

Robust geotechnical design of rock bolts for stability of rock slopes using response surface

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Abstract— Stability of rock slopes in many civil engineering projects involves installation of rock bolts for stabilization. Uncertainty in input parameters that determine the stability of rock slopes may lead to uncertainty in evaluating the factor of safety. Design of installed rock bolts must be optimized for safety requirements, cost efficiency and robustness. A design is considered robust if it is least sensitive to variation in input parameters. A well accepted quantity, Signal-to-Noise Ratio (SNR) is selected to be a robustness measure. Factor of safety of slope is evaluated using commercial software Phase2D. General purpose response surface as a surrogate model to Phase 2D simulation is created to model the system response for various designs. Reliability index for all the designs was calculated from the response surface via Monte Carlo simulations. Further analysis was performed on those designs which satisfy a threshold value of reliability index. A simplified Robust Geotechnical Design (RGD) methodology is adopted in which a number of single objective optimizations were performed to locate the position of acceptable design w.r.t. utopia point in a plot of cost vs. robustness. Finally the global knee point is taken to be the optimum design whose distance is minimum from the utopia point.

Keywords—Response Surface; Reliability Index, Design Robustness; Pareto Front

I. INTRODUCTION

Proper design of rock slope is a challenging problem faced by civil and mining engineers since long time. Several analytical and numerical techniques have been developed to determine the Factor of safety (FOS). Rock slope reinforcement is required in many construction projects like dams, for bridge abutments etc. It always has been difficult to choose a reinforcement design which may account for the uncertainties present in the rockmass. Conventional designs are based on deterministic approach based on FOS. From multiple candidate designs, those satisfy code specified safety (typically FOS for engineered slope is 1.5) along with serviceability requirements, are shortlisted and optimization procedure is carried out to select a design with least cost. Uncertainties in material properties are not explicitly considered in this analysis, rather a cushion based on threshold FOS is assumed to account for these uncertainties.

Another approach which received much attention recently from many researchers is probabilistic analysis and Load Resistance Factored Design (LRFD) based approaches

which include uncertainty in the analysis. This assumes that the design is acceptable if the reliability index of reinforced slope exceeds a commonly accepted threshold.

Both these approaches are aimed at achieving certain safety and costs targets and do not take into account the robustness of the design. Robustness based approach seeks a design which is insensitive to noise parameters (hard to control) by adjusting (easy to control) design parameters. This approach originated from industrial product design engineering. [20]. The term Robust Geotechnical Design (RGD) is coined by [10] in 2013. This approach has been implemented in drilled shaft design, braced excavations in soil etc. In rock slopes, noise parameters are joint and rock properties which are inherently variable and cannot be controlled. Example of design parameters are bolt length, bolt spacing, tensile strength etc. which can be selected easily by the engineers. So by appropriately adjusting the design parameters, the reliability index of FOS of slope is made least sensitive to changes in noise parameters. Previously [12] had implemented RGD approach for design of rock slope and making it stable by further excavation. But the slope had only two discontinuities, hence analytical solution was available.

This paper deals with selection of design for a rock slope which has to be reinforced with rock bolts. It does not have analytical solution and is solved using FEM in plane strain mode. To evaluate the reliability index by Monte Carlo Simulations (MCS), finite element model will not be feasible. So a response surface which acts as surrogate to numerical model, relating explicitly the output and input parameters is created and then MCS is carried out on response surface. Finally an optimization is carried out based on minimum distance (MD) algorithm, which is quite simple as compared to multi-objective optimization adopted [10].

II. ROBUSTNESS MEASURE OF A DESIGN

Since the RDG approach aims at cost and robustness optimization, robustness index of the design needs to be defined. Note that this index is meaningful only if the design satisfies acceptable safety and serviceability requirements. Many measures of robustness has been proposed [11]. In this paper we employ a widely accepted robustness measure known as signal to noise ratio [13].

$$SNR = 10 \log_{10} \left(\frac{E[g(d, \theta)]^2}{\sigma[g(d, \theta)]} \right) \quad (1)$$

where $E[g(d, \theta)]$ is the mean and $\sigma[g(d, \theta)]$ is the standard deviation of the output at a given design set d . Higher is the value of SNR, more robust the design would be.

