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Mechanism to synthesize a 'moving optical mark' at solid-ambient interface for the estimation of thermal diffusivity of solid

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A novel mechanism is proposed, involving a novel interaction between solid-sample supporting unsteady heat flow with its ambient-humidity; invokes phase transformation of water-vapour molecule and synthesize a 'moving optical-mark' at sample-ambient-interface. Under tailored condition, optical-mark exhibits a characteristic macro-scale translatory motion governed by thermal diffusivity of solid. For various step-temperature inputs via cooling, position-dependent velocities of moving optical-mark are measured at a fixed distance. A new approach is proposed. 'Product of velocity of optical-mark and distance' versus 'non-dimensional velocity' is plotted. The slope reveals thermal diffusivity of solid at ambient-temperature; preliminary results obtained for Quartz-glass is closely matching with literature.
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INTRODUCTION

Thermal diffusivity is a thermo-physical parameter of paramount importance while addressing issues relevant to thermal design/management and control in real world applications. It appears as a constant of proportionality in the Fourier's equation, for the conduction of heat in presence of non-stationary temperature field. One of the most popular techniques to estimate thermal diffusivity is Flash method. It utilizes a pulsed heat input at the front surface and monitors the time required for the temperature at the rear surface of sample to reach a maximum; the constitutive equation relates time taken for fraction of Non-dimensional temperature, sample thickness and thermal diffusivity.[1] For dielectric solids with low thermal diffusivity and low absorption, estimation of thermal diffusivity with the support of ambient is preferable[2] and most conventional techniques are Photoacoustic technique[3] and Photothermal beam deflection technique.[4,5]

The mechanism involved in Photothermal beam deflection technique is as follows. When sample surface is exposed to Electromagnetic radiation resulting damped thermal wave is generated at sample surface extended spatially in the neighbour air medium and produces an invisible optical mark. For various input of chopping frequencies, the zero deflection of probe beam (at thermal wave length $\lambda/2$) is obtained experimentally. The plot of wavelength dependent parameter (x_0) versus square root of frequency, the slope of the curve reveals square-root of thermal diffusivity.[4]

In this letter, a novel mechanism is proposed to synthesize a 'moving optical mark' at sample-ambient-interface resulting from unsteady heat flow in solid via step-temperature excitation (cooling). A new approach and a new analytical model are proposed to estimate thermal diffusivity of solid based on measurement of 'position-dependent velocity' of a moving optical mark at a given distance and non-dimensional velocity.

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ANALYTICAL MODEL

Let $T(x,t)$ be the one-dimensional temperature distribution in homogeneous semi-infinite bulk-solid that results, after a time lapse of t seconds. If α is the thermal diffusivity of the solid, in the absence of sources/sinks of heat inside its surfaces the heat conduction in such sample is governed by the partial differential equation

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \tag{1}$$

For a step-temperature scheme of excitation with the initial condition $T(x,0) = T_i$ and the boundary conditions $T(0,t) = T_0$ and $T(x \to \infty,t) = T_i$, the expression for the temperature $T(x,t)$ can be obtained from the exact solution of the above equation and can be written as[6]

$$T(x,t) = T_i + erfc(\eta) \times (T_0 - T_i) \tag{2}$$

Where $'erfc'$ is complementary error function evaluated at for a numerical constant $'\eta'$ defined by

$$\eta = \frac{x}{\sqrt{4\alpha t}} \tag{3}$$

Hitherto, the equations (2) and (3) have been used to predict the temperature distribution $T(x,t)$ resulting in a material of known thermal diffusivity 'α' from step-temperature thermal excitation under initial and boundary conditions specified by T_i and T_0. It can also be used to estimate 'α', by curve fitting the experimentally measured data on $T(x,t)$ into the above equations.

