Routing and Scheduling in Packet Radio Networks

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Abstract: - A New approach for channel assignment in packet radio networks is presented. To arrive at this: 1) we show the difficulty with traditional graph coloring method for link scheduling. 2) An algorithm for generating maximal independent sets of links is presented. 3) Importance of combined routing and scheduling is shown and an LP problem is formulated for combined routing and scheduling.

Key Words: Radio network, scheduling, and graph coloring method, evacuation.

I. INTRODUCTION

A network of processor that communicate using broadcast radio is a *radio network*. A typical example is a packet radio network (PRN). The *stations* constituting a radio network share a common *radio channel* over which communication takes place. The multi-hop nature of most radio networks makes spatial reuse possible in the sharing or *assignment* of channels. The channel assignment considered here assigns transmission rights using time division multiplexing. In this method, transmission that will not collide in time may overlap in time, there by obtaining channel reuse in time. This is typically done by constructing a schedule: that is, a sequence of fixed length time slots[1], where each possible transmission is assigned a slot in such a way that transmissions assigned to the same slot do not collide. It is shown that the problem of finding an optimum schedule is NP-complete [3]. In order to properly discuss the concept of scheduling, we first consider what is meant by a *collision*. In particular, depending on the signaling mechanism, transmissions may collide in two ways—these are typically referred as *primary* and *secondary interference*. *Primary interference* occurs when the schedule is such that a station[2] does more than one thing in a single time slot—for instance, receiving from two different transmitters. *Secondary interference* occurs when a receiver R tuned to a particular transmitter T is with in the transmission range of another transmitter whose transmissions, though not intended for R, interfere with the transmission of T.

[1]One slot duration is equal to transmission time of a packet plus the maximum propagation delay. Here the maximum is over all the links in the network.

[2]A station can be in one of the following states: idle, transmitting, and receiving states.

[3]Link schedule has also been referred as point-to-point scheduling, link activation scheduling, and receiver directed schedule.

Depending on the service required by network, there are two kinds of scheduling —*broadcast* and *link*[3]. In a *broadcast schedule,* the entities scheduled are the stations themselves. The transmission of a station is intended for, and must be received collision free by, all of its neighbors. Here, primary interference is not tolerated, and it follows from the definitions that secondary interference does not arise. Thus two stations may not be assigned the same slot if they are adjacent or have a common neighbor. In a *link schedule,* the *links*[4] between station are scheduled. The transmission of a station is intended for a particular neighbor, and we require that there be no collision at the receiver. Here neither *primary* nor *secondary interference* is tolerated. Thus two links may not be assigned same slot if either they are adjacent or there exists a third link from the transmitter of a link to the receiver of the other link.

We address the problem of routing and scheduling[5] that minimizes the number of slots required to *evacuate* the network for given end-to-end demands. Here evacuation is the transfer of all the packets residing initially at various nodes to their respective destinations.

Previous work on link scheduling includes [2], [3], [5], [6], [8], [9], [10] and [11]. The work in these papers coves the aspect of link scheduling such as static and distributed implementations. The algorithms in these papers are based on the graph coloring methods and does not consider the mutual interference among the links assigned the same slot. This is explained in detailed in section III. The problem link scheduling by considering end-to-end demands [11] was not considered adequately in the literature. Our interest in this work is to produce near optimal solution for link scheduling by considering interference among the links assigned in the same slot and end-to-end demands of the nodes of the network.

This paper is organized as follows. Section II gives the definitions of terms and explanation of concepts that we use in later sections. Section III shows that graph-coloring method of scheduling is not suitable. And an algorithm is presented to generate maximal independent sets[6] (MISs) [4] of links in a PRN. Section IV shows the importance of combined routing and scheduling. An LP problem is formulated which minimizes the schedule length for given end-to-end demands. Results have been observed for arbitrary networks with combined routing and scheduling. Section V concludes the summary of the work.

[4]If x can transmit a message to y, there is a link from x to y.

[5] Here scheduling refer to link scheduling.

[6]An independent set of links is the set of links which can be assigned the same slot. Maximal independent is one which is not contained in any other independent set.

II PRELIMINARIES

In this section, we provide definition of, and explanation for, some of the notions that we employ throughout later sections. These include graph-coloring method of link scheduling, the network model used in our approach, and an independent set of links.

A. *Graph Coloring Method for Link Scheduling*

The work in [2, 3, 5, 6, 8] covers the aspect of link scheduling using the algorithms based on edge coloring of graphs. The network model used is a standard representation of *radio network* by a directed graph $G = (V, A)$ [13]. Here V is a set of vertices denoting the stations in the network, and A is a set of directed edges between vertices such that for any two distinct vertices $u, v \in V$, $(u, v) \in A$ if and only if v can receive u's transmission.