III. DESCRIPTION OF THE ROCK SLOPE

A sample rock slope of height 82m "Fig. 1" is to be reinforced by end anchored rock bolts. The slope has 2 sets of joints: Joint set 1 and joint set 2. They are very widely spaced as per description adopted from ISRM [9]. Mechanical properties of intact rock and joints with their variability are given in table 1. FOS of this slope without reinforcement is 1.31. End anchored rock bolts have been suggested for installation so as to bring FOS of slope under static conditions exceeding or equal to 1.5.

A. Probabilistic distribution of discontinuity parameters

Joint set J1 and J2 both are assumed to be rough and persistent with no infill. Cohesion value for both of them have been assigned zero. Joint friction angle has been found to be normally distributed with COV as 10% [14]. Joint orientation (dip and dip direction) follows fisher distribution in 3D, in which dip values are almost normally distributed as shown by [2]. [19] also reported dip angles of joint as normally distributed. Hence dip angles of both joint sets are assumed to follow normal distribution with standard deviation (SD) of 5°. Spacing between joints of a particular joint set follows mostly lognormal distribution as mentioned in many literatures [19] and [14].

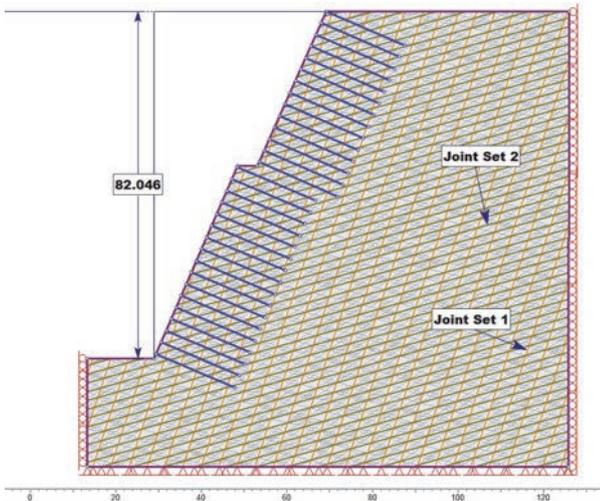


Fig.1. Rock slope geometry with reinforcement

A Coefficient of variation (COV) of 20% is taken to represent the uncertainty in joint spacing of both the joint sets. Normal and shear stiffness of joint are assumed deterministic.

B. Intact Rock parameters

Since FOS of slope is very less sensitive to the intact rock properties, they are not treated as random variables in

probabilistic analysis [2]. Refer to table 2 for deterministic intact rock property.

C. Plane strain FEM analysis

Numerical analysis of slope was performed in Phase² which is a 2D elasto-plastic Finite element stress analysis

TABLE I PROBABILITY DENSITY FUNCTION OF ALL INPUT VARIABLES (NOISE FACTORS)

S No.	Parameter	Distribution	Mean	SD
1	J1 Friction (ϕ_1)	Normal	37°	3.7°
2	J2 Friction (ϕ_2)	Normal	33°	3.3°
3	J1 dip (ψ_1)	Normal	72°	5°
4	J2 dip (ψ_2)	Normal	16°	5°
5	J1 spacing (S_1)	Lognormal	4m	0.8m
6	J2 spacing (S_2)	Lognormal	2m	0.4m

TABLE II DETERMINISTIC PARAMETERS USED IN THE MODELS

INTACT ROCK		
S. No.	Parameter	Value
1	Density	2700 kg/m ³
2	Young's modulus	65 GPa
3	Poisson's ratio	0.2
4	Uniaxial compressive strength	80 MPa
5	Hoek Brown constant m_i	20
JOINT STIFFNESS		
Joint set	Parameter	Value
1	Normal stiffness	4.27 GPa/m
1	Shear stiffness	1.989 GPa/m
2	Normal stiffness	23.63 GPa/m
2	Shear stiffness	9.94 GPa/m

programme. Continuum with joints approach has been adopted which includes Hoek Brown yield criterion proposed by [8] for intact rocks and Mohr-Coulomb constitutive law for joints which were modelled with the help of interface elements. Three noded constant strain triangular element was used to discretize the intact rock and interface elements were modeled as four noded quadrilateral element of zero thickness [7]. Total no. of elements in the model is variable depending on reinforcement design adopted, but typically it is in range of 6500. Both x and y direction were fixed at the base of the model and lateral boundary was fixed in x direction as shown in "Fig 1". FOS was calculated using shear strength reduction technique.

