangleq (r_1, \ldots, r_K)$ be the rates allocated to the K-users in the F-MAC via the algorithms described in Sections 2.1, 2.3 and 2.2. Jain's index is defined as [28]

$$\mathcal{J}(\mathbf{r}) = \frac{\left[\sum_{i=1}^{K} r_i\right]^2}{K \sum_{i=1}^{K} r_i^2} \tag{18}$$

Algorithm 3 Distributed Algorithm to obtain Nash Bargaining Solution

```
User i: choose a_i ∈ P_i uniformly.
Use a_i for T time slots.
procedure DISAGREEMENT VALUE
    User i computes disagreement value
    δ_i = (1/T_δ) Σ_{t=1}^{T_δ} ω_i^(t)(a_i, H_i(t))
    User i now updates weight as
    Ω̂(a) ← Π_{i=1}^{K} (ν_i(a) − δ_i)
    After T slots: w.p ρ_i user i experiments
    procedure ACTION UPDATE
        w.p ε choose a'_i ≠ a_i, a'_i ∈ P_i
        w.p. 1 − ε
            choose a'_i ≠ a_i s.t. h_i with high α_i gets higher power level
            If α_i same for all h_i, then higher value of channel state gets higher power level.
    end procedure
    Call new action â_i
    User i: use â_i for T time slots.
    â_j = a_j if user j is not experimenting.
    User i: find Ω̂(â_i, a_{−i})

    if Ω̂(â_i, â_{−i}) > Ω̂(a_i, â_{−i}) then a_i ← â_i
        P_benchmark = Ω̂(â_i, â_{−i})
    else
        Randomly select another action
    end if
end procedure
```

It can be easily shown that $1/K \leq \mathcal{J}(\mathbf{r}) \leq 1$. An allocation policy is fair if $\mathcal{J}(\mathbf{r})$ is close to 1, and policy is called unfair if it is close to $1/K$.

3 Fading MAC: with security constraints

In this section, we consider a time-slotted fading MAC-WT channel with K-users who have messages to transmit confidentially to a legitimate receiver (Bob), while a passive eavesdropper (Eve) is listening to the conversation and trying to decode. The notation corresponding to Bob is same as in the previous sections. Here, we define the notation for the channel to Eve. Let $\{\widetilde{G}_i(t)\}$ be the channel gain process from user i to Eve (Table 1). At time t, Eve receives

$$\widetilde{Z}(t) = \sum_{i=1}^{K} \widetilde{G}_i(t) \widetilde{X}_i(t) + \widetilde{\eta}_e(t), \tag{19}$$

where $\widetilde{\eta}_e(t)$ is white Gaussian noise, with distribution $\mathcal{N}(0, \sigma_e^2)$ and independent of $\{\widetilde{\eta}_b(t)\}$ and the channel gain processes and $\{\widetilde{X}_i(t)\}$.

Table 1 Nomenclature

Symbol	Definition
K	Number of transmitting users
$\widetilde{H}_i(t)$	Channel gain to Bob
$\widetilde{G}_i(t)$	Channel gain to Eve
M	Possible values of channel gain
$\boldsymbol{\mathcal{P}}_i$	Action space
$\overline{P}_i$	Power constraint for user i
$\pi(i)$	ith element of permutation of index set
$\alpha_i^{(j)}$	pmf of $H_i(t)$
n	Maximum no. of action of a user
$\omega_i^{(t)}\left(a_i^{(t)}, H_i(t)\right)$	Instantaneous reward for user i for given action $a_i^{(t)}$
r_i	Rate of user i
$\beta_i^{(j)}$	pmf of Eve's channel state, $G_i(t)$
$\Phi_i(t)$	Empirical distribution over action space for user i
a_i	action choosen by user i
δ_i	Disagreement value for user i
$\eta_b(t)$	AWGN at Bob
$\eta_e(t)$	AWGN at Eve
$\mathcal{J}(\mathbf{r})$	Jain's index
$\mathfrak{C}(a_i, a_{-i})$	Cost of each user
$c(a_i)^{(t)}$	Average cost of user i up to time t
ϵ	Regret for cost minimization game
ϵ'	Weight update factor
$\mathbb{1}_{\{A\}}$	Indicator function
$\boldsymbol{\mathcal{V}}$	Utility set for Nash bargaining
Δ	Disagreement strategy for Nash bargaining solution
δ_i	Disagreement value for user i

We transform this relation in the same way as was done in transforming (1) to (2). We get

$$Z(t) = \sum_{i=1}^{K} \widehat{G}_i(t) X_i(t) + \eta_e(t), \tag{20}$$

where $X(t) = \widetilde{X}(t)/\sigma_e$, $\widehat{G}(t) = \widetilde{G}(t)/\sigma_e$ and $\eta_e(t) = \widetilde{\eta}_e(t)/\sigma_e$.