A natural interpretation of link scheduling in this context is a one of edge coloring the corresponding graph G = (V, A). That is, any pair of directed edges (a, b), (c, d) may be colored the same if and only if

1) a, b, c, d are all mutually distinct.
2) $(a, d) \notin A$ and $(c, b) \notin A$.

Here, when the first (second) condition fails to hold, there is a primary (secondary) edge conflicts between (a, b) and (c, d).
In section ----, we show that this method is not suitable for link scheduling.

B. *The Experimental Model*

We assume that the *radio network* is immobile in which all of the stations have the same transmitted power. That is, the transmission radius for each radio unit for given signal to interference threshold. In this context, the network may be represented by a three-tuple (N, R, P) where N is the number of stations, R is the transmission radius for given signal-to-interference threshold, and $P = \{ (x_i, y_i), 1 \leq i \leq N \}$ is the set of locations for each of the stations. The location of a station is generated randomly, using a uniform distribution for its X and Y coordinates, in a given area. We convert this network into a graph $G = (V, A)$, so that $|V| = N$, and $(u, v) \in A$ if and only the Euclidean distance between (x_u, y_u) and (x_v, y_v) is less than or equal to R. Under this model, all the edges in the graph are bi-directional. We use undirected edges for bi-directional edges.

C. *An Independent set of links*

In section III, we present an algorithm to generate maximal independent sets of links for given signal-to-interference threshold in a *radio network*. For this we use the following notions and the definition for an independent set of links. The assumed model of interference is as follows.

1) R is the coverage range of a radio unit for given transmitted power and signal-to-noise threshold.
2) γ is the path loss exponent, which we assumed to be 4.
3) S_o is the transmitted power by a radio unit and is the same for all radio units. We take the ambient noise power as unit in our approach.
4) $S_{ij} = S_o/d_{ij}^{\gamma}$ is the signal-to-noise ratio at receiver j of link (i, j).
5) $I_{ij} = \sum_k S_o/d_{kj}^{\gamma}$ is the interference power at the receiver j of link (i, j) from all other links assigned the same slot along with link (i, j). Where the summation is over all other links assigned the same slot along with link (i, j).

With this notation, a given set X of links is said to be independent under the constrained that the signal-to-interference of each link is greater than a given threshold SI_o. This can be written as

$$S_{ij}/(I_{ij}+1) \geq SI_o, \ \forall (i, j) \in X.$$

where $I_{ij} + 1$ is the interference plus ambient noise, which we assumed to be unity.

The same definition holds for an *independent set*, in subsequent sections.

III.GENERATION OF MAXIMAL INDEPENDENT SETS OF LINKS IN A PRN

In this section, we show the difficulty with graph coloring method of link scheduling. And an algorithm is presented to generate maximal independent sets of links in a PRN. The difficulty with graph coloring method for link scheduling is explained through the following examples.

Example 3.1

Consider the network on 4 nodes as shown in Figure.3.1. Using the graph coloring method, links (a, b) and (d, c) can be assigned the same slot.

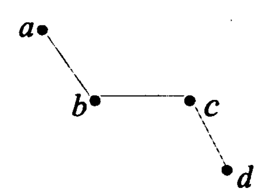

Figure 3. 1: An example of a PRN on 4 node

Suppose that these two links are not independent according to the definition of an independent set, given in section II, i. e.

$$S_{ab}/(S_{db}+1) < x \text{ or } S_{dc}/(S_{ac}+1) < x$$

where x is the signal-to-interference threshold, then these two links cannot be assigned the same slot.

Example 3.2

Consider the network on 4 nodes as shown in Fig. 3.2. Using the graph coloring method, no two links can be assigned the same slot. Suppose that links (a, b) and (d, c) can be assigned the same slot according to the definition of an independent set, given in section II, i. e. $S_{ab}/(S_{db}+1) > x$ or $S_{dc}/(S_{ac}+1) > x$ where x is the signal-to-interference threshold, then these two links can be assigned the same slot

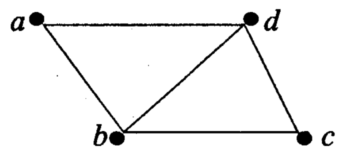

Figure 3.2: An example of a PRN on 4 nodes

From the above two examples, we can generalize that graph-coloring methods are not suitable for finding maximal independent sets of links in PRNs. No algorithm could be found in literature which consider the mutual interference among the links assigned the same slot. The algorithm described below overcome this problem. This algorithm has been used in [4] to generate maximal independent sets in cellular system modeled by hypergraphs.