IV. ROBUST DESIGN OF ROCK SLOPE

Previous section characterized the variability in rock and joint properties. Reliability of this slope will be different for different designs. A collection of acceptable design which is called 'design pool' is selected and reliability index for each design is evaluated via Monte Carlo simulation using response surface.

A. Design pool and cost of design

Design pool consists of set designs that engineer thinks is feasible for reinforcement of the slope. End anchored bolt has been selected for analysis in this paper. Rock bolt's length, tensile strength and in-plane spacing are chosen as design parameters. Table 3 lists feasible values that each parameter can take. It is assumed that higher the diameter of the bolt, higher is its tensile strength and correspondingly high pre-tension force can be applied. Pre tension applied to a bolt is 50% of its tensile strength. Thus design pool consists of all the permutations of design parameters mentioned in table 3 which evaluates to 75 (${}^5P_1 * {}^3P_1 * {}^5P_1$).

TABLE III. FEASIBLE VALUES OF DESIGN PARAMETERS

S No.	Bolt Length (L) (m)	In-plane spacing (S) (m)	Bolt tensile strength (TS) (MN)
1	10	2	0.1
2	15	3	0.25
3	20	4	0.4
4	25		0.55
5	30		0.7

A model to estimate cost of rock bolt has been proposed. This is just for illustration and one may adopt a different suitable cost model for their analysis. No. of bolts has been decided as per in-plane spacing and subsequently total bolt length is obtained by multiplying it with length of single bolt. Drilling cost has been implicitly included in the total bolt length. Cost per meter of lowest tensile strength bolt i.e. bolt with least diameter, be 1 unit. It is assumed that cost per meter of bolt with subsequent higher diameter will be 0.5 units higher than previous one. This model has been used to optimize the cost in RGD.

B. Response surface

It is well recognized that Monte Carlo simulation is computationally expensive when there is not explicit equation relating input and output variables. Response surface is replaces complicated numerical model like FEM, FDM etc. by simpler functions which acts as surrogate to actual input output relationship. Here a conventional response surface [17] and [15] which consists of a multivariate polynomial based on regression analysis on data obtained from an appropriate design of experiments. The response surface function (RSF) based on quadratic polynomial chaos expansion is usually adopted [20] and [21].

$$y = a_0 + \sum_{i=1}^m a_i \theta_i + \sum_{i=1}^m a_{m+i} \theta_i^2 \quad (2)$$

Where y is output i.e. FOS, m is no. of noise factors (six in this case), θ_i is a component of vector of noise parameters $\theta = \{\phi_1, \phi_2, \psi_1, \psi_2, S_1, S_2\}$ and a_0, a_1, \dots, a_{12} are 13 constant coefficients which need to be determined. To increase the accuracy of reliability estimate from this RSF, one may

either increase the no. of higher order terms or include cross terms in the analysis but it will increase the no. of constants to be determined. Determination of the 13 constants of RSF requires 13 no. of FE model evaluations. It is very important to appropriately locate the sampling points in the input parameter space so that the response surface is best represented throughout the domain.

A sample design method proposed by [3] is adopted to determine unknown coefficients. It suggests that FOS of rock slope be evaluated deterministically at the following 13 evaluation points (Note mean of θ_i is denoted as μ_{θ_i} and standard deviation as $\sigma_{\theta_i} : \{\mu_{\theta_1}, \mu_{\theta_2}, \dots, \mu_{\theta_6}\}, \{\mu_{\theta_1 \pm k\sigma_{\theta_1}}, \mu_{\theta_2}, \dots, \mu_{\theta_6}\}, \dots, \{\mu_{\theta_1}, \mu_{\theta_2}, \dots, \mu_{\theta_6 \pm k\sigma_{\theta_6}}\}$. Usually $k=1$ is chosen by many researchers (Xu and Low [20], Zhang et al. [21]). This will result into solving $2m+1$ linear equations to determine 13 no. of coefficients. A MATLAB program was written to solve the system of equations.