We define $G_i(t) \triangleq \left|\widehat{G}_i(t)\right|^2$. The channel gains of Eves' channels are assumed discrete valued, in the set $\mathcal{G}_i \triangleq \left\{g_i^{(1)}, \ldots, g_i^{(M)}\right\}$. Also, $\{G_i(t), t \geq 0\}$ are *iid* independent of each other and also of the sequences $\{H_i(t)\}$, with distribution $\left\{\beta_i^{(1)}, \ldots, \beta_i^{(M)}\right\}$ respectively. User i transmits at a fixed rate r_i via wiretap coding. If the receiver successfully decodes (see details below in this subsection), it sends an ACK to that particular user. Otherwise, it sends a NACK. We assume that the NACK and ACK are transmitted at low rates so that these can be received with negligible error at the intended transmitter. The goal of each user is to maximize the probability of successful transmission.

Each user i is assumed to know its own channel gains $H_i(t)$ and $G_i(t)$ at time t. Since the receiver can estimate the channel gain of all the users (either by receiving known pilots or by using initial data received), the receiver can use successive decoding strategy to decode all the users.

We define

$$\begin{aligned} &C_e\left(P_{\pi(i)}, P_{-\pi(i)}, H_{\pi(i)}, G_{\pi(i)}\right) \\ &\quad \triangleq \frac{1}{2}\log\left(1 + \frac{G_{\pi(i)} P_{\pi(i)}\left(H_{\pi(i)}, G_{\pi(i)}\right)}{1 + \sum_{j\neq i}^{K} G_{\pi(j)} P_{\pi(j)}\left(H_{\pi(j)}, G_{\pi(j)}\right)}\right) \end{aligned} \tag{21}$$

Then the receiver will send an ACK to the transmitting user $\pi(i)$ if

$$r_{\pi(i)} \leq \left(C_b(P_{\pi(i)}, H_{\pi(i)}, G_{\pi(i)}) - C_e(P_{\pi(i)}, H_{\pi(i)}, G_{\pi(i)})\right)^+, \tag{22}$$

otherwise a NACK, where $(a)^+ = \max(0, a)$. The above constraint follows from the achievable secrecy-rate region of a Gaussian MAC-WT as discussed in [13]. Each user i takes action (allocating power) $P_i^{(j)}$ when its channel gains are $H_i^{(j)}$ and $G_i^{(j)}$ to transmit at its rate.

Now, we can use all the algorithms of Sections 2.1, 2.2 and 2.3 to obtain a CCE, PP and NBS.

3.1 Fading MAC-WT with individual main channel CSI only

We consider now the case where the users as well as the receiver do not know Eve's channel gain, but only its distribution. Also, the transmitters do not know even the distribution of Eve's channel gains. In this scenario, the natural metric for the receiver to decide whether to send an ACK or a NACK will be outage based. First, we define the secrecy outage, when $H_1, \ldots, H_K$ are given, as

$$\begin{aligned} &P_O^S(\pi(i)) \triangleq \\ &Pr\left\{r_{\pi(i)} > \log\left(1 + \frac{H_{\pi(i)} P_{\pi(i)}\left(H_{\pi(i)}\right)}{1 + \sum_{j=i+1}^{K} H_{\pi(j)} P_{\pi(j)}\left(H_{\pi(i)}\right)}\right)\right. \\ &\quad \left. - \log\left(1 + \frac{G_{\pi(i)} P_{\pi(i)}\left(H_{\pi(i)}\right)}{1 + \sum_{j\neq i}^{K} G_{\pi(j)} P_{\pi(j)}\left(H_{\pi(j)}\right)}\right)\right\}, \end{aligned} \tag{23}$$

where probability is over the channel gains $G_{\pi(i)}$. The receiver sends an ACK if $P_o^S < \epsilon$, else the receiver sends a NACK. Hence, we define utility of user i as

$$\omega_i\left(a_i^{(t)}, h_i(t)\right) = \mathbb{1}_{\{P_O^S(i)<\epsilon\}}, \tag{24}$$

where $\mathbb{1}_{\{C\}}$ is an indicator function. With these utility functions, we can use the algorithms provided in Sections 2.1, 2.2 and 2.3.

3.2 Avoiding security breach

In the previous sections, we assumed that when the legitimate receiver cannot securely decode the message, it sends a NACK. This is useful for the transmitters to learn the equilibrium point. But the messages transmitted during those slots may be decoded by Eve (with probability $> \epsilon$ in Section 3.1). Now, we modify the system a little so as to use the above coding scheme but mitigate this secrecy loss also.

We assume that each slot is comprised of two subslots. The channel gain does not change during the whole slot. In the first part of the slot, we transmit a dummy (random) message. If Bob sends an ACK to user i, then the actual confidential message can be transmitted by user i in the second subslot at the same power. If Bob sends a NACK, then user i should not use the second subslot. We can make the second subslot much larger than the first subslot so that the rate loss due to the dummy messages is minimal.

4 Transmission at multiple rates

Till now, we have considered the case where the users are transmitting at fixed rates. Now, we consider the more realistic scenario where the users can transmit at different rates, depending on their channel gains. We assume that user i can choose any rate from the rate set $\mathcal{R}_i = \left\{r_i^{(1)}, \ldots, r_i^{(M_R)}\right\}$. We now define a new strategy set such that choosing the rate of transmission becomes part of the action taken along with the power chosen. Hence, we define the modified strategy set for F-MAC as

$$\mathcal{A}_i \triangleq \left\{ \left(r_i, P_i^{(1)}, \ldots, P_i^{(M)}\right) : r_i \in \mathcal{R}_i, P_i^{(k)} \in \left\{p_i^{(1)}, \ldots, p_i^{(M)}\right\}, \sum_{j=1}^{M} \alpha_i^{(j)} P_i^{(j)} \le \overline{P}_i \right\} \tag{25}$$

and for F-MAC-WT as

$$\mathcal{A}_i \triangleq \left\{ \left(r_i, P_i^{(1)}, \ldots, P_i^{(M)}\right) : r_i \in \mathcal{R}_i, P_i^{(k)} \in \left\{p_i^{(1)}, \ldots, p_i^{(M)}\right\}, \sum_{j=1}^{M} \alpha_i^{(j)} \beta_i^{(j)} P_i^{(j)} \le \overline{P}_i \right\}. \tag{26}$$

We can now use all the existing algorithms to compute CCE, PP and NBS.

5 Numerical results

In this section, we provide several examples using the algorithms developed in this paper. We divide our examples into two parts: (1) F-MAC (without security constraint) and (2) F-MAC-WT.

We first discuss the quantization of channel gains from various continuous fading distributions.

5.1 General distribution

Suppose that we want to quantize the channel state to n states, such that the quantized channel state value takes values from a known probability mass function

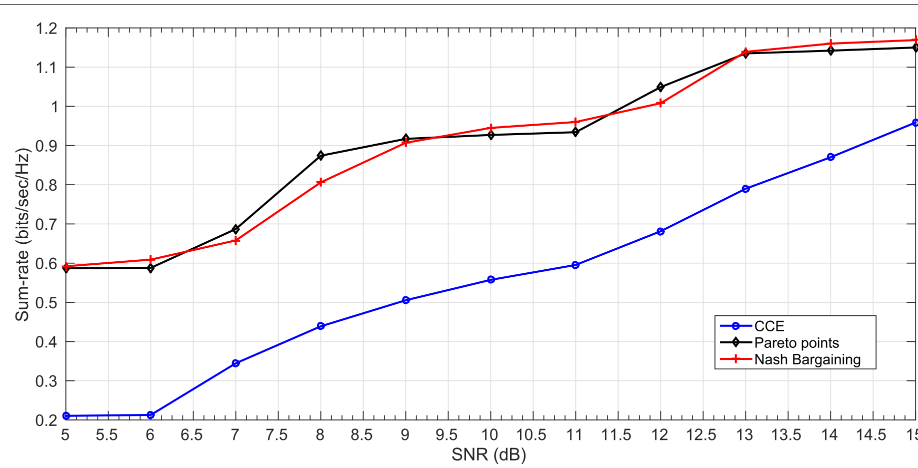

Fig. 1 Sum-rate comparison: CCE vs NBS vs PP (F-MAC, fixed rate case)

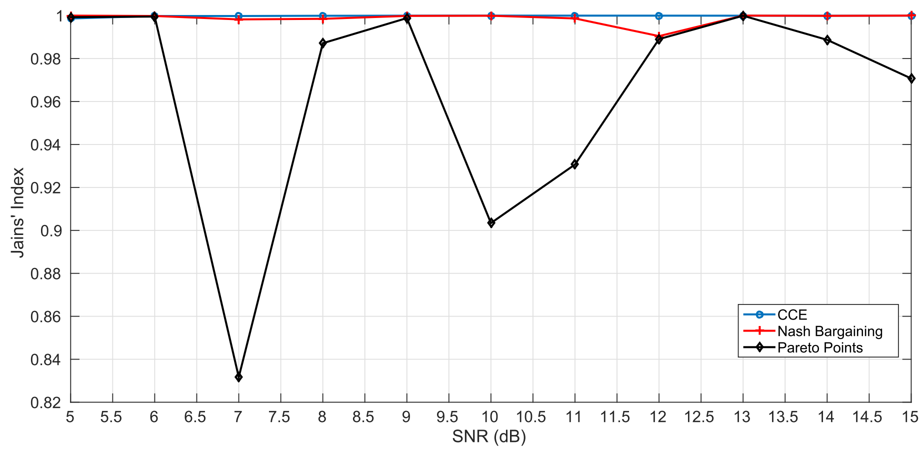

Fig. 2 Fairness comparison for FMAC fixed rate

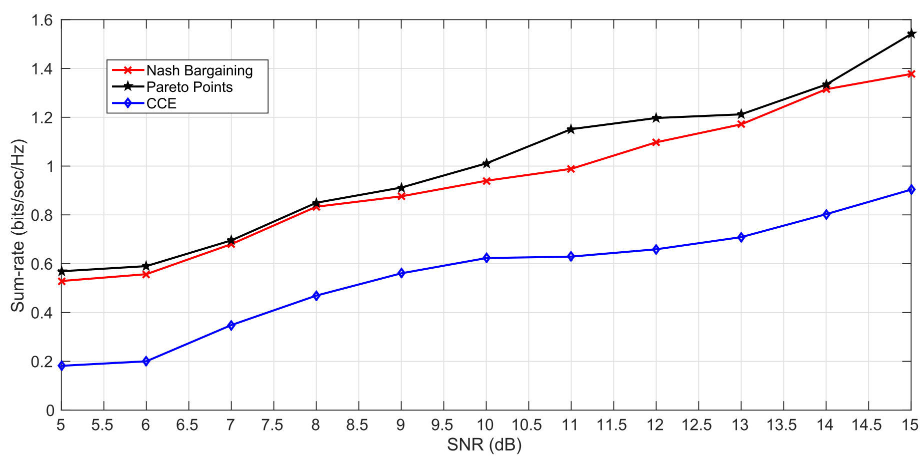

Fig. 3 Sum-rate comparison for FMAC: multiple transmission rates

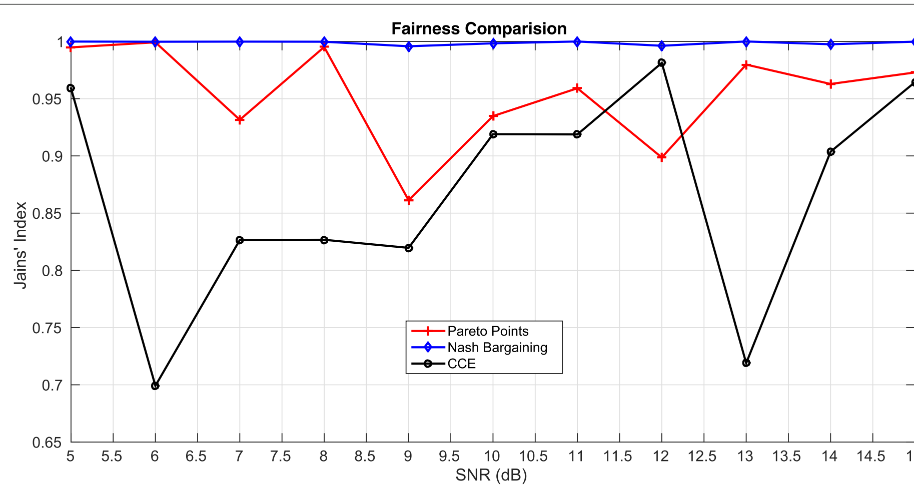

Fig. 4 Fairness comparison for F-MAC: multiple transmission rates

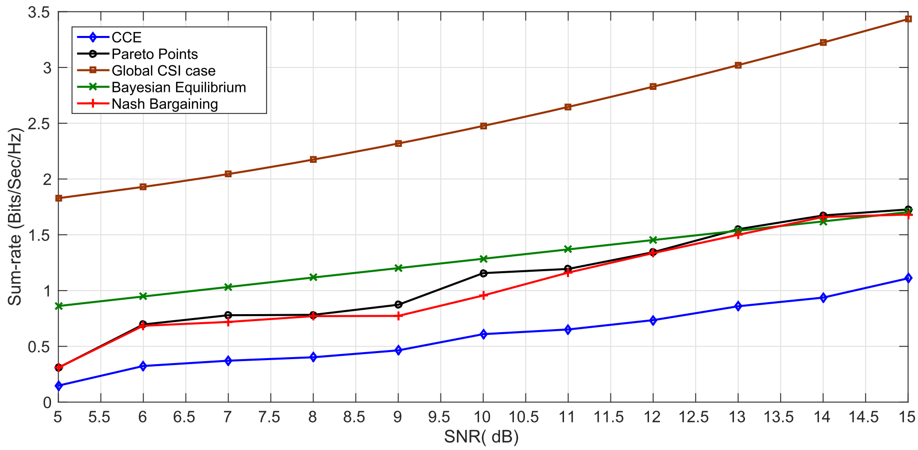

Fig. 5 F-MAC: sum-rate comparison for our scheme vs existing schemes

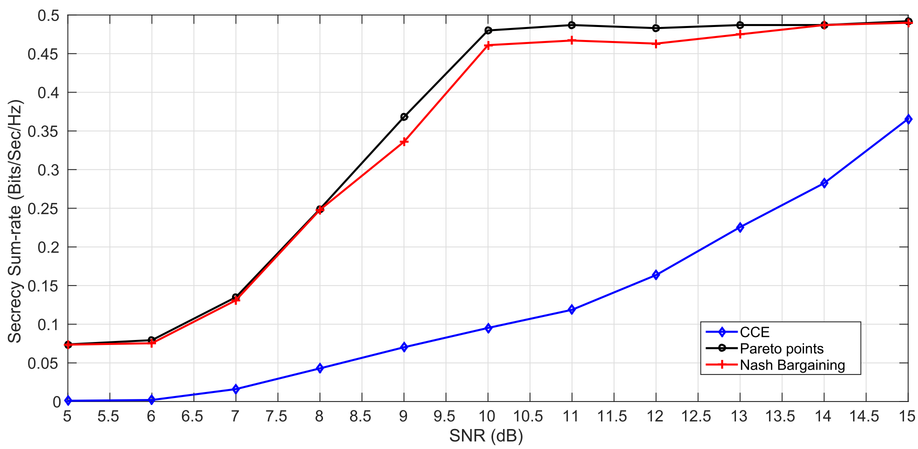

Fig. 6 Sum rate with security constraints: comparison of CCE, PP and NBS at fixed transmission rate (with CSI of Eve)

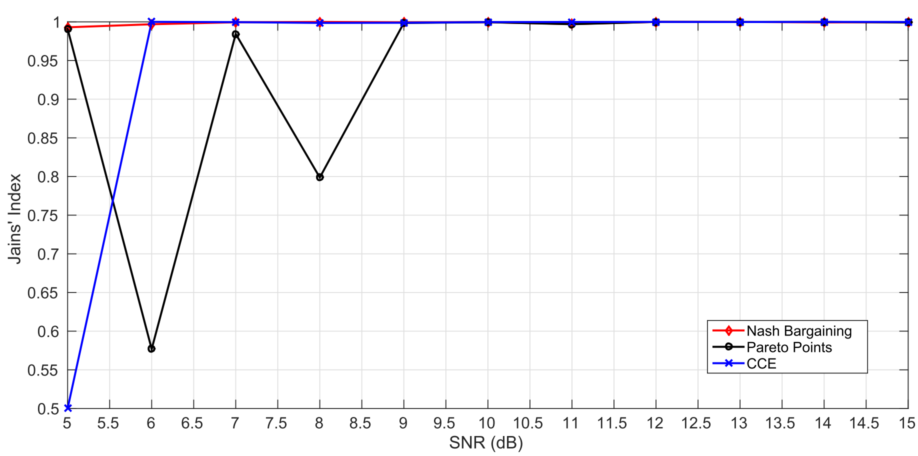

Fig. 7 Fairness of CCE, PP and NBS at fixed transmission rate (with CSI of Eve)

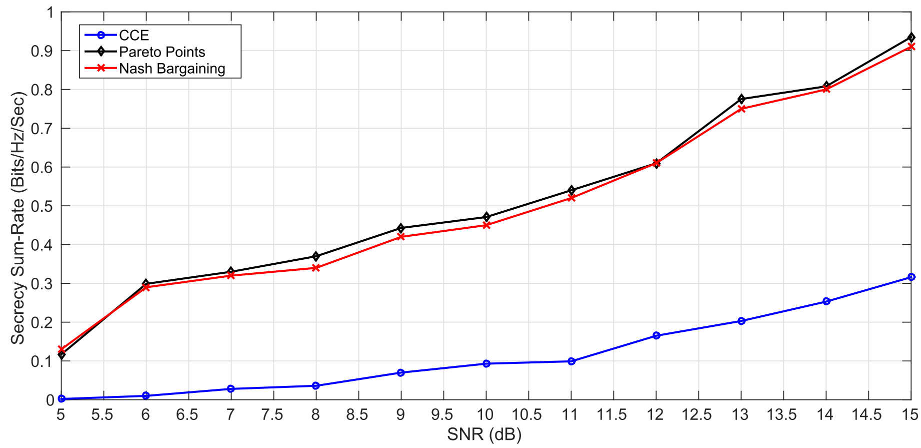

Fig. 8 Comparison of CCE, PP and NBS for F-MAC-WT, with no CSI of Eve (fixed transmission rate)

$\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. Let $\widehat{H}_1, \ldots, \widehat{H}_n$ denote the quantized values of channel state H. We can compute these from the following equations:

$$\int_{\widehat{H}_{i-1}}^{\widehat{H}_i} f_H(h)dh = \alpha_i, \text{ for } i = 2, 3, \ldots, n-1 \tag{27}$$

Now, the required values for channel gain coefficients are

$$H_i = \int_{\widehat{H}_{i-1}}^{\widehat{H}_i} |h| f_H(h)dh, \; i = 2, 3, \ldots, n-1, \tag{28}$$

$$H_n = \int_{\widehat{H}_{n-1}}^{\infty} |h| f_H(h)dh. \tag{29}$$

By taking n large enough, we can have the solution obtained by quantization, accurate to any degree. For Rayleigh distribution,

$$f_H(h) = \frac{h}{\mu} e^{-h^2/2\mu}, \; h \geq 0 \tag{30}$$

where μ is the mean of the fading process. For lognormal shadowing,

$$f_H(h) = \frac{1}{h\sigma\sqrt{2\pi}} e^{-\frac{\ln(x)-\mu}{2\sigma^2}} \tag{31}$$

with mean e^{μ} and variance $e^{\mu+\sigma^2/2}$.

5.1.1 F-MAC (without security constraint)

We first consider a fading MAC where we take $\mathcal{H} = \{0.1, 0.5, 0.9\}$ chosen with uniform distribution over the set, for all users, and we assume that a user can choose any power from the power set $\{1, 5, \ldots, 100\}$. In this scenario, we first consider the case when users are transmitting at fixed rate, 1 bit/sec. In this scenario, we compare the sum rate obtained by our three algorithms, i.e. Algorithm 1 for CCE, Algorithm 2 for PP and Algorithm 3 for NBS (see Fig. 1). We note that NBS and PP are better than CCE in terms of sum rate. Also, regarding the fairness among the users, we see from Fig. 2 that NBS is fairer than PP, since

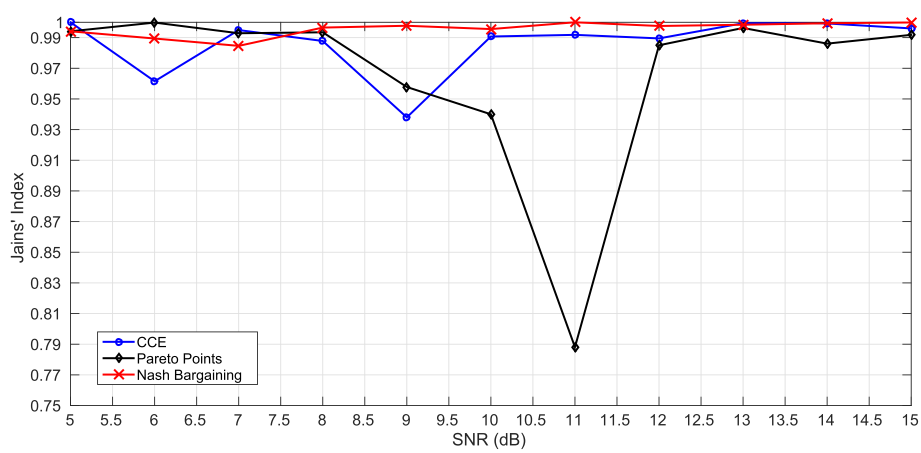

Fig. 9 Comparing fairness of CCE, PP and NBS for F-MAC-WT, with no CSI of Eve (fixed transmission rate)

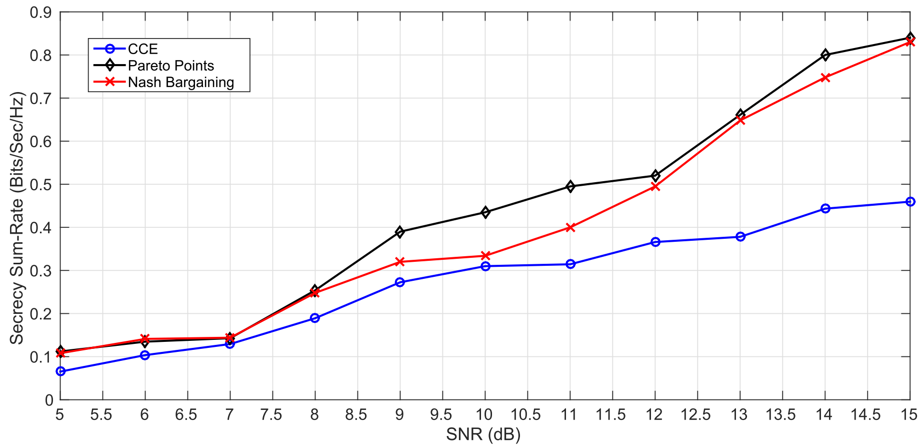

Fig. 10 FMAC-WT: sum rate comparison of CCE, PP and NBS for multiple rate case (with CSI of Eve)

the Jain's index of the rate pair for NBS is close to 1 and that of PP is away from 1. From this, we conclude that NBS is the best scheme to use, as it achieves a much higher sum rate than CCE and each user individually gets better rates than via CCE and PP.

Next, we consider a more practical case where users can choose transmission rates from the set $\{0.4, 0.8, 1, 1.5, 2, 2.3\}$. Here also, we compare the sum rate obtained via CCE, PP and NBS. To get the result for CCE, all users use Algorithm 1. For finding Pareto points, all users use Algorithm 2, with the weights $\gamma_i = 1$. For obtaining Nash Bargaining solution, all users use Algorithm 3. As expected, we observe that PP and NBS give much better rates than CCE (Fig. 3). From Fig. 4, we also observe that here also, NBS is fairer among the three algorithms.

Finally, to compare the performance with the existing schemes, we take an example where we assume $\mathcal{H} = \{0.1, 0.9\}$ and the power set is $\{1, 5, \ldots, 100\}$. Also, as in the previous example, the users can choose transmission rates from the set $\{0.4, 0.8, 1, 1.5, 2, 2.3\}$. We compare our algorithms (viz. CCE, PP and NBS) with the case where global knowledge of CSI is assumed. We also compare our schemes with that of [8], where each user knows its own channel and distribution of other users' channel gains. The authors in [8] posed the problem of power allocation as a Bayesian game and obtained a Bayesian equilibrium. We observe that our PP and NBS give sum rates close to this scheme (Fig. 5).

We also observe the following behaviours of the three schemes (Figs. 2 and 4). At any given SNR, the three schemes allocate resources (power, rate) to each user. In CCE, each user optimizes its own objective function (here probability of successful transmission), without worrying about the other users. Hence, Jain's index will be sometimes fair and sometimes unfair. Thus, there will be fluctuations. In Pareto policy, the sum throughput of all the users is optimized. But the allocation of resources to

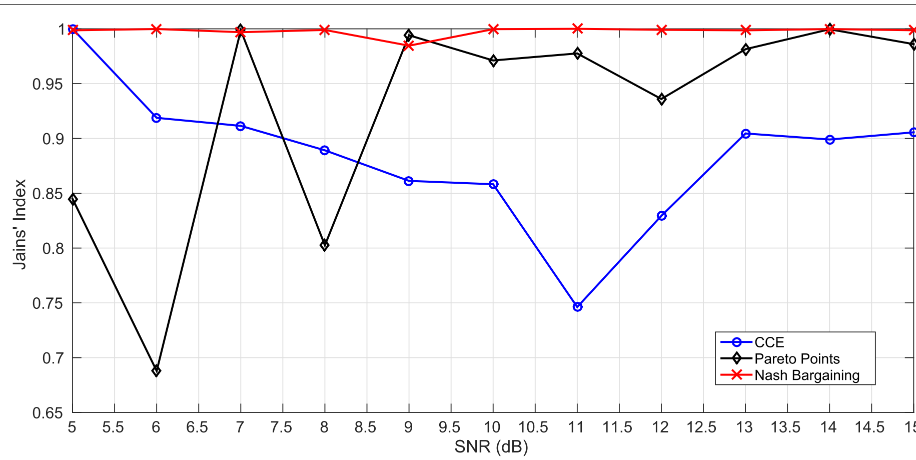

Fig. 11 F-MAC-WT: fairness comparison of CCE, PP and NBS for multiple rate case (with CSI of Eve)

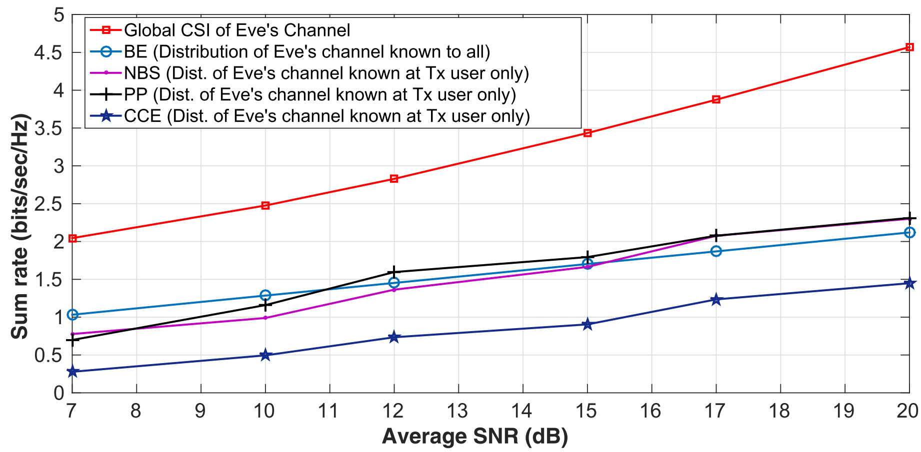

Fig. 12 F-MAC-WT: Comparison with existing schemes

individual users can often be unfair. This is a known weakness of Pareto points. Hence, Jain's index is above 0.5. Nash Bargaining is designed to be a fairer scheme. Hence, Jain's index for it is close to 1. At any given SNR, we expect it to provide a fairer solution than the CCE and a Pareto point.

5.1.2 F-MAC-WT (with security constraint)

Next, we consider a 2-user fading MAC-WT with full CSI. We let $\mathcal{H} = \{0.1, 0.5, 0.9\}$ and $\mathcal{G} = \{0.05, 0.4, 0.8\}$ for both the users. We assume that the probability with which any state can occur is equal, i.e. $\alpha_i^{(j)} = \beta_i^{(j)} = 1/3$ for $i = 1, 2$ and $j = 1, 2, 3$. A user can choose any power from the power set $\{1, 5, \ldots, 100\}$. We first consider a fixed rate scenario. Each user knows its channel gain to Bob and Eve. We again use Algorithm 1 for obtaining CCE, Algorithm 2 for PP and Algorithm 3 for NBS. We observe that the PP and the NBS obtain much higher sum rate than the CCE (Fig. 6). Also, we observe that the NBS is fairer than the PP (Fig. 7), by observing that Jains' index is close to 1 in NBS and away from a in case of PP.

Next, we consider the case where the users do not have CSI of Eve available but only the distribution is known, which is more realistic as Eve is passive. Here also, with modified utility, we compute CCE, PP and NBS via Algorithms 1, 2 and 3, respectively. As in previous cases, the same trend is observed, i.e. NBS and PP give better rates than CCE and NBS is fairer than PP (Figs. 8 and 9).

Next, we consider the case when users have CSI of Eve available to them and can transmit at multiple rates choosing from $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6\}$. From Fig. 10, we note that PP and NBS give better secrecy sum rates, and from Fig. 11, we observe fairness of NBS.

We take one more example with $\mathcal{H} = \{0.1, .9\}$ and $\mathcal{G} = \{0.05, 0.8\}$. We compare the NBS and the PP with the case when CSI of the transmitters is known globally but only the distribution of Eve's channel gains are known at all transmitters. This case is studied in [17] for continuous channel states and a centralized solution which maximizes the sum rate is found. We also find the Bayesian equilibrium (BE) for the case when each user knows the distribution of all the channel gains to Eve, as done in [8] for F-MAC without security constraints. Here, we observe that the NBS and the PP outperform BE at high SNR (Fig. 12). At low SNR, the sum rate for the NBS and the PP are quite close to that of BE. We also observe here that the CCE performs the worst.

6 Conclusions

In this paper, a K-user fading multiple access channel with and without security constraints is studied. First, we consider a F-MAC without the security constraints. Under the assumption of individual CSI of users, we propose the problem of power allocation as a stochastic game when the receiver sends an ACK or a NACK depending on whether it was able to decode the message or not. We have used multiplicative weight no-regret algorithm to obtain a coarse correlated equilibrium (CCE). Then we consider the case when the users can decode ACK/NACK of each other. In this scenario, we provide an algorithm to maximize the weighted sum utility of all the users and obtain a Pareto optimal point. PP is socially optimal but may be unfair to individual users. Next, we consider the case where the users can cooperate with each other so as to disagree with the policy which will be unfair to individual user. We then obtain a Nash bargaining solution, which in addition to being Pareto optimal is also fair to each user. We use Jain's index to quantify the fairness between different algorithms.

Next, we study a K-user fading multiple access wiretap channel with CSI of Eve available to the users. We use the previous algorithms to obtain a CCE, PP and NBS. Next, we consider the case where each user does not know the CSI of Eve but only its distribution. In that case, we use secrecy outage as the criterion for the receiver to send an ACK or a NACK. Here also, we use the previous algorithms to obtain a CCE, PP or NBS. Finally, we show that our algorithms can be extended to the case where a user can transmit at different rates. At the end, we provide a few examples to compute different solutions and compare them under different CSI scenarios.

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Authors' contributions
The author names are in order of contribution, i.e. first author has the major contribution, second author helped in writing the algorithms and third author is a research advisor. All authors read and approved the final manuscript.

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