

The derivative of the topological susceptibility at zero momentum and an estimate of η' mass in the chiral limit

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Abstract

The anomaly-anomaly correlator is studied using QCD sum rules. Using the matrix elements of anomaly between vacuum and pseudoscalars π , η and η' , the derivative of correlator $\chi'(0)$ is evaluated and found to be $\approx 1.82 \times 10^{-3} \text{ GeV}^2$. Assuming that $\chi'(0)$ has no significant dependence on quark masses, the mass of η' in the chiral limit is found to be $\approx 723 \text{ MeV}$. The same calculation also yields for the singlet pseudoscalar decay constant in the chiral limit a value of $\approx 178 \text{ MeV}$.

In QCD, it is known that the vacuum gauge field configurations are significant, in particular the instanton solutions corresponding to self-dual fields ($G_{\mu\nu}^a = \pm(1/2)\tilde{G}_{\mu\nu}^a$) play a role in solving the so called $U(1)$ problem, i.e., the ninth pseudoscalar η' remains massive even in the chiral limit when all quark masses are zero. The axial vector current in QCD has an anomaly

$$\partial^\mu \bar{q} \gamma_\mu \gamma_5 q = 2 i m_q \bar{q} \gamma_5 q - \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad \text{where, } \tilde{G}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a. \quad (1)$$

The topological susceptibility $\chi(q^2)$ defined by

$$\chi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ Q(x), Q(0) \} | 0 \rangle, \quad \text{with, } Q(x) = \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \quad (2)$$

is of considerable theoretical interest and has been studied using a variety of theoretical tools like lattice gauge theory, QCD sum rules, chiral perturbation theory etc. In particular the derivative of the susceptibility at $q^2 = 0$

$$\chi'(0) = \left. \frac{d\chi(q^2)}{dq^2} \right|_{q^2=0} \quad (3)$$

enters in the discussion of the proton-spin problem [1, 2, 3, 4, 5]. In the QCD sumrule approach, one can determine $\chi'(0)$ as follows. Using dispersion relation one can write

$$\frac{\chi'(q^2)}{q^2} - \frac{\chi'(0)}{q^2} = \frac{1}{\pi} \int ds \Im(\chi(s)) \left[\frac{1}{s(s-q^2)^2} + \frac{1}{s^2(s-q^2)} \right] + \text{subtractions}. \quad (4)$$

Defining the Borel transform of a function $f(q^2)$ by

$$\hat{B}f(q^2) = -q^2, n \rightarrow \infty \left[\frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2} \right)^n f(q^2) \right]_{-q^2/n=M^2} \quad (5)$$

one gets from Eq.(4)

$$\chi'(0) = \frac{1}{\pi} \int ds \frac{\Im(\chi(s))}{s^2} \left(1 + \frac{s}{M^2} \right) e^{-s/M^2} - \hat{B} \left[\frac{\chi'(q^2)}{q^2} \right]. \quad (6)$$

According to Eq.(2), $\Im(\chi(s))$ receives contribution from all states $|n\rangle$ such that $\langle 0|Q|n\rangle \neq 0$. In particular we have [6]

$$\langle 0|Q|\pi^0\rangle = i f_\pi m_\pi^2 \left(\frac{m_d - m_u}{m_d + m_u} \right) \frac{1}{2\sqrt{2}}. \quad (7)$$

The matrix elements, when $|n\rangle$ is $|\eta\rangle$ or $|\eta'\rangle$, can be determined as follows. It is known from both theoretical considerations based on chiral perturbation theory as well as phenomenological analysis that one needs two mixing angles θ_8 and θ_0 to describe the coupling of the octet and singlet axial vector currents to η and η' [7, 8, 9]. Introduce the definition

$$\langle 0|J_{\mu 5}^a|P(p)\rangle = i f_P^a p_\mu; \quad a = 0, 8; \quad P = \eta, \eta', \quad (8)$$

where $J_{\mu 5}^{8,0}$ are the octet and singlet axial currents :

$$J_{\mu 5}^8 = \frac{1}{\sqrt{6}} (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s) \quad (9)$$

$$J_{\mu 5}^0 = \frac{1}{\sqrt{3}} (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s). \quad (10)$$