It must be noted that the previous response surface created was for a particular design set $\mathbf{d} = \{L, S, TS\}$. As mentioned earlier that combinations of \mathbf{g} makes 75 sets of designs. Now to construct a general response surface, which would be valid for all set of designs, each of 13 coefficients are represented as a function of three design parameters. To achieve that, sample design parameters are suitably selected from the discrete design pool so that it best represents the domain of design. It has been suggested by [16], to have one sample design consisting to be middle values of all three design parameters. For selecting other designs, two sampling points corresponding to lower and upper bound for each parameter is selected one at time. Thus for our case a total of 7 designs are sampled (note subscript L denotes lower value and U denotes upper value): $\{\mu_1, \mu_2, \mu_3\}, \{\mu_{1U}, \mu_2, \mu_3\}, \{\mu_{1L}, \mu_2, \mu_3\}, \dots, \{\mu_1, \mu_2, \mu_{3L}\}$. For each of these 7 designs all 13 coefficients must be evaluated. These coefficients were arranged in form of a matrix named A of size $n*m$ ($7*13$).

Now the general response surface also known as quasi response surface is established by second order polynomial fit between each a_i 's and design parameters.

$$a_i = b_0 + \sum_{j=1}^n b_j d_j + \sum_{j=1}^n b_{n+j} d_j^2 \quad (3)$$

Where i ranges from 0 to 12, n is no. of design parameters (3 in this case), and b_0, b_1, \dots, b_6 are unknown coefficients. Determination of these 7 coefficients require 7 values of a_i 's. Columns of matrix A are set of each a_i 's which is used to calculate b_j 's. Thus equation (3) is solved 13 times to obtain a matrix B of size $13*7$ "Fig 2". Each row of matrix B can be used to evaluate corresponding a_i 's given a set of design. This matrix B is the building block of response surface. If one have to estimate FOS of a reinforced slope with given design, one needs to evaluate all 13 a_i 's for a given design with the help of matrix B. After obtaining a_i 's, substitute it in equation (2) and the state of input parameters to obtain FOS.

C. Monte Carlo simulation

Although there are accurate methods in literature to find the reliability index of a system once RSF is obtained and performance function is defined eg. First Order Reliability Method (FORM), Second Order Reliability Method [1] Monte Carlo analysis has been chosen because the former two methods involve non-linear optimizations. This may invoke additional complicity in analysis. Since our failure region is not expected to be very small (range 10^{-5} to 10^{-7}), Monte Carlo simulation will not be computationally expensive on RSF. Once RSF is created, input parameters are derived by taking random realizations out of associated probability density function which is then substituted into equation (2). If FOS is greater than one then it is assumed to be in safe region otherwise in failure zone. 10^5 number of Monte Carlo simulations have been conducted on each of 75 response surfaces and probability of failure is determined by

$$P_f = \frac{F}{N} \quad (4)$$

where N is total no. of simulations i.e. 10^5 , and F is no. of realizations whose FOS has been in failure region. Reliability index is calculated as

$$R = \Phi^{-1}(1 - P_f) \quad (5)$$

where Φ^{-1} is standard normal inverse and R is reliability index. Based on reliability index calculation, those designs whose reliability exceeds a certain safe value is shortlisted for further analysis. Certain target reliability index is difficult to estimate as it depends on consequence of failure and relative cost of safety measures. Safe value is chosen to be 2.5 ($P_f = 0.0062$) that has been suggested by [5].

107.3815	-5.4472	0.1816	-16.8996	2.562	-234.787	259.8867
-0.2179	0.0359	-0.0012	-0.1635	0.027	1.1276	-1.5165
0.0031	-0.0005	0	0.0022	-0.0004	-0.015	0.0203
0.8162	0.065	-0.0022	-0.847	0.1197	0.8131	-0.3535
-0.0117	-0.001	0	0.0129	-0.0018	-0.0117	0.0051
-3.3148	0.0974	-0.0032	1.0447	-0.1569	5.2179	-5.8811
0.0234	-0.0007	0	-0.0073	0.0011	-0.0366	0.0411
0.0804	0.0146	-0.0005	-0.1967	0.0283	0.5183	-0.5833
-0.0039	-0.0005	0	0.0061	-0.0009	-0.015	0.0167
-1.2111	0.0666	-0.0022	0.1344	-0.0156	4.441	-5.1736
0.1667	-0.0082	0.0003	-0.0273	0.0039	-0.5599	0.651
2.7528	0.0281	-0.0009	-2.125	0.375	0.5903	0.0694
-0.7153	-0.0047	0.0002	0.5312	-0.0937	-0.0174	-0.1736

Fig 2: Matrix B- values used for establishing response values

TABLE IV. ACCEPTABLE DESIGNS

S No.	Bolt length (m)	Bolt spacing (m)	Bolt tensile strength (MN)	Reliability index
1	30	2	0.1	2.8338
2	30	2	0.25	2.6997
3	30	2	0.7	2.5663
4	30	3	0.1	2.9165
5	30	3	0.25	4.2649
6	30	3	0.4	3.8461
7	30	3	0.55	3.8906
8	30	3	0.7	3.3598
9	30	4	0.7	2.6174

A MATLAB code was written to implement above mentioned procedures and 9 designs were obtained having desired safety Reliability index (Table 4). Cost model explained previously is applied to the designs in table 4 and along with SNR it is listed in table 5.

TABLE V COST AND SNR OF ACCEPTABLE DESIGNS

S. No.	No. of bolts	Total bolt length (m)	Cost per unit length	Total Cost	SNR
1	45	1350	1	1350	10.5823
2	45	1350	1.5	2025	11.9673
3	45	1350	3	4050	11.8618
4	31	930	1	930	10.5301
5	31	930	1.5	1395	11.8884
6	31	930	2	1860	12.5398
7	31	930	2.5	2325	12.6041
8	31	930	3	2790	11.9793
9	22	660	3	1980	11.411

D. Multi-objective optimization

Objective of this optimization is to obtain a lowest cost design among the design pool which has highest robustness based on SNR. Now if one observes carefully, these two are mutually conflicting requirements. Higher the cost of design, more robust the design would be [10] has implemented multi- objective optimization algorithms like NSGA-II [6]. Solution through this algorithm leads to set of designs that from a Pareto front. Thus a final compromised design is obtained by locating the knee point "Fig 3".

In this paper, NSGA-II algorithm is avoided and multi objective problem is solved through a series of single objective optimizations which does not require estimating Pareto front [16]. To locate the knee point, Minimum Distance (MD) approach based on utopia design is adopted [4].

For all designs that this optimization is carried out, values of minimum cost C_{min} , maximum cost C_{max} , lowest value of SNR SNR_{min} and highest value of SNR SNR_{max} is obtained. Now as mentioned previously it is impossible to obtain a design that simultaneously satisfies C_{min} and SNR_{max} , it is decided to view the objective in slightly different fashion. Optimization will now aim to minimize ($1/SNR$) i.e. maximize SNR. So we can theoretically locate two points in Fig 3 such that its coordinates are C_{min} and ($1/SNR_{max}$). This point is termed as utopia design.

The MD approach suggests that the best optimized design is the design point having coordinates ($C_k, 1/SNR_k$) whose distance from the utopia point is minimum. The computed distance can be viewed as an additional cost required for practical selection of design from the utopia point, which is best option but it is theoretical (exists only in the plot, practically impossible).

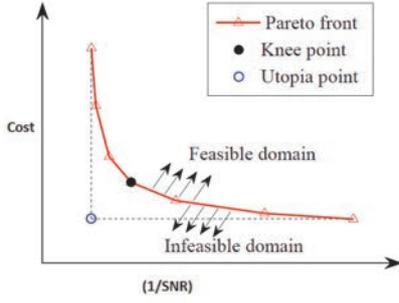


Fig.3. Pareto front with utopia and knee point

It is very important to note that before measuring the distance, one must normalize the cost and (1/SNR) values. It is carried out as:

$$f^i(p) = \frac{f^i(p) - [f^i(p)]_{min}}{[f^i(p)]_{max} - [f^i(p)]_{min}} \quad (6)$$

Where $f^i(p)$ may be elements of cost or (1/SNR) and subscripts max and min denotes maximum and minimum values respectively. After normalization, origin becomes the utopia point.