This algorithm generates the maximal independent sets of links in PRNs modeled by graphs. It consists of three sets:

1) *compsub;*
2) *candidates*
3) *not*

The set *compsub* is a set of links (edges) all of which form an independent set. The set *candidates* is the set of all links that are eligible to extend the *compsub*, i.e., each of which form an independent set with *compsub*.

The set *not* is a set of vertices, all of which at an earlier stage already defined as an extension of the present configuration *compsub* and are now explicitly excluded.

A recursively defined extension operator generates all extensions of the given configuration of *compsub* that it can make with the given set of candidates and that do not contain any vertex in not. All extensions of *compsub* containing any vertex in not have already been generated. The basic mechanism now consists of the following five steps:

1) Selection of the first vertex in *candidates.*
2) Adding the selected candidate to *compsub.*
3) Creating a new set candidates from the old set by removing each vertex which does not form an independent set with the set with the selected candidate and *compsub* and forming a new set not in a similar manner from the old set not.
4) If both *not* and *candidates* set are empty, no further extension of the present configuration of *compsub* is possible, nor is there a larger independent set including the present configuration of *compsub* in graph since not is empty. Hence, *compsub* contains a maximal independent set, which is generated. If only candidates is empty, no further extension of the present configuration of *compsub* is possible and there exists a larger independent set including the present configuration of *compsub*. This independent set has been generated before. Thus, the algorithm backtracks. If *candidates* is nonempty (irrespective of whether *not* is nonempty), the extension operator is called to operate on the sets just formed.
5) Upon return, removal of the selected candidate from the *compsub* and its addition to the old set *not.*

The worst case time complexity of this algorithm is exponential in number of links in the network since the number of MISa can grow exponentially with the number of links. The memory requirement is P + |E|P, where P is the maximum size of a MIS and |E| is the number of links in the network. The algorithm generates MISs progressively. So, if a certain number of MISS are required for some application, the procedure can be stopped after that number has been obtained.

IV. COMBINED OPTIMAL ROUTING AND SCHEDULIND

In this section, we show how combined routing and scheduling improves the network throughput significantly over the scheduling with shortest-path (S-P) routing. An LP problem [7] is formulated which minimizes the schedule length for given end-to-end demands. We consider the following example to explain how combined routing and schedule minimizes the schedule length.

Example 4.1

Consider the following PRN on 6 nodes as shown in Fig 4. 1. The end-to-end demands are shown in the figure 4. 1 by the numbers on the arrows. All the remaining end-t-end demands are zeros. Assume that the nodes positions are such that the following are some of the maximal independent sets . *{(e, a), (c, d)}, {(e, a), (b, c)}, (b, d), {(f, c), (a, b)}, {(f, c), (d, a)}, {(e, a), (f, c)}.*

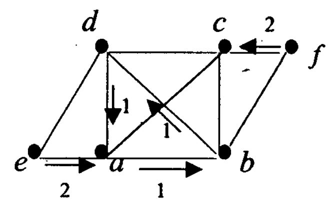

Figure 4. 1: An example of a PRN on 6 nodes.

Assume that there are no other MISs among the links specified in the above MISs. With the shortest routing between every pair of nodes, the number of slots required to evactuate the network is 5. This is obtained from the following slots assignment.

Max. Independent Set	Number of Slots Assgned
{(e, a), (f, c)}	2
{(f,c), (a, b)}	1
{(f, c), (d, a)}	1
(b, d)	1

By re-routing the traffic between b and d via b to c and then to d, the number of slots required to evacuate the network is 4.

This is obtained from the following slots assignment.

Max. Independent Set	Number of Slots Assigned
{(e, a), (b, c)}	1
{(e, a), (c, d)}	1
{(f, c), (a, b)}	1
{(f, c), (d, a)}	1

So, there is an improvement for this scenario of traffic patron.

From the above example, we arrive at the combined routing and scheduling (CRS).

A. Combined Routing and Scheduling as an LP

Consider the first K shortest paths [12] between every pair of nodes (if thet exist). Lew W be the set of all source-destination (s-d) pairs and w be any *s-d* pair. Let P_w be the set of all K (or less if K don't exist) paths for any s-d pair w and x_p be the trafic on the path p for any $p \in P_w$, $w \in W$. Let Y_{ij} be the set of all paths which contain link (i, j). Let there be a total of n *MISs* generated using the algorithm stated in Section III. Denote the *MISs* as $S_1, S_2, S_3, \ldots\ldots S_n$.

Define

$$S_i^{(j,k)} = \begin{cases} 1 & if \quad link(j,k) \in S_i \\ 0 & o.w \end{cases} \tag{1}$$

Let T be the traffic matrix, where T_{ij} is the number of packets to be transferred from node i to node j.

Then the load on any link $(i, j) = \sum_{p \in Y_{ij}} x_p$ (2)

Here the objective is to evacuate the network for given end-to-end demands with minimum schedule length.

$$i.\ e.,\ min \sum_{i=1}^{n} n_i \tag{3}$$

Subject to,

$$\sum n_i S_i^{(j,k)} \geq \sum_{p \in Y_{j,k}} x_p, \quad \forall (j,k) \in E \tag{4}$$

$$and \sum_{p \in P_w} x_p \geq T_{ij}, \quad \forall w = (i,j) \in W \tag{5}$$

where $(n_1, n_2, n_3, \ldots\ldots n_n)$ is the activation vector. Here each component n_i of the activation vector gives the number of times that the maximal independent set S_i to be activated in order to evactuate the network for given end-to-end demands given by the traffic matrix T.

The objective function given by equation (3) is the number of slots required to evacuate the network for given end-to-end demands. The set of constraints given by equation (4) ensures that the number of slots allocated to each link (j, k) is not less than the load (number of packets) on the link (j, k). And the set of constraints given by equation (5) ensures that the total traffic over all the paths between each *s-d* pair (i, j) is not les than the given T_{ij}.

B. On the performance of CRS

We study the performance of combined routing and scheduling an arbitrary network on 9 nodes as shown in Fig 4.2. Here we have not shown the details about node position, transmitted power, signal-to-interference threshold, and the set of maximal independent sets. We have cosidered all MISs and the first 3 S-Ps between every *s-d* pair. The table below shows the performance of CRS over scheduling with shortest routing. The elements of the traffic matrices T_1, T_2, and T_3 are generated using uniform distribution over 0 to 100. We have not relaxed the LP. If n_i's and x_p's are large in valuse then the error resulting from not relaxing the LP can be neglected.

In large problems (networks), we may not be able to generate all the MISs and may consider a few of them, say a set which constitute a *minimum cover*[7][10]. In this case, the improvement of CRS over scheduling with S-P routing may be even more significant. For example, by considering the MISs which constitute the *minimum cover* for this 9-node network and for the traffic matrix T_2, the improvement in throughput is 23.6 percent.

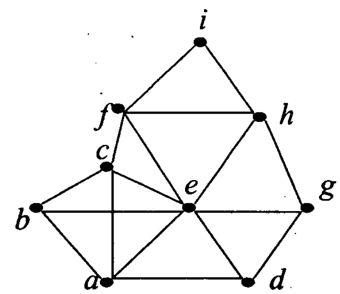

Fig 4. 2: An example of a PRN on 9 nodes

Traffic Matrix	No. of slots Required with S-P routing.	No. of slots required with CRS	Improvement in throughput
T_1	5715	4990	14.5%
T_2	5068	4624	9.6%
T_3	5045	4490	12.4%

Table showing the schedule lengths for different traffic demands

[7]A cover by maximal independent sets of the link set E of a *graph* $G = (V, E)$ is a set of maximal independent sets $S = \{E_1, E_2, E_3, \ldots\ldots, E_k\}$ such that $\bigcup_i^k E_i = E$ In this case, the size of the cover is k. A cover by maximal independent sets S = { $E_1, E_2, E_3, \ldots\ldots, E_k$} is said to be *minimal cover* if $S - E_i$ is not a cover for any i, $1 \leq i \leq k$. A *minimal cover* whose size is smallest over all possible *minimal covers* is the *minimum cover*.

V. CONCLUSIONS

A new approach for channel allocation in packet radio networks is presented. We have shown that graph-coloring method of finding maximal independent sets of links is not suitable. An algorithm is presented to find maximal independent sets, which overcomes the disadvantages with graph coloring method. We have shown how the combined routing and scheduling improves the throughput significantly. An LP problem is formulated which minimizes the schedule to evacuate the network for given end-to-end demands. Results have been observed for arbitrary networks with uniformly distributed end-to-end demands. The limitation of this approach is that the LP problem converges very slowly as the number of links in the network. This is because; the number of maximal independent sets may increase exponentially with nodes in the network.

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