The $|P(p)\rangle$ represents either η or η' with momentum p_μ . The couplings f_P^a can be equivalently represented by two couplings f_8, f_0 and two mixing angles θ_8 and θ_0 by the matrix identity

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos \theta_8 & -f_0 \sin \theta_0 \\ f_8 \sin \theta_8 & f_0 \cos \theta_0 \end{pmatrix} \quad (11)$$

Phenomenological analysis of the various decays of η and η' to determine f_P^a has been carried out by a number of authors [7, 8, 9]. In a recent analysis [9] Escribano and Frere find, with

$$f_8 = 1.28 f_\pi \quad (f_\pi = 130.7\text{MeV}), \quad (12)$$

the other three parameters to be

$$\theta_8 = (-22.2 \pm 1.8)^\circ, \quad \theta_0 = (-8.7 \pm 2.1)^\circ, \quad f_0 = (1.18 \pm 0.04) f_\pi. \quad (13)$$

The divergence of the axial currents are given by

$$\partial^\mu J_{\mu 5}^8 = \frac{i 2}{\sqrt{6}} (m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s) \quad (14)$$

$$\partial^\mu J_{\mu 5}^0 = \frac{i 2}{\sqrt{3}} (m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s) + \frac{1}{\sqrt{3}} \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \quad (15)$$

Since $m_u, m_d \ll m_s$, one can neglect them [10] to obtain

$$\langle 0|\frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}|\eta\rangle = \sqrt{\frac{3}{2}} m_\eta^2 (f_8 \cos \theta_8 - \sqrt{2} f_0 \sin \theta_0) \quad (16)$$

$$\langle 0|\frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}|\eta'\rangle = \sqrt{\frac{3}{2}} m_{\eta'}^2 (f_8 \sin \theta_8 + \sqrt{2} f_0 \cos \theta_0). \quad (17)$$

Using Eqs.(7), (16) and (17) we get the representation of $\chi(q^2)$ in terms of physical states as

$$\begin{aligned} \chi(q^2) &= -\frac{m_\pi^4}{8(q^2 - m_\pi^2)} f_\pi^2 \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 - \frac{m_\eta^4}{24(q^2 - m_\eta^2)} (f_8 \cos \theta_8 - \sqrt{2} f_0 \sin \theta_0)^2 \\ &\quad - \frac{m_{\eta'}^4}{24(q^2 - m_{\eta'}^2)} (f_8 \sin \theta_8 + \sqrt{2} f_0 \cos \theta_0)^2 + \text{higher mass states.} \end{aligned} \quad (18)$$

On the other hand, $\chi(q^2)$ has an operator product expansion [11, 12, 1, 5]

$$\begin{aligned}
\chi(q^2)_{OPE} = & - \left(\frac{\alpha_s}{8\pi} \right)^2 \frac{2}{\pi^2} q^4 \ln \left(\frac{-q^2}{\mu^2} \right) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{83}{4} - \frac{9}{4} \ln \left(\frac{-q^2}{\mu^2} \right) \right) \right] \\
& - \frac{1}{16} \frac{\alpha_s}{\pi} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \left(1 - \frac{9}{4} \frac{\alpha_s}{\pi} \ln \left(\frac{-q^2}{\mu^2} \right) \right) + \frac{1}{8q^2} \frac{\alpha_s}{\pi} \langle 0 | \frac{\alpha_s}{\pi} g_s G^3 | 0 \rangle \\
& - \frac{15}{128} \frac{\pi \alpha_s}{q^4} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle^2 + 16 \left(\frac{\alpha_s}{4\pi} \right)^3 \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle \left[\ln \left(\frac{-q^2}{\mu^2} \right) + \frac{1}{2} \right] \\
& - \left[\frac{q^4}{2} \int d\rho n(\rho) \rho^4 K_2^2(Q\rho) + \text{screening correction to the direct instantons} \right]. \quad (19)
\end{aligned}$$

In Eq.(19), the first term arises from the perturbative gluon loop with radiative corrections [12], the second, third and fourth terms are from the vacuum expectation values of G^2 , G^3 and G^4 . The $\langle 0 | G^4 | 0 \rangle$ term has been expressed as $\langle 0 | G^2 | 0 \rangle^2$ using factorization [11]. The fifth term proportional to the quark mass has been computed by us and is indeed quite small compared to other terms numerically. Finally, the last two terms represent the contribution to $\chi(q^2)$ from the direct instantons [11]. $n(\rho)$ is the density of instanton of size ρ , K_2 is the Mc Donald function and $Q^2 = -q^2$. In a recent work [13], Forkel has emphasized the importance of screening correction which almost cancels the direct instanton contribution (cf. especially Fig.8 and Secs. V and VI of Ref. [13]). For this reason we shall disregard the direct instanton term and screening correction for the present and return to it later.

From Eq.(6), we now obtain

$$\begin{aligned}
\chi'(0) = & \frac{f_\pi^2}{8} \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 \left(1 + \frac{m_\pi^2}{M^2} \right) e^{-\frac{m_\pi^2}{M^2}} + \frac{1}{24} \left(f_8 \cos \theta_8 - \sqrt{2} f_0 \sin \theta_0 \right)^2 \left(1 + \frac{m_\eta^2}{M^2} \right) e^{-\frac{m_\eta^2}{M^2}} \\
& + \frac{1}{24} \left(f_8 \sin \theta_8 + \sqrt{2} f_0 \cos \theta_0 \right)^2 \left(1 + \frac{m_{\eta'}^2}{M^2} \right) e^{-\frac{m_{\eta'}^2}{M^2}} \\
& - \left(\frac{\alpha_s}{4\pi} \right)^2 \frac{1}{\pi^2} M^2 E_0(W^2/M^2) \left[1 + \frac{\alpha_s}{\pi} \frac{74}{4} + \frac{\alpha_s}{\pi} \frac{9}{2} \left(\gamma - \ln \frac{M^2}{\mu^2} \right) \right] \\
& - 16 \left(\frac{\alpha_s}{4\pi} \right)^3 \frac{1}{M^2} \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle - \frac{9}{64} \frac{1}{M^2} \left(\frac{\alpha_s}{\pi} \right)^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
& + \frac{1}{16} \frac{1}{M^4} \frac{\alpha_s}{\pi} \left\langle g_s \frac{\alpha_s}{\pi} G^3 \right\rangle - \frac{5}{128} \frac{\pi^2}{M^6} \frac{\alpha_s}{\pi} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle^2. \quad (20)
\end{aligned}$$

Here $E_0(x) = 1 - e^{-x}$ and takes into account the contribution of higher mass states, which has been summed using duality to the perturbative term in χ_{OPE} , and W is the effective continuum threshold. We take $W^2 = 2.3 \text{ GeV}^2$, and Fig.1 plot the r.h.s. of Eq.(20) as a function of M^2 . We take $\alpha_s = 0.5$ for $\mu = 1 \text{ GeV}$ and

$$\begin{aligned}
\langle 0 | g_s^2 G^2 | 0 \rangle = 0.5 \text{ GeV}^2, \quad \langle 0 | \bar{s}s | 0 \rangle = 0.8 \langle 0 | \bar{u}u | 0 \rangle \text{ with } \langle 0 | \bar{u}u | 0 \rangle = -(240 \text{ MeV})^3, \\
m_s = 150 \text{ MeV and } m_u/m_d \approx 0.5. \quad (21)
\end{aligned}$$

Writing

$$\langle 0 | g_s^3 G^3 | 0 \rangle = \frac{\epsilon}{2} \langle 0 | g_s^2 G^2 | 0 \rangle, \quad (22)$$

as in Ref. [5], we take $\epsilon = 1 \text{ GeV}^2$. We also have the PCAC relation,

$$-2(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle = f_\pi^2 m_\pi^2. \quad (23)$$

For f_0, f_8, θ_8 and θ_0 we use the central values given in Eqs.(12) and (13).

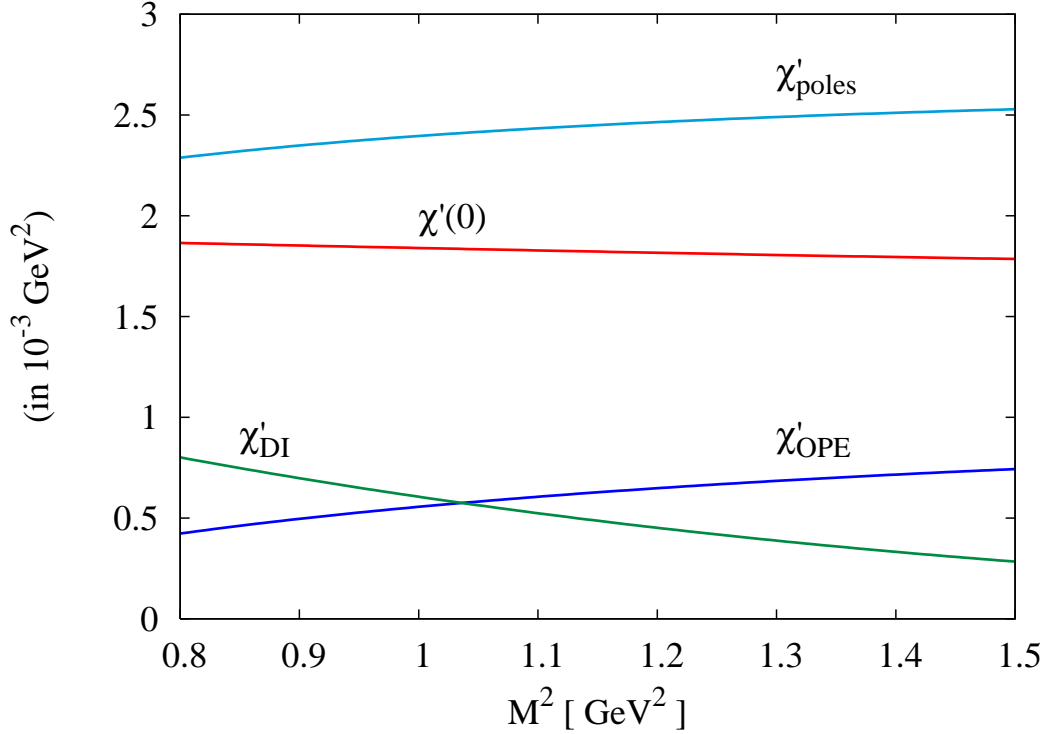


Fig. 1: Various terms contributing to $\chi'(0)$, Eq.(20). The value of $\chi'(0)$ is the one obtained without the direct instantons. The latter, see Eq.(29), is given by χ'_{DI} , which is larger than χ'_{OPE} and also has the wrong M^2 behaviour suggesting that screening corrections are important.

Let us now examine how the various terms in the r.h.s. of Eq.(20) add up to remain a constant. The pion term is small and has little variation because of its low mass, η and η' are significantly larger and η is even larger than η' . In Fig.1 the upper line gives the combined contribution of π , η and η' which we denote as χ'_{poles} and it is seen that it has a gentle increase with M^2 . The OPE term given by the last three lines in Eq.(20), which we denote by χ'_{OPE} , so that

$$\chi'(0) = \chi'_{poles} - \chi'_{OPE}$$

is also plotted in Fig.1. It is seen that χ'_{OPE} is roughly about 25% of χ'_{poles} also increases with M^2 , with the result that $\chi'(0)$ is nearly a constant w.r.t M^2 .

We expect this trend of compensating variation in χ'_{poles} and χ'_{OPE} to be maintained when variation in χ'_{poles} due to uncertainties in θ_8 , θ_0 , f_8 , f_0 [see Eqs.(12)] and (13) and the variations in χ'_{OPE} due to uncertainties in the estimates of the vacuum condensates are taken into account. We can then obtain, from Fig.1, the value

$$\chi'(0) \approx 1.82 \times 10^{-3} \text{ GeV}^2. \quad (24)$$

We note that the above determination, Eq.(24), is in agreement with an entirely different calculation by two of us [14] from the study of the correlator of isoscalar axial vector currents

$$\begin{aligned} \Pi_{\mu\nu}^{I=0} &= \frac{i}{2} \int d^4x e^{iq \cdot x} \langle 0 | \{ \bar{u} \gamma_\mu \gamma_5 u(x) + \bar{d} \gamma_\mu \gamma_5 d(x), \bar{u} \gamma_\mu \gamma_5 u(0) + \bar{d} \gamma_\mu \gamma_5 d(0) \} | 0 \rangle \\ \Pi_{\mu\nu}^{I=0} &= -\Pi_1^{I=0}(q^2) g_{\mu\nu} + \Pi_2^{I=0}(q^2) q_\mu q_\nu. \end{aligned} \quad (25)$$

$\Pi_1^{I=0}(q^2 = 0)$ can be computed from the spectrum of axial vector mesons. In Ref. [14] a value

$$\Pi_1^{I=0}(q^2 = 0) = -0.0152 \text{ GeV}^2 \quad (26)$$

was obtained. It is not difficult to see that when $m_u = m_d = 0$

$$\chi'(0) = -\frac{1}{8} \Pi_1^{I=0}(q^2 = 0) \quad (27)$$

which is consistent with Eqs.(24). Let us now return to Eq.(19) and consider the effect of incorporating the direct instanton term in Eq.(20) in the spike approximation [5]:

$$n(\rho) = n_0 \delta(\rho - \rho_c) \quad (28)$$

with $n_0 = 0.75 \times 10^{-3} \text{ GeV}^4$ and $\rho_c = 1.5 \text{ GeV}^{-1}$. The contribution of the direct instanton to $\hat{B}[\chi'(q^2)/q^2]$ can be found using the asymptotic expansion for $K_2(z)$ and $K_2'(z)$ and we find it to be

$$\chi'_{DI} = \frac{n_0}{4} \sqrt{\pi} \rho_c^4 M^2 \left[M \rho_c + \frac{9}{4} \frac{1}{M \rho_c} + \frac{45}{32} \frac{1}{M^3 \rho_c^3} \right] e^{-M^2 \rho_c^2}. \quad (29)$$

We have plottend this term separately in Fig.1. We note that unlike χ'_{poles} and χ'_{OPE} , which increase with M^2 and therefore compensate each other, the contribution of χ'_{DI} , Eq.(29), decreases rapidly with M^2 . It is not difficult to see that $\chi'(0)$ will no longer remain constant. This strongly suggests that screening corrections to χ'_{DI} are important just as they are for $[\chi(q^2)/q^2]$ as found by Forkel [13].

We now turn to an estimate of η' mass in the chiral limit: $m_u = m_d = m_s = 0$. In this limit $SU(3)$ flavor symmetry is exact and, we have $m_\pi = m_\eta = 0$ while η' is a singlet. Let us denote by $\eta_\chi = \eta'(m_s = 0)$ and $m_\chi = m_{\eta'}(m_s = 0)$, the singlet particle and its mass in the chiral limit. Returning to Eq.(19), we first note that the explicitly quark mass dependent term in χ_{OPE}

$$-16 \left(\frac{\alpha_s}{4\pi} \right)^3 \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle \approx 1.9 \times 10^{-6} \text{ GeV}^4$$

is numerically much smaller than, for example

$$\frac{9}{64} \left(\frac{\alpha_s}{\pi} \right)^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \approx 4.5 \times 10^{-5} \text{ GeV}^4$$

which itself is much smaller than the perturbative term. In the chiral limit $\langle 0|Q|\pi \rangle = \langle 0|Q|\eta \rangle = 0$. If we assume that the quark mass dependence of $\chi'(0)$ is negligible then $\chi'(0)$ in Eq.(20) can also be expressed in term of f_{η_χ} and m_χ as:

$$\chi'(0) = \frac{1}{12} f_{\eta_\chi}^2 \left(1 + \frac{m_\chi^2}{M^2} \right) e^{-\frac{m_\chi^2}{M^2}} - \hat{B} \left[\frac{\chi'_{OPE}(q^2)}{q^2} \right]. \quad (30)$$

From Eqs.(20) and (30) we may then write

$$\begin{aligned} \frac{1}{12} f_{\eta_\chi}^2 \left(1 + \frac{m_\chi^2}{M^2} \right) e^{-\frac{m_\chi^2}{M^2}} &\approx \frac{1}{24} f_\pi^2 \left(1 + \frac{m_\pi^2}{M^2} \right) e^{-\frac{m_\pi^2}{M^2}} \\ &+ \frac{1}{24} \left(f_8 \cos \theta_8 - \sqrt{2} f_0 \sin \theta_0 \right)^2 \left(1 + \frac{m_\eta^2}{M^2} \right) e^{-\frac{m_\eta^2}{M^2}} \\ &+ \frac{1}{24} \left(f_8 \sin \theta_8 + \sqrt{2} f_0 \cos \theta_0 \right)^2 \left(1 + \frac{m_{\eta'}^2}{M^2} \right) e^{-\frac{m_{\eta'}^2}{M^2}}. \end{aligned} \quad (31)$$

We can find f_{η_χ} and m_χ from Eq.(31) using the least “chi-squared” criterion in the range $0.8 \text{ GeV}^2 < M^2 < 1.5 \text{ GeV}^2$. We find $m_\chi \approx 723 \text{ MeV}$ and corresponding $f_{\eta_\chi} = 178 \text{ MeV}$. In Fig.2 we have plotted the l.h.s. and r.h.s. of Eq.(31) as a function of M^2 for best-fit values of m_χ and f_{η_χ} . The decay constant f_{η_χ} is of the same order as physical decay constants f_8 and f_0 .

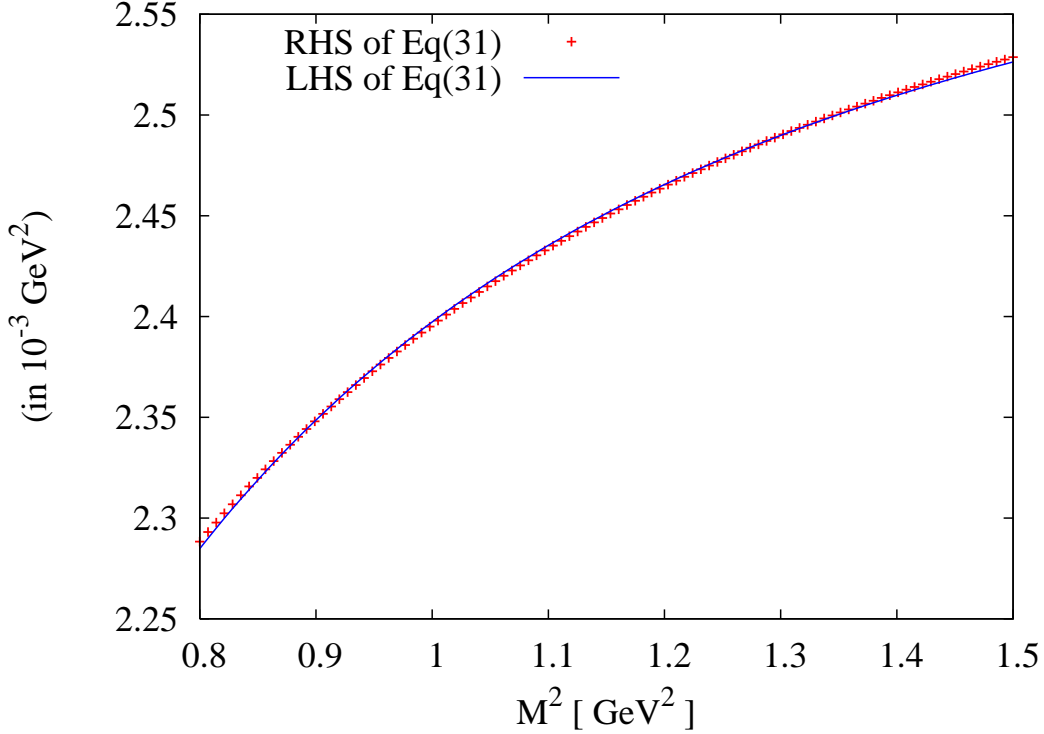


Fig. 2: Estimate of η' mass and coupling in the chiral limit, see Eq.(31). The continuous curve corresponds to $m_\chi = 723$ MeV.

We now compare our result for $\chi'(0)$ with some earlier results. In Ref. [1], Narison *et al* obtained a value for $\chi'(0) \approx 0.7 \times 10^{-3} \text{ GeV}^2$, substantially different from the value derived here. Since the expression for χ_{OPE} used by us is identical to theirs, albeit the estimate used for the gluon condensates are slightly different, we need to explain the difference in the end result for $\chi'(0)$. The most important difference is in the expression of $\chi(q^2)$ in terms of physical intermediate states. We have seen that both η and η' contribute, and in fact η makes a larger contribution than η' . In Ref. [1] only $\eta'(958)$ state is taken into account. We have also seen that if we were to take the chiral limit then η and η' contribution to $\chi(q^2)$ is representable by η_χ with mass $m_\chi = 723$ MeV, which is substantially different from the physical η' mass. This also explains why Narison *et al.* find stability in the sum rule only for rather larger $W^2 (=6 \text{ GeV}^2)$ instead of our $W^2 = 2.3 \text{ GeV}^2$. We must also add that while our Eq.(6) involves only $[\chi'(q^2)/q^2]$, Narison *et al.* use the linear combination of two sum rules (cf. Eq.(6.22) of Ref. [1]).

In Ref. [2], Ioffe and Khodzhamiryan's claim that the OPE for $\chi(q^2)$ does not converge is based on the following. They computed the correlators

$$i q_\mu q_\nu \int d^4x e^{iq \cdot x} \langle 0 | T \{ J_{\mu 5}^0(x), J_{\nu 5}^q(0) \} | 0 \rangle$$

where, $J_{\mu 5}^q = \bar{q} \gamma_\mu \gamma_5 q$ ($q = u, d, s$) with $m_u = m_d = 0$ but $m_s \neq 0$ and $j_{\mu 5}^0$ is the flavor singlet axial current. Introducing the definition

$$\langle 0 | J_{\mu 5}^q(x) | \eta'(p) \rangle = i p_\mu g_{\eta'}^q$$

they estimated

$$g_{\eta'}^s / g_{\eta'}^u \approx 2.5. \quad (32)$$

If $SU(3)$ symmetry were to be exact, this ratio would be unity. Insisting that the ratio in Eq.(32) should be close to unity even when $m_s \neq 0$, they concluded that their result signals a breakdown of OPE [2]. As discussed earlier, $\langle 0 | J_{\mu 5}^8 | \eta' \rangle \neq 0$. In fact using the values given in Eqs.(12) and (13), it is easy to obtain

$$\frac{g_{\eta'}^s}{g_{\eta'}^u} = \frac{\sqrt{2}(f_0 \cos \theta_0 - \sqrt{2} f_8 \sin \theta_8)}{f_8 \sin \theta_8 + \sqrt{2} f_0 \cos \theta_0} \approx 2.24 \quad (33)$$

which is close enough to the estimate of Ref. [2]. As in the case of Narison *et al* [1], Ioffe and Samsonov [5] and Forkel [13] also do not take into account the π , η matrix element of the anomaly in their sum rules involving $\chi(q^2)$.

In conclusion we find a value of $\chi'(0) \approx 1.82 \times 10^{-3} \text{ GeV}^2$ without incorporating direct instantons. Screening corrections to the latter appears to be significant. We also obtain an estimate $m_\chi = 723 \text{ MeV}$ and $f_{\eta_\chi} = 178 \text{ MeV}$.

ACKNOWLEDGEMENT: JPS and AU thank CHEP, IISc, Bangalore, for their hospitality at the Center where part of this work was done.

References

- [1] S. Narison, G. M. Shore and G. Veneziano, Nucl. Phys. B **546** (1999) 235 and references therein.
- [2] B. L. Ioffe and A. Yu. Khodzhmiryan, Yad. Fiz. **55** (1992) 3045.
- [3] B. L. Ioffe and A. G. Oganesian, Phys. Rev. D **57** (1998) R6590.
- [4] B. L. Ioffe, hep-ph/9511401; hep-ph/9804328.
- [5] B. L. Ioffe and A. V. Samsonov, hep-ph/9906285.
- [6] D. I. Gross, S. B. Treiman and F. Wilczek, Phys. Rev. D **19**, (1997) 2188.
- [7] H. Leutwyler, Nucl. Phys. Proc. Suppl. **64** (1998) 223; R. Kaiser and H. Leutwyler, hep-ph/9806336.
- [8] Th. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58** (1998) 114006; hep-ph/9812269.
- [9] R. Escribano and J. M. Frere, hep-ph/0501072 and references therein.
- [10] R. Akhaury and J. M. Frere, Phys. Lett. B **220** (1989) 258.
- [11] V. A. Novikov *et al.*, Phys. Lett. B **86** (1979) 347; Nucl. Phys. B **191** (1981) 301.
- [12] A. L. Kataev, N. V. Krasnikov and A. A. Pivovarov, Nucl. Phys. B **198** (1982) 508; hep-ph/9612326.
- [13] H. Forkel, Phys. Rev. D **71** (2005) 054008.
- [14] J. Pasupathy and R. K. Singh, hep-ph/0312304.