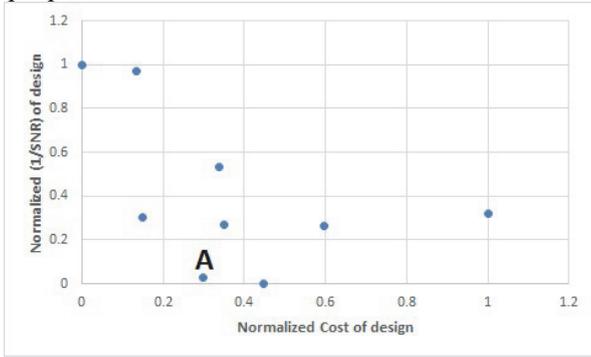


Fig. 4. Minimum distance approach adopted for RGD of rock slope

TABLE VI DISTANCE FROM UTOPIA POINT FOR ACCEPTABLE DESIGNS

S. No.	Original Values		Normalized Values		Distance from utopia point
	Cost	1/SNR	Cost	1/SNR	
1	1350	0.094497	0.134615	0.970023	0.979319
2	2025	0.083561	0.350962	0.270166	0.442904
3	4050	0.084304	1	0.317726	1.049262
4	930	0.094966	0	1	1
5	1395	0.084116	0.149038	0.305655	0.340055
6	1860	0.079746	0.298077	0.026034	0.299212
7	2325	0.079339	0.447115	0	0.447115
8	2790	0.083477	0.596154	0.264809	0.652322
9	1980	0.087635	0.336538	0.530856	0.628543

"Fig 4", shows location of acceptable designs w.r.t utopia point for the analysis done in this paper. "Table 6" tabulates all the data for normalized values and distances from utopia point. Point A denotes the optimized design which has minimum distance. Thus point A corresponds to 6th design i.e. bolt length of 30m, bolt spacing of 3m and bolt tensile strength of 0.4MN. It has a reliability index of 3.84. Monte Carlo simulation on its response surface results in following probability density function (PDF) "Fig 5" – Bar graph). Probability distribution functions have been fitted to the simulated data (see legend in fig 5) using negative of the log-likelihood method. Generalized extreme value distributions fits best to the data. Lognormal distribution stands at fourth.

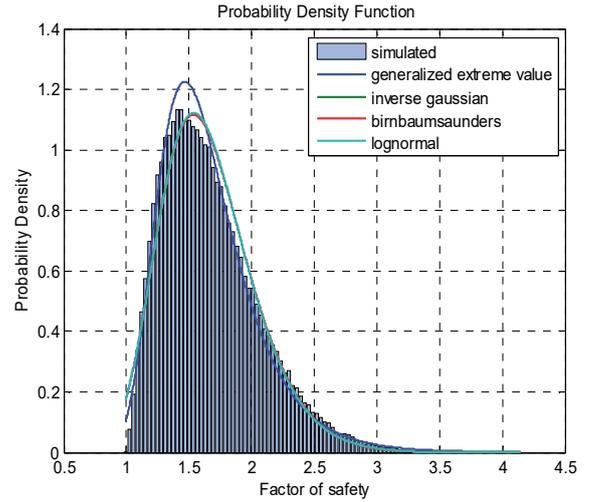


Fig.5. PDF of optimized design

The best fit for this PDF is generalized extreme distribution with parameters location, scale and shape are 1.4771, 0.3003 and 0.0298 respectively. The statistical description of data generated from Monte Carlo simulated values are: Mean – 1.6605, Standard deviation – 0.3920, Skewness – 0.9420 and Kurtosis – 4.0108.

V. CONCLUSIONS

An efficient technique RGD is implemented for designing of reinforcement of rock slopes. The approach presented in this paper can be applied to problems whose analytical solutions do not exist and FEM solutions take more time making Monte Carlo simulation computationally expensive. Implementation of MD algorithm avoids the use of a more complex NSGA-II optimization procedure. This design approach can be viewed as practical engineering tool which incorporates conventional reliability based approach in conjunction with additional intelligent decision making tool i.e. multi-objective optimization. A more general response surface methodology which is valid for whole design pool is developed to obtain the reliability index. After obtaining the optimized design, one may either increase the order of the RSF polynomial or implement stochastic response surface to obtain more accurate estimates of reliability index and sensitivity.

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