Let ΔT_c represent the 'change in temperature' above/below the initial temperature inside the sample, for every choice of (x,t) defined by

$$\Delta T_c = T(x,t) - T_i \tag{4}$$

The equation (3) can be rewritten as

$$(x_{T_B})^2 = 4(\eta_{T_B})^2(\alpha t) \tag{5}$$

For 1D unsteady heat flow via step-temperature excitation, the notation x_{T_B} and η_{T_B} is to emphasize that after a definite time elapse 't', at every location along heat ray path, there is a unique thermal disturbance with a magnitude specified by T_B and this thermal disturbance has a time varying non-dimensional quantity η_{T_B} that specifies the strength of ΔT_c relative to the strength of thermal disturbance at $x = 0$, and is given by

$$\Delta T_c = (T_B - T_i) = erfc(\eta_{T_B})(T_0 - T_i) \tag{6}$$

$$\text{Defining } \zeta_{T_B} = 2(\eta_{T_B})^2 \tag{7}$$

The equation (5) can be rewritten as

$$(x_{T_B})^2 = 2\zeta_{T_B}(\alpha t) \tag{8}$$

Differentiating the above equation w.r.t 't' and using

$$v_{T_B} = \frac{dx_{T_B}}{dt} \tag{8a}$$

Equation (8) can also be written as

$$v_{T_B} x_{T_B} = \zeta_{T_B}\alpha \tag{9}$$

Equation (9) emphasizes that in a sample supporting non-stationary heat flow, there exists a unique speed v_{T_B} directed along the ray path of heat flow and a unique non-dimensional constant ζ_{T_B} (derived from η_{T_B}), in addition to the unique thermal disturbance T_B. It can also be interpreted to suggest that the thermal disturbance at every location is in non-uniform translatory motion possessing but also has a unique speed at every location, designating v_{T_B} and ζ_{T_B} as non-stationary and

stationary attributes of temperature disturbance of specific magnitude.

$$\frac{v_{T_B}}{\zeta_{T_B}} * x_{T_B} = \alpha \tag{10}$$

A new relationship for the thermal diffusivity of the solid is given in the above equation. At a distance x_{T_B}, the velocity of optical mark is modulated by ζ_{T_B} so it is called as Non-Dimensional Velocity (NDV). If NDV is unity, then moving optical mark indicates the 'motion of effective temperature point' discussed by Settu Balachandar *et al*.[7] It paves a way for direct measurement of thermal diffusivity i.e., measuring velocity of optical mark at a unit distance.

Let us consider an optical mark with temperature T_B in the heat ray path. Using chain rule, in Fourier equation (1) becomes,

$$\alpha \frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial x}{\partial t} \frac{\partial T(x,t)}{\partial x} \tag{11}$$

$$\text{Velocity} = v = \frac{\partial x}{\partial t} \tag{11a}$$

Then,

$$\alpha * \frac{\partial^2 T(x,t)}{\partial x^2} = v * \frac{\partial T(x,t)}{\partial x} \tag{12}$$

Velocity is readily obtained by 1st order and 2nd order differentiation of equation (2) and substitute in above equation. The velocity of optical mark is given by[7]

$$v(x,t) = \frac{x}{2t} \tag{13}$$

It should be noted that the 'velocity of temperature wave' is well known in literature. The velocity of optical mark due to step-temperature excitation (given in equation (13)) is different in the following manner. The 'velocity' of temperature- wave resulting from periodic thermal excitation is same for all moving isothermal surfaces, and not depends on either its magnitude or its position. Because in quasi steady state, all isothermal surfaces are marching with same velocity governed by its wavelength and its input frequency.[6,8] In the case of unsteady heat flow via step-temperature excitation, all isothermal surfaces are emerging from its origin exhibits rectilinear translatory motion with a position dependent velocity i.e., in transient heat flow all isothermal surfaces are moving with unique velocities at a given location.

NDV is readily obtained using the following equation obtained from equation (6),

$$\zeta_{T_B} = 2\left[inverfc\left[\frac{T_B - T_i}{T_0 - T_i}\right]\right]^2 \tag{14}$$

Where, inverfc is inverse complementary error function. From the knowledge of T_i, T_0, and T_B, NDV can be computed directly. For the estimation of diffusivity of solid, the position-dependent velocity of optical mark is measured for the given distance x and NDV using equation (10).

NOVEL MECHANISM

The mechanism and principle of hygrometer is well known.[9] The standard ambient-air is characterized by an ambient temperature, relative humidity and the dew-point temperature (T_{dp}). At T_{dp}, saturation water vapour pressure is equal to partial pressure of water vapour (in an ambient-air). The proposed mechanism is based on the interaction between a 'unique temperature point T_B' (equivalent to T_{dp}) in the time varying temperature field (inside the sample) resulting from step-temperature excitation via cooling and water vapour molecules present in the close proximity of surface of the sample. The said interaction induces phase transformation of water vapour molecules from gas-phase to liquid-phase, and deposited as a thin dew-film on lateral surface of the sample. The

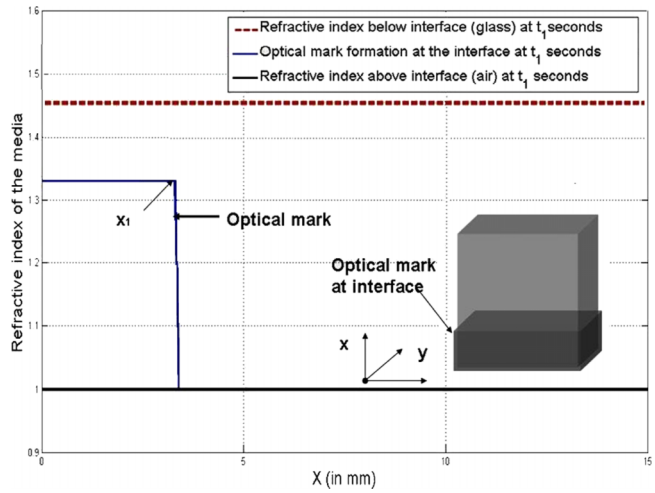

FIG. 1. The refractive index profile below the interface, at the interface, and the above the interface. The optical mark located at x_1 on lateral surface of the sample at time elapse t_1 seconds.

refractive index of Quartz glass is 1.457 and the refractive index of air is 1. The 'moving boundary of thin dew-film' is called as a 'moving optical mark'. Optical mark indicates a unique temperature point T_B with a new refractive index 1.33 i.e., as a consequence of said interfacial interaction, the refractive index of the ambient changed from 1.0 to 1.33. It can be considered as a giant refractive index transformation compare to refractive index change inside the solid (4 orders of magnitude higher). The refractive index profile below the interface, at the interface, and the above the interface after a delay time t_1 seconds are shown in figure 1.

The proposed mechanism separates a unique moving temperature point in time varying temperature profile with a huge optical contrast. The boundary of the transformed region is a 'sharply falling step' and executes a translatory motion as long as, the unsteady heat *flows* in semi-infinite solid-sample. The proposed mechanism is corroborated in the following.

The photograph of experimental setup is shown in the figure 2. It consists of LED light source, Quartz glass sample, Peltier module with power supply, and camera controlled by computer. For the given optical configuration, sample height is calibrated in terms of pixels, and using Digital hygrometer the ambient temperature (equal to sample initial temperature), and dew point temperature are measured. Sample is uniformly illuminated with LED light and the reflected light from the sample is captured by CCD camera. Experiment starts when sample is brought into thermal contact with Peltier cold arm at time t=0. The optical mark formed instantaneously at the sample-ambient interface and started moving away from its origin (at x=0), undergoes rectilinear translatory motion. The motion of optical mark at one of its lateral surfaces is recorded using CCD camera. For various step-temperature inputs (T_0), the thermal diffusivity dependent velocity of the optical mark

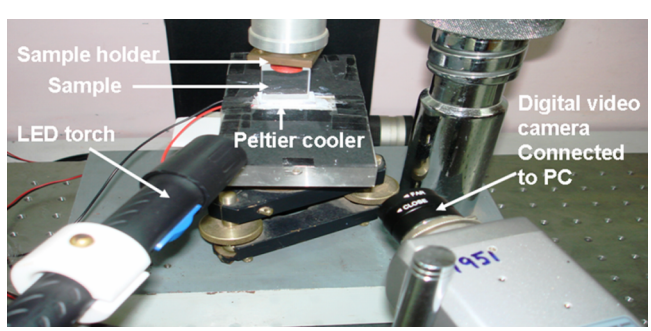

FIG. 2. Photograph of experimental setup.

TABLE I. Experimental values and theoretical value using equation (9).

s.no	T_0	ζ_{T_B}	V*x measured value in mm^2/s	Theoretical value ζ_{T_B}*α mm^2/s
1	6.4	0.6548	0.354	0.5684
2	5.5	0.7150	0.4088	0.6206
3	4	0.8115	0.5302	0.7044
4	3.5	0.8427	0.5471	0.7315
5	1.5	0.9627	0.6668	0.8356

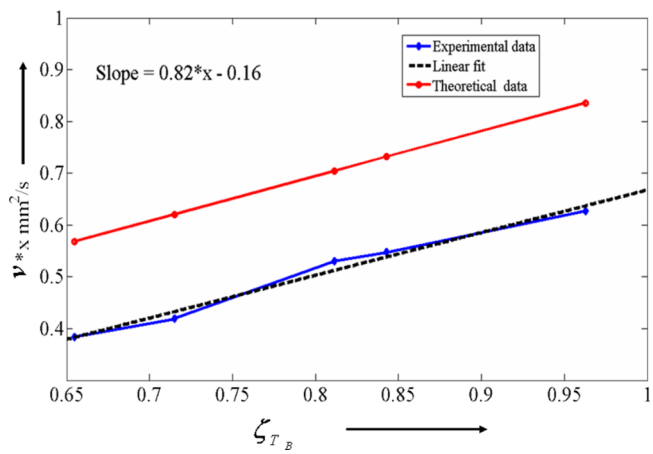

FIG. 3. Plot between 'the product of experimentally measured velocity and distance υ*x in mm^2/s' against non-dimensional velocity ζ_{T_B}.

is measured at the fixed distance away from the cooler. The location of optical-mark is estimated with suitable image processing. The experimental values are tabulated in table I.

In table I theoretical value is higher because the thermal resistance (between Peltier cooler and sample) and heat losses are not included in the preliminary results. For every experimental reading, the experiment is repeated for 5 times and mean value is taken. The graph is plotted between 'product of measured velocity and its distance x' against NDV ζ_{T_B}, the slope of the curve gives thermal diffusivity of solid shown in figure 3. The preliminary result obtained through the proposed method is 0.82 mm^2/s, and closely matching with literature value for Quartz sample is 0.868mm^2/s.[10]

The experimental setup is simpler compare to conventional optical techniques photothermal beam deflection technique. The moving optical mark is visible to naked eye, and the refractive index change at interface is giant compare to refractive index change in photo thermal beam deflection technique. So the requirement of high sensitive detector is eliminated. The proposed approach is based on velocity of moving optical mark therefore dynamic temperature measurement is eliminated. By conditioning the ambient-humidity the value of T_B can be altered.

Like Mirage effect, and Photoacoustic effect, the moving optical mark resulting from 'condensation of water vapour' molecule is introduced for the first time for the estimation of thermal diffusivity of solid. It is named as '*Dew-film boundary translatory motion technique*'. The temperature associated with optical mark is completely governed by ambient parameters so the mark formation is sample independent and only position of the mark has governed by diffusivity of sample.

In conclusion, a novel mechanism for the estimation of thermal diffusivity based on 'position-dependent translatory motion-velocity of optical-mark' at sample-ambient-interface is proposed for the first time with the support of ambient-humidity. The proposed approach is suitable for optically transparent solid with low thermal diffusivity and for samples which are not sensitive to moisture. The main advantage of the proposed method is; it eliminates sample preparations like optically smooth/ parallelism. Optical mark can be seen fairly visible with high contrast with an unaided eye. The measurement of thermo-optic coefficient of sample, and temperature dependent ambient

parameters are eliminated. The preliminary result of the proposed approach is reported and close agreement with literature.

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