Practical Routing and Wavelength Assignment algorithms for All Optical Networks with Limited Wavelength Conversion

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Abstract— We present an Integer Linear Program based algorithm and a K shortest path based heuristic algorithm for solving the Routing and Wavelength Assignment problem in All Optical Networks with Limited Wavelength Conversion. These algorithms are executed on a random mesh National Science Foundation Network(NSFNET). Their performances are compared.

I. INTRODUCTION

Wavelength Routed All Optical Networks are the most promising candidates for meeting the growing high capacity and varied needs of the telecommunication industry. Lightpaths are logical channels which provide an end-to-end connectivity in the all optical network [5]. The performance of the Wavelength Division Multiplexing(WDM) networks can be improved by allowing wavelength translation(conversion) at the routing nodes [1],[6]. The key issue in the all optical network design process once the physical realization has been achieved, is to properly dimension the network with respect to wavelength numbers and to determine a wavelength allocation plan. The above functionality is done by provisioning algorithms.

For circuit switched lightpath service on an optical network, each lightpath will carry traffic related to only one source-destination pair. A session is a set of connections between source-destination pairs. For example (A,B : 1),(B,C : 2) is a session wherein the number of connections required from node A to node B is one and from node B to node C is two.

A. Limited Wavelength Conversion

In this paper we assume that for any given input wavelength, it is possible to translate it to a limited range of output wavelengths at any node in the network where wavelength converters are deployed. If λ_i is the input wavelength then it can be converted to any of the following outgoing wavelengths $\lambda_i, \lambda_{i+1}, \dots, \lambda_{(i+(d-1)) \bmod F}$. Where d is the degree

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of wavelength conversion and F is the number of wavelengths supported by the fiber.

II. ROUTING AND WAVELENGTH ASSIGNMENT(RWA) PROBLEM

The physical network topology is represented as $G(N, E, F)$, in which N represents the set of nodes, E represents the set of directional fibers, and F represents the set of wavelengths on each link. The physical topology, the traffic matrix are given as input for the problem. Our objective is to maximize the number of lightpaths to be established from the traffic matrix. Since we try to maximize connections in a given session or traffic matrix, for a fixed set of wavelengths, it is called Max-RWA problem.

Any algorithm that solves the above problem in general should:

(a). Maximise the number of lightpaths established using the minimum number of wavelengths. (b). Assign two lightpaths the same wavelength in a link.

The following assumptions are made in our RWA problem:

(a). The number of wavelengths in each link of the fiber is assumed to be same. Each call requires a full wavelength on each link of its path. The capacity of each wavelength is assumed to be same. (b). Wavelengths are assigned uniformly randomly from the set of free wavelengths on the associated path. (c). Simplex connections are considered. (d). We assume that all the nodes in the network are equipped with wavelength converters with limited conversion capability.

A. Linear Program

We formulate the Max-RWA problem as a Mixed Integer Linear Program (MILP). In the formulation the paths for a connection are not specified before hand, the linear program solver is allowed to choose any possible path and any possible wavelength for a source-destination pair. Thus the logical topology design and wavelength assignment are inbuilt in the formulation itself, which is not in the formulations presented in [2],[3]. The constraints involve the edges or arcs of the network. By this approach

the solution for the RWA problem tends to optimality.

We use the following notations:

' i ' and ' j ' denote originating and terminating node of a lightpath. ' l ' and ' m ' denote the endpoints of a physical link. ' q ' used as subscript denotes the q th lightpath between a source-destination pair.

The *Parameters* of the formulation are:

N = Number of nodes in the network. ρ = the traffic matrix, i.e., $\rho(i, j)$ is the number of connections that are to be established between node i and node j . $P_{l,m}$ denotes the existence of a link in the physical topology. If $P_{l,m} = 1$ then there is a fiber link between node l and m , otherwise $P_{l,m}$ is 0. F denotes the number of wavelengths the fiber can support.

The *Variables* of the formulation are:

$\lambda_{(l,m),q}^{i,j} = 1$, if a light path setup between node i and node j uses the q th multiple link or q th wavelength of the edge (l, m) else $\lambda_{(l,m),q}^{i,j} = 0$ and $q \in \{0, 1, 2 \dots F - 1\}$.

$\eta_{q,\hat{q}}^{(i,j),m}$ denotes the number of connections of source-destination pair (i, j) that undergo wavelength conversion, from wavelength q to $\hat{q}$, at the node m . This variable takes only positive integral values.

$b(i, j)$ denotes the number of lightpaths established between node i and node j . It takes positive integral values.

A.1 Objective

$$\max \sum_i \sum_j b(i, j)$$

The objective here is to maximize the number of connections to be established from the given traffic matrix.

