A 4-State Asymmetric 8-PSK TCM Scheme for Rayleigh Fading Channels Optimum at High SNR's

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Abstract - We use an asymmetric 8-PSK signal set in a 4-state rate 2/3 Trellis-Coded Modulation (TCM) scheme for fading channels to show performance gain over known TCM schemes with symmetric and known asymmetric constellations. This scheme is an example of TCM schemes over fading channels where performance gain can be achieved at high SNRs, without increasing the minimum squared product distance, effective length or decreasing the multiplicity. Simulations for Rayleigh fading channels, using Monte Carlo simulation, are presented.

I. Introduction

TCM schemes achieve significant coding gain without bandwidth expansion by treating coding and modulation as single entity [1]. The code design criteria for fading channels specifies that the effective length and the minimum product distance(rather than Euclidean distance) should be maximized for a code and the multiplicity should be minimized [2].

Based on the above design rules TCM schemes for fading channels were constructed by Wilson and Leung [3], Jamali and LeNgoc[4], Du and Vucetic[5], Divsalar and Simon [6], Venkat, Sundar Rajan and Bahl [7] [8] and Zafar and Sundar Rajan[9]. The maximum value of minimum squared product distance using symmetric 8-PSK signal set and effective length 2 (maximum value of effective length) is $2.344E_s^2$. The symmetric 8-PSK constellation is shown in Fig. 1 (a) and the corresponding signal point assignment is shown in Fig. 2 [4].

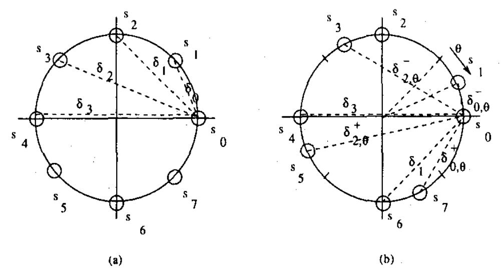

Figure 1: (a) Symmetric 8-PSK with distances marked (b) Asymmetric 8-PSK signal set with angle of asymmetry θ (used in VSB and the code of this paper).

The use of asymmetric signal sets for trellis coding has been considered by Divsalar and Simon in [6], Venkat, Sundar Rajan and Bahl [8] and Zafar and Sundar Rajan[9]. The asymmetric M-PSK signal set used by Divsalar and Simon does not give any improvement for 4 and 8-state rate 2/3 8-PSK schemes. Venkata Subramaniam, Sundar Rajan and Bahl(VSB) used the asymmetric (single rotation) 8-PSK constellation shown in Fig.1(b), obtained by rotating the signal points by an angle θ, and the signal point assignment given by Jamali and LeNgoc [4], to improve the minimum squared product distance to $4E_s^2$ at $\theta = 15°$ [7][8]. Zafar and Sundar Rajan use a new type of asymmetric signal set called Double Rotation 8-PSK (DRPSK) signal set and corresponding signal point assignment rules to increase the minimum squared product distance to

$6.11E_s^2$[9]. It is also shown that significant coding gain can be achieved at high SNRs by decreasing the higher order product distances.

In this paper we show that following the design rules of Venkata Subramaniam, Sundar Rajan and Bahl (VSB)[7][8], and the 8-PSK signal set with asymmetry (as shown in Fig. 1(b)), better bit error performance can be achieved at high SNRs even as the value of $d_p^2(2)$ remains constant by decreasing the higher order product distances .

Simulation results for Rayleigh fading channels, are used to verify that the asymmetric 8-PSK TCM scheme of this paper indeed outperform the known 8-PSK TCM schemes. Throughout, it is assumed that the effect of the fading on the phase of the received signal is fully compensated and that the fading amplitude is known, or in other words, it is assumed that the channel state information (CSI) is available. Also ideal interleaving is assumed.

This paper is organized as follows. The asymptotic error analysis of TCM schemes for fading channels is described in Section II. In Section III we discuss code design for asymmetric PSK signal sets and in Section IV simulation results are presented along with explanation for the observed results.

II. Asymptotic Error Analysis

For the performance analysis we use the system model given in [7][4]. Using this model, the channel output y_i, assuming coherent detection and ideal interleaving, can be represented as,

$$y_i = \rho_i x_i + n_i. \tag{1}$$

where, $n_i = \mathrm{Re}(n_i) + \mathrm{Im}(n_i)$ is a complex noise process whose real($\mathrm{Re}(n_i)$) and imaginary part ($\mathrm{Im}(n_i)$) are uncorellated, zero mean Gaussian r.v.'s each with variance $\sigma^2 = N_o/2$, ρ_i is a random variable representing the random amplitude of the received signal and x_i's are input symbols, ouput from the rate 2/3 convolution encoder and mapped to 8-PSK signal set, before ideal interleaving.

An upper bound on the average error probability of this system, using Viterbi decoding, is obtained as [2]

$$P_e = \sum \sum_{\mathbf{x},\hat{\mathbf{x}}\in \mathbf{C}} P(\mathbf{x}) Pr(\hat{\mathbf{x}}/\mathbf{x}), \tag{2}$$

where C is the set of all valid sequences, $P(\mathbf{x})$ is the a priori probability of transmitting $\mathbf{x}$, $Pr(\hat{\mathbf{x}}/\mathbf{x})$ is the pairwise error probability, i.e. the probability that the decoder wrongly decodes to $\hat{\mathbf{x}}$ when $P(\mathbf{x})$ was the transmitted sequence. The pairwise error probability can be approximated for Rayliegh fading channels as

$$Pr(\hat{\mathbf{x}}/\mathbf{x}) \leq \prod_{i\in\eta} \frac{1}{1+\frac{E_s}{N_0}|x_i - \hat{x}_i|^2}. \tag{3}$$

The average error probability, for Rayliegh Fading Channels may be approximated, by using the above equation, as[4].

$$P_e \leq \sum_{l_\eta} \sum_{d_p^2(l_\eta)} \alpha(l_\eta, d_p^2(l_\eta)) \frac{1}{(\frac{1}{4N_o})^{l_\eta} d_p^2(l_\eta)}. \tag{4}$$

where, $d_p^2(l_\eta)$ denotes the minimum squared product distance between signal points of error event paths with effective length l_η and $\alpha(l_\eta, d_p^2(l_\eta))$ denotes the average number of code sequences having effective length l_η and squared product distance $d_p^2(l_\eta)$. At high SNR's the term due to minimum effective length L dominates. The P_e in this case can be written as

$$P_e \approx \frac{\alpha(L, d_p^2(L))}{(\frac{1}{4N_o})^L d_p^2(L)}. \tag{5}$$

In view of the above equations the code design criteria for Rayleigh fading channels with perfect CSI are,

1. maximize the effective length of the code L, and
2. maximize the smallest product of the squared distances of signals along the error events of effective length L.
3. minimize the multiplicity of error events, $\alpha(L, d_p^2(L))$, at the smallest product distance

Equations similar to those in Sec. II-B can be obtained when no CSI is present at the receiver. The code design criteria for both of these case are also as above.

III. Code Design

The TCM codes for fading channels are designed based on the above criteria. Based on the code design criteria for fading channels 4-state rate 2/3 symmetric 8-PSK TCM schemes have been constructed in [4] [3] [8]. We show here that following the design rules of Venkat, Sundar Rajan and Bahl for 8-PSK signal set with asymmetry as shown in Fig. 1(b) further improves the performance at high SNRs.

As the VSB code is derived from the Jamali and LeNgoc code, we first consider the Jamali and LeNgoc code obtained by the labelling shown in Fig. 1(b). In this code

	S_0	S_1	S_2	S_3
S_0	s_0	s_4	s_2	s_6
S_1	s_3	s_7	s_1	s_5
S_2	s_6	s_2	s_4	s_0
S_3	s_5	s_1	s_7	s_3

Figure 2: Transition matrix used in VSB and the code of this paper.

the minimum squared product distance is achieved by the combination [4]

$$\delta_3^2\delta_0^2 = 2.344E_s^2. \qquad (6)$$

Observing columns of the matrix shown in Fig. 2, it follows that the δ_0^2 term in the above expression appears only in one of the following ways:

$$s_5s_6 \qquad or \qquad s_1s_2$$

and in no other way, i.e., no other combination of the form s_is_{i+1} can appear as branch labels in the same level, since both s_i and s_{i+1} do not appear in any column in Fig. 2(c). The nearest squared product distance to the minimum product distance is given by the term $\delta_1^2\delta_1^2 = 4$. Introduction of asymmetry as shown in Fig. 1(b), where the signal points $\{s_1,\ s_3,\ s_5,\ s_7\}$ are rotated clockwise with respect to the other 4 signal point, increases the distance among the pairs (s_5s_6) and (s_1s_2) while the $\delta_1^2\delta_1^2$ is undisturbed and the signal points s_0, s_2, s_4 and s_6 do not change their position. If the angle of asymmetry introduced is θ as shown in Fig. 2(b), let the new distances after introducing asymmetry be denoted as $\delta_{0,\theta}^-, \delta_{0,\theta}^+, \delta_1, \delta_{2,\theta}^-, \delta_{2,\theta}^+, \delta_3$ (marked in Fig. 2(b)). Due to introduction of asymmetry the distances between error events of effective length and actual length 2 squared product distances either decrease or increase, while minimum product distance of higher effective lengths decreases. The minimum squared product distance $\delta_3^2(\delta_0^+)^2$ given by,

$$\delta_3^2(\delta_0^+)^2 = (4)(4sin^2(\frac{\pi}{8}+\frac{\theta}{2})) \qquad (7)$$

goes on increasing as θ increases. But as the next higher squared product distance is $\delta_1^2\delta_1^2 = 4$, the maximum angle of rotation in the VSB code is $\frac{\pi}{12}$. In other words θ is increased up to that value for which

$$4sin^2(\frac{\pi}{8}+\frac{\theta}{2}) = 1.$$

As we have observed above, the introduction of asymmetry decreases the minimum squared product distances of higher effective lengths i.e. $min\ d_p^2(3)$, $min\ d_p^2(4)$ *etc.* decrease. This observation is the key to further performance improvement over the VSB code. Although increase of angle of rotation above $\theta = \frac{\pi}{12}$ dose not improve the minimum squared product distance of minimum effective length, the multiplicity of this error event is reduced from 48 to 32 and minimum squared product distances of higher effective lengths decreases (see Table 1). Due to this decrease in higher length minimum squared product distances the performance of the code deteriorates at low SNR's and

improves at high SNR's. This suggests a trade of between the performance at low S-NR's and high SNR's. Also the decrease in multiplicity results in asymptotic performance gain as predicted by equation (5). The angle of asymmetry however should not be too high ($\theta \to 45°$). As the signal points tend together the asymptotic bound on the error probability is no longer well approximated by its first term. This results in catastrophic trellis codes i.e.one where more and more long paths appear which have a squared product distance slightly greater than or equal to the minimum squared product distance of effective length 2.
The free Euclidean distance for the VSB scheme as also the code of this paper introduced above is

$$(\delta_0^-)^2 + (\delta_0^+)^2 + \delta_1^2 = 3.268. \quad (8)$$

The free Euclidean distance of is a function of θ, and for $\theta > \frac{\pi}{12}$ is higher than the Free Euclidean distance for both the Jamali and LeNgoc code and the VSB code (see Table 3).

Table 1: Comparison of 4-state rate 2/3 8-PSK schemes

	JN [9]	VSB	Code of this paper	
	$\theta = 0°$	$\theta = 15°$	$\theta = 25°$	$\theta = 30°$
L	2	2	2	2
$d_p^2(L)$	$2.344E_s^2$	$4E_s^2$	$4E_s^2$	$4E_s^2$
$d_p^2(3)$	$0.686E_s^3$	$0.536E_s^3$	$0.317E_s^3$	$0.202E_s^3$
d_{free}^2	$3.172E_s$	$3.268E_s$	$3.44E_s$	$3.55E_s$

IV. Simulation Results

Fig. 3 shows simulation results for Wilson and Leung code (WL), Jamali and LeNgoc code (JN), Venkat, Sundar Rajan and Bahl code (VSB) and the code of this paper for Rayleigh fading channels. The multiplicity of the VSB 8-PSK ($\theta = \pi/12$) TCM scheme for length two events is $\alpha(2, d_p^2(2)) = 48$[7]. If we increase the angle of asymmetry beyond 15^o this number comes down to 32. The multiplicity for length 3 error events is given by $\alpha(3, d_p^2(3)) = 64$. The minimum length 3 product distance is achieved by the combination

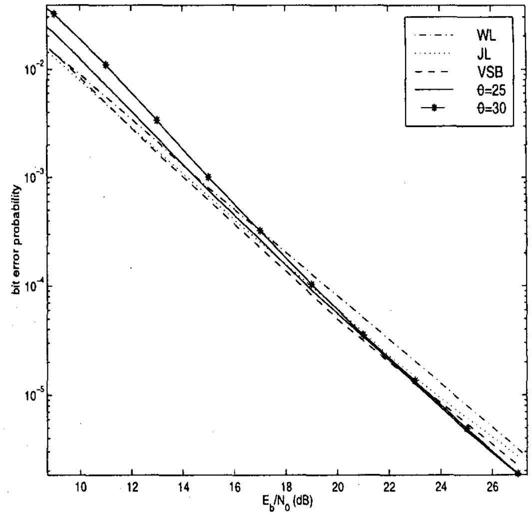

Figure 3: Bit error rate performance of the code of this paper over a Rayleigh fading channel with perfect CSI

$$d_p^2(3) = (\delta_0^-)^2 (\delta_0^+)^2 \delta_1^2.$$

On increasing θ this term becomes smaller and smaller and adversely effects the performance of the code at lower SNRs, but improves the performance at high SNRs as can be seen from Fig. 3.
This asymmetric TCM scheme ($\theta = 25^o$) shows improvement of more than 0.6 dB over VSB for SNR's greater than 26 dB over Rayleigh fading channels.

V. References

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