Estimation of State of Charge of Lead Acid Battery using Radial Basis Function

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Abstract: A Radial Basis Function based learning system method has been proposed for estimation of State of Charge (SOC) of lead Acid battery. Coulomb metric method is used for SOC estimation with correction factor computed by Radial Basis Function method. Radial Basis function based technique is used for learning battery performance variation with time and other parameters. Experimental results are included.

I. INTRODUCTION

Estimation of State of charge (SOC) of Lead Acid Batteries has been a subject of active research for a long time and continues to be so even today. A large number of techniques and algorithms have been proposed to predict SOC of Lead Acid batteries, each with its own limitations. In one of the earliest reported works (around 1897) Peukart [1] proposed a semi empirical formula for calculating SOC using the discharge current. Following the first impedance measurement of battery by Willihnganz [2] in 1941, several ideas were proposed for estimating SOC battery by impedance measurement. However, these parameters vary with type of battery and experimental conditions and their variation is not uniform for all battery systems. Review papers by Hampson et al. [3] and Heut [4] provide excellent insight into the applicability of impedance measurement as a test of SOC. Mathematical model based SOC prediction methods were proposed by Salameh and Cassaca [5], among others. A SOC indicator derived by extending the known open-circuit voltage-charge relation to the operating conditions and combining it with the coulomb metric method was proposed by Weiss and Appelbaum [6]. Modified coulometric methods for SOC estimation was proposed by Aylor et.al. [7], Alzieu et.al.[8] and Caumont et.al.[9], T.H. Liu et.al.[10], Torikai et.al.[11] suggested a method combining the available data of battery discharge characteristics with an identification technique based on battery's mathematical model. Several researchers have proposed SOC indicators based on neural networks, the work reported by Chan et al. [12] is one such effort. In this paper a new method for estimation of SOC of lead Acid battery is proposed which uses Radial Basis Functions combined with coulomb metric method.

II. RADIAL BASIS FUNCTION: AN OVERVIEW

Radial Basis function is used to approximate the real value function $\{f(x) : x \in \mathbf{R}^d\}$ of d variable by $\{S(x) : x \in \mathbf{R}^d\}$ with scattered data position. If $\mathbf{x}_j$ a set of point in $\mathbf{R}^d$ and $f(x)$ is a function $(f(x) : \mathbf{R}^d \rightarrow \mathbf{R})$ such that $\{f(x_j) : j = 1, 2, \dots, n\}$ and

$$f(\mathbf{x}_j) = e_j \quad (1)$$

then there is a interpolating function S such that

$$S(\mathbf{x}_j) = f(\mathbf{x}_j) \quad (2)$$

Now S has a form

$$S(x) = \sum_{j=1}^n \lambda_j \phi(\|\mathbf{x} - \mathbf{x}_j\|) \quad (3)$$

$$r = \|\mathbf{x} - \mathbf{x}_j\| \quad (4)$$

Where S is a linear combination of translates of a function ϕ . Function ϕ is called Radial Basis Function (RBF), which is a continuous spline depending upon the distances of data centers $\mathbf{x}$ ($\mathbf{x} \in \mathbf{R}^d$). As they are spherically symmetric about the centers, they are called radial. The norm is usually Euclidean. ϕ is a fixed function $\mathbf{R}^+ \rightarrow \mathbf{R}$. Some typical form of ϕ is

$$\phi(r) = r \quad (5)$$

$$\phi(r) = r^3 \quad (6)$$

$$\phi(r) = r^2 \log r \quad (7)$$

$$\phi(r) = \exp(-r^2) \quad (8)$$

$$\phi(r) = (r^2 + c^2)^{-1/2} \quad (9)$$

$$\phi(r) = (r^2 + c^2)^{1/2} \quad (10)$$

λ_j are Radial Basis function Coefficient. λ_j can be calculated with help of $\mathbf{x}_j$ and $f(\mathbf{x}_j)$, as follows

$$f(\mathbf{x}_k) = e_k \quad (11)$$

$$e_k = \sum_{j=1}^n \lambda_j \phi(\| \mathbf{x}_k - \mathbf{x}_j \|) \quad (12)$$

In matrix form (12) can be written as

$$\mathbf{A} \boldsymbol{\lambda} = \mathbf{E} \quad (13)$$

Where $\boldsymbol{\lambda}$, $\mathbf{E}$ are

$$\boldsymbol{\lambda} = [\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \dots \lambda_j \dots]^T \quad (14)$$

$$\mathbf{E} = [e_1 e_2 e_3 e_4 e_5 \dots e_k \dots]^T \quad (15)$$

Where A_{jk} is an element of $\mathbf{A}$ matrix

$$A_{jk} = \phi(\| \mathbf{x}_k - \mathbf{x}_j \|) \quad (16)$$

Radial basis function coefficients λ_j can be calculated by solving (13) (linear simultaneous equations). It may be noted that the matrix $\mathbf{A}$ must be non singular to solve (13) for calculation of λ_j . For many of the radial basis functions the non singularity of matrix $\mathbf{A}$ is guaranteed as mentioned by Powell [13]. Some of references for Radial Basis Function are mentioned [14], [15].

III. PROPOSED METHOD

Coulomb metric method is one of the methods to measure SOC of Lead Acid Battery. In coulomb metric method, SOC estimated by subtracting charge flow out of battery from the initial existing charge as described by (17).

$$SOC = S_{initial} - \text{Chargeflow out of battery} \quad (17)$$

$S_{initial}$ is initial state of charge of battery (i.e. Before discharge takes place, if battery is full charge $S_{initial} = 100$). However, the non-linear and time varying behavior of battery poses a severe problem with the coulomb metric method. As for example, a battery of capacity C Amph at C/20 rate would suggest different capacities with different discharge rates. The capacity of battery is also a function of battery temperature [16]. Also a battery display a variation in performance with aging and other operational factors (Charging pattern, Depth of Discharge). Thus preventing a look up table based procedure to estimate SOC, unless the look up table is modified with aging of battery and other variation. In this paper, a learning system has been proposed using Radial Basis Function Interpolation method for on line learning of battery characteristics with coulomb metric method. A parameter ε which is a function of discharge rate and temperature is introduced to compensate the effect of non linearity discuss earlier. Therefore, (17) can be written as (18)

$$SOC = S_{initial} - \varepsilon * \text{Chargeflow out of battery} \quad (18)$$

$$SOC = S_{initial} - \frac{1}{N_d} \int \varepsilon(i, T) i dt \quad (19)$$

Equation (19) is written in integral form, N_d is a normalizing factor, such that SOC can be expressed in percentage form. Functional form of ε is not available in general and there is a need of correcting ε with the variation of battery performance for error free estimation. These two objectives are met by using Radial Basis Function. Radial Basis function is used to map ε from discharge and temperature data. When battery performance is altered due to aging and other factors, the ε automatically adapts itself and minimizes the errors in estimation of SOC. Radial Basis Function system must learn the initial nature of ε from battery manufacturer data or by experiment at different temperature and discharge rate or through some empirical formula given by corresponding battery manufacturer. Therefore, the system is initialized by the knowledge of characteristics of a battery from a specific manufacturer and type.

IV. LEARNING PHASE

A. Calculation of Error

Initially two types of errors are considered for the estimation of SOC. The positive error occurs when the estimated charge is zero and actual battery charge is non zero. The negative error occurs when the estimated charge is non zero while the actual charge is zero. A cutoff voltage is considered as a marking for zero SOC, for a single cell lead acid battery nominal V_{cutoff} is 1.75 V (For C/20 rate discharge). The cutoff voltage also depends upon the discharge current [17]. In case of positive error, SOC estimation gives zero SOC reading before reaching cut-off voltage. Therefore, an estimation of error is taken as follows.

$$AE = \frac{V_{Battery} - V_{cutoff}}{V_{cutoff}} \quad (20)$$

In the second case when battery reaches cut-off voltage but SOC reading is higher than zero the SOC reading itself is taken as error term (21).

$$AE = SOC \text{ indicates} \quad (21)$$

AE is multiplied by correction factor whose values depends on As value itself as well as AE is positive or negative introducing non-linearity to As value for controlled convergence of algorithm and fail safe operation. For example, if As value is too small but positive As is made equal to zero to avoid unnecessary oscillation. If AE is negative it is multiplied by a high value so that next cycle AE is either zero or positive as a fail-safe condition. A negative value of As indicates that the battery possesses charge but in reality has no charge. This is a critical situation. On contrary, the converse is not so critical.

B. Input Space Partitioning

To reinitialise Radial Basis Function coefficient, information of battery discharge rate and temperature is required with error term $\Delta\epsilon$.

Fig. 1 Input Variable Space I and T Partition

Because ϵ is dependent on the magnitude of discharge rate and temperature during discharge process and most likely ϵ at this discharge rate and temperature has error. The system keeps a record of the discharge rate and temperature. To keep record of discharge rate and temperature information the following procedure is adopted. Input space (Discharge rate and Temperature) partitioned in to n number of division for each variable as shown in fig 1 (P_I denotes partition for current and P_T for temperature). The number of partitions may not be the same width for all input variables in general. Discharge current and temperature collected through each sampling is inspected for which partition it belongs. Two parameters are allotted for each partition. The first parameter counts the number of times the sampled data fall into particular partition and the second parameter keeps the sum of corresponding data. After completion of full discharge, average is calculated for each partition. Partition average of the maximum fired partition in temperature and discharge current may be taken as discharge current and temperature information for Radial Basis Function coefficient calculation

C. Learning

Learning vector set consists of current temperature and corresponding value of ϵ . Current i and temperature T are the input variables to the RBF system and are denoted through vector $\mathbf{x}_k$ ($k=1, n$) where $\mathbf{x}_k = (i, T)^T$. RBF coefficients λ is calculated from learning vector set through (22). Equation (22) is generated from (13)

$$\lambda = A^{-1}E \quad (22)$$

A matrix is calculated by (16). E is a column vector whose element e_k is the value of ϵ for the vector $\mathbf{x}_k$. Initial learning process is accomplished by battery information available from the data provided by manufacturer or through experiment). An incorrect prediction of SOC after a fill

charge-discharge cycle calls for the modification of RBF system such that in next cycle, the prediction is correct. Therefore, a modification of Radial Basis Function parameters λ are required.

The Radial Basis Function parameter λ ($\lambda = [\lambda_1, \lambda_2, \dots, \lambda_n]^T$) are recalculated with the current rate, temperature and error information. Modification of λ can be done locally or globally. Local change is defined as re-initialization of Radial Basis Function system (i.e. calculation of λ_j) such that ϵ of previous discharge (most occurring discharge rate and temperature) rate and temperature is corrected by the error amount. In local change the nearest (measuring Euclidean norm) of initial learning set input vector changed to observe discharge rate and temperature and corrected ϵ and then λ is calculated by (22). Global change is defined as re-initialization of Radial Basis Function system such that ϵ is corrected by error amount irrespective of current and temperature information (i.e. correction in ϵ is made for all discharge rate and temperature) and then λ are calculated by (22).

D. Multi Rate discharge

In the previous paragraph the algorithm is described to find a new ϵ when the discharge rate and temperature are limited to a single partition for most of time. In case they are not limited to a single partition, it is difficult to evaluate the error in ϵ . If the battery discharge at 10 Amp rate for a time duration of t_1 and at a rate 20 Amp for the rest of time the division of error amount in ϵ into respective component is not known. Hence, the problem is to solve for two variables with a single equation, describe by the (23). Here ΔC_1 is the error in estimation of SOC .

$$\int_{I_1} i_1 (\epsilon + \Delta\epsilon_1) dt + \int_{I_2} i_2 (\epsilon + \Delta\epsilon_2) dt = C + \Delta C_1 \quad (23)$$

Where integral I_1 is evaluated for a time duration of t_1 and integral I_2 for the rest of the time. However, this situation can be overcome if in another discharge event takes place. In which most of time current rate and temperature belongs to same partition as in the case of previous one but for different interval describe by (24)

$$\int_{I_3} i_1 (\epsilon + \Delta\epsilon_1) dt + \int_{I_4} i_2 (\epsilon + \Delta\epsilon_2) dt = C + \Delta C_2 \quad (24)$$

Here $\Delta\epsilon_1, \Delta\epsilon_2$ denotes error in ' ϵ ' and $\Delta C_1, \Delta C_2$ are error in SOC for discharge current i_1, i_2 respectively. If there is no error in ϵ the equations will be like (25).

$$\int_{I_1} \epsilon dt + \int_{I_2} \epsilon dt = C \quad (25)$$

From (23), (24) and (25) a linear set of equations can be formed in terms of $\Delta\epsilon$ as described (26) and (27)

$$I_1 \Delta \varepsilon_1 + I_2 \Delta \varepsilon_2 = \Delta C_1 \quad (26)$$

$$I_3 \Delta \varepsilon_1 + I_4 \Delta \varepsilon_2 = \Delta C_2 \quad (27)$$

$\Delta \varepsilon$ can be found solving (26) and (27).

This procedure can be extended for 'n' different discharge rate and temperature. Expressed in matrix form

$$I \Delta \varepsilon = \Delta C \quad (28)$$

Where

$$\Delta C = [\Delta C_1, \Delta C_2, \Delta C_3, \dots, \Delta C_N]^T$$

$$\Delta \varepsilon = [\Delta \varepsilon_1, \Delta \varepsilon_2, \Delta \varepsilon_3, \dots, \Delta \varepsilon_N]^T$$

Equation (28) is solvable provided matrix I is not a singular.

V. EXPERIMENTAL RESULT

Fig. 2 shows the experimental setup for testing the algorithm. A 486 PC based data acquisition system has been used with 12 bit ADC (AD674) and sampling time of 500 ms. First Radial Basis Function system is initialized with current temperature and ε data form (29).

$$\varepsilon(i, T) = \frac{1}{(1 - k \log(\frac{i}{i_r})) + a(T - 300)} \quad (29)$$

Equation (29) is the modified form referred in [15], (29) is least square fitted with the experimental data of a 80Amp 12 V battery. i_r is reference current, $\varepsilon(i_r, 300K)$ equal to one, k and a are two parameters, k can be determined by least square fitting from second source information. Initial ε plot with current and temperature is shown in fig 3.1. A number of experiments are carried out for validation of proposed method. In each experiment initial learning vectors is perturbed to test the effectiveness of algorithm.

Fig. 2 Experimental Setup

Fig. 3.1 Initial Interpolated $\varepsilon(I, T)$ by Radial Basis Function

Fig. 3.2 $\varepsilon(I, T)$ After fifth Learning Cycle

First experiment has been done with 80 Amp 12V battery with discharge rate C/10 at room temperature. Before starting the each experiment, battery is in full Charge State. Fig. 3.2

shows the ' ϵ ' plot after five learning cycles. In this learning process, local correction is done for discharge rate only, and global correction for temperature. (i.e. correction is made for all temperature data) with the assumption that the temperature performance of battery altered very slowly with time while the discharge rate performance is most responsible for the error. Fig. 4 shows the approximate error with each cycle of learning for the estimation of SOC in experiment. The long cycles experiment results are shown in fig. 5, fig. 6 and fig. 7. Fig 5 described the error in SOC estimation for a 20Ah 12V VRLA battery with discharge rate about C/3 at room temperature. Fig 6 describe error in SOC estimation for a 80Ah 12V VRLA battery with multiple discharge rate of random duration. Where discharge rates are between C/3 and C/10. Fig 7 shows error in SOC prediction for a 80 Ah 12V sealed lead acid battery with discharge rate of C/6.

Fig. 4 Error in SOC Estimation with each Learning cycle

Fig 5 Error in SOC for 20Ah 12V VRLA battery (C/3 discharge rate)

Fig 6 Error in SOC for 20Ah 12V VRLA battery (Multiple discharge rate)

Fig 7 Error in SOC for 20Ah 12V VRLA battery (C/6 discharge rate)

VI. CONCLUSION

A new approach has been described to estimate the SOC of Lead Acid battery using Radial Basis Function based learning method. The proposed method considers battery non-linearity due to discharge rate, with temperature and corrects itself for aging and other variations of the battery characteristics to estimate SOC. Experimental results suggest that proposed method give excellent prediction of SOC assuming that the initial charging state of battery is known and is able to learn performance variation. The proposed algorithm can further be extended to include factors such as incomplete charging and interrupted discharging.

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