A.2 Constraints

- - *Flow conservation constraints at source and destination nodes of a logical link*

$$\sum_q \sum_m \lambda_{(i,m),q}^{i,j} P_{(i,m)} = \sum_q \sum_l \lambda_{(l,j),q}^{i,j} P_{(l,j)} = b(i, j) \quad \text{for all } (i, j).$$

$$b(i, j) \leq \rho(i, j), \quad \text{for all } (i, j).$$

Remark: These constraints ensure that demand at the origin and destination node for a source-destination pair (i, j) is satisfied.

- - *Limited wavelength conversion constraints*

$$\sum_l \lambda_{(l,m),q}^{i,j} P_{(l,m)} = \sum_{\hat{q} \in I'(q)} \eta_{q,\hat{q}}^{(i,j),m} \quad (A1)$$

for all $(i, j), q$ and $m \neq i, m \neq j$.

$$\sum_{\hat{q} \in I'(q)} \eta_{q,\hat{q}}^{(i,j),m} = \sum_l \lambda_{(m,l),q}^{i,j} P_{(m,l)} \quad (A2)$$

for all $q, (i, j)$ and $m \neq i, m \neq j$.

Remark: Eqn A1 ensures that the number of lightpaths at a node m on wavelength q for a source-destination pair (i, j) is equal to the number of lightpaths summed over all wavelengths in $I(q)$ at node m for source-destination pair (i, j) . $I(q)$ is the set of wavelengths to which the traffic from wavelength q can be distributed. Constraint A2 ensures that number of light paths that are converted to wavelength q at node m for source-destination pair (i, j) is equal to the number of lightpaths summed over all wavelengths in $I'(q)$ at node m for source-destination pair (i, j) . $I'(q)$ is the set of wavelengths from which traffic can be collected to wavelength q .

The concept behind the above wavelength conversion constraints is represented in figure 1.

Fig. 1. Representation of wavelength conversion at a node according to the ILP formulation. Here two input and output fibers are taken each with two wavelengths. Degree of conversion allowed in this figure is 2. The parameters of the η variable in the figure represent only the wavelengths.

- - *Wavelength clash constraints*

$$\sum_i \sum_j \lambda_{(l,m),q}^{i,j} P_{(l,m)} \leq 1 \quad \text{for all } (l, m), q$$

Remark: This ensures that no two lightpaths are assigned the same wavelength on link (l, m) .

A.3 Rounding Heuristic

The above formulation is exactly solvable for small sized networks of less than 6 nodes. For large networks and large traffic matrix the MILP formulation is intractable. Hence we relax the integer constraints for the λ , η and $b(i, j)$ variables in the formulation to get a feasible solution. Heuristic algorithms are developed to get feasible solution to the MILP formulation. The rounded results are not optimal but close to optimal solution obtained when MILP is solved exactly.

There are many heuristics available [2] for obtaining a feasible solution for the relaxed MILP. We have followed the approach of [7] with modification suited to this formulation. We present here the maximum flow path algorithm.

– Step 1: Let $\max_{(x,z)} \lambda_{(i,x),z}^{i,j}$ be $\lambda_{(i,l),q}^{i,j}$.

Let $temp = \lambda_{(i,l),q}^{i,j}$.

– Step 2: If $l = j$ then stop. Else

let $\max_{q'} \eta_{q,q'}^{(i,j),l} = \eta_{q,\hat{q}}^{(i,j),l}$ and

let $\max_y \lambda_{(l,y),\hat{q}}^{i,j} = \lambda_{(l,p),\hat{q}}^{i,j}$

If $\lambda_{(l,p),\hat{q}}^{i,j}$ is not available then choose the next maximum of eta variable and the corresponding $lambda$ in the above statement in Step 2.

If $temp \geq \lambda_{(l,y),\hat{q}}^{i,j}$ then $\eta_{q,q}^{(i,j),l} = \eta_{q,\hat{q}}^{(i,j),l} - temp$

else $\eta_{q,q}^{(i,j),l} = 0$

Let $temp = \lambda_{(l,p),\hat{q}}^{i,j}$

– Step 3: If $p = j$ then stop. Else assign p to l , $\hat{q}$ to q and continue with step 2.

Once the path is found for a logical connection between (i, j) , round the $lambda$ variables to 1 and remove them for further consideration of algorithm execution.

Repeat the above three steps for every logical connection between the source-destination pair (i, j) . For a set of connection between (i, j) the above algorithm terminates when either all the connections between (i, j) are routed and wavelength assigned or when there is no wavelength available for establishing lightpaths.

Then by rearranging the wavelength assignments for some paths we can realize more connections from the connection matrix.

A.4 Complexity

In the MILP formulation the number of variables grow approximately as $O(2N^4F)$. The number of constraints grow approximately as $O(2N^3F)$. On the average the LP solver took around seven minutes on an IBM 43P/RS6000 machine to solve one iteration of the relaxed MILP problem. The running times of the rounding heuristics were less than a minute.

B. Heuristic Algorithm for RWA problem

Here we present a computationally less intensive heuristic algorithm based on K shortest paths for solving the Max-RWA problem.

Assuming that in the lightpath request matrix, the largest lightpath request between any source-destination pair is m , we find the K shortest paths, wherein K is greater than m . The algorithm proceeds in following steps:

Step 1: Finding K shortest paths [4] in terms of hop – length between all source-destination pairs in traffic matrix:

The K shortest paths are stored in $lightpaths1, lightpaths2, \dots, lightpathsK$ arrays. Consider the first shortest path array i.e., $lightpaths1$ for processing and go to Step2.

Step 2 : Wavelength Assignment to the Lightpaths

For the lightpath which is not wavelength assigned in the traffic matrix, choose the path for that from the chosen K shortest path array. A typical lightpath between nodes (1) and (N) is represented as $node[1], node[2], \dots, node[Q], \dots, node[N]$; where nodes $node[2], \dots, node[Q]$ are nodes along the lightpath. The physical fiber links along the lightpath ($node[1], node[2]$) labelled as link 1, ($node[2], node[3]$) as link 2 and the last link ($node[N-1], node[N]$) as link N-1. The first physical link of the lightpath (here ($node[1], node[2]$)) is taken and scanned for a free wavelength. If a wavelength λ_j is free, then we try to find in all links of that lightpath for the availability of the wavelength λ_j . Then the algorithm proceeds further, differently for the following cases

i. For the case of no conversion of wavelength along the lightpath, if the wavelength λ_j is available in all physical links along the lightpath, then we allocate that wavelength for the lightpath. If the continuity of the wavelength λ_j along the links in the lightpath is not possible then, we scan for the next free wavelength λ_j on the link ($node[1], node[2]$). As before the availability of the wavelength λ_j on all the physical links on the lightpath is checked, if the wavelength is available then it is assigned, else we scan for the next free wavelength on the link ($node[1], node[2]$) and the above procedure for wavelength assignment is repeated till the lightpath is wavelength assigned or the wavelengths in the link ($node[1], node[2]$) is exhausted.

ii. For the case of limited wavelength conversion, if wavelength λ_j is blocked on any link ' n ', then go back one physical link towards the source node of the lightpath ' $n - 1$ ' and try to obtain a free wavelength by wavelength conversion. If wavelength is not available go back further one link ' $n - 2$ ' and try to obtain a free wavelength by wavelength conversion. Repeat the above procedure till a free wavelength is obtained or link 2 is reached on the lightpath. If a free wavelength is not available at link 2 then go to link 1 and choose a new free wavelength and traverse the physical links of the lightpath towards the destination node assigning wavelengths with or without conversion.

While back tracking for a free wavelength if at any

intermediate link if we get a free wavelength after conversion, then traverse from that link towards the destination node assigning wavelength with or without conversion. The wavelength conversion allowed at any node depends on the degree of conversion allowed.

Step 3: Repeat the step 2 till all the lightpaths in the chosen array are exhausted.

Step 4: If any of the lightpaths in given traffic matrix is not wavelength assigned, then choose the next of the K shortest path arrays and goto step 2. If all the lightpaths are wavelength assigned or all the K shortest path arrays are exhausted, then stop the algorithm.

III. NUMERICAL RESULTS AND DISCUSSION

A. Linear Program approach

For small networks say less than seven nodes the MILP was solved exactly but for larger networks we solve the relaxed ILP and developed heuristics to obtain a feasible solution for the MILP. To solve the MILP and the LP's we used IBM's Optimization Sub-routine Library (OSL) routines on an IBM 43P/RS6000 platform.

A.1 NSFNET

The skeleton of the NSFNET shown in figure 2 is a 14 node network with 21 edges. In this network each edge represents a pair of directed edges. The pair of directed edges represent a pair of fibers, one in each direction.

The traffic matrix that has to be realised over the NSFNET is shown in the table I. We assumed that at most 3 multiple connections were permitted for a source-destination pair. The number of connections are chosen from 0,1,2,3 with equal probability for a source destination pair. The total number of connection to be set up is 268.

The results after rounding the solutions of Linear Program is tabulated for cases of wavelengths conversion degree 1,2 and 3 in table II. We note that the number of connections increases for a given number of wavelengths as the degree of conversion increases. For a given wavelength, we find that the LP upper bound remains constant for various degrees of wavelength conversion. This happens because of the relaxed integer constraints. However we find that the number of connections realised after rounding varies with the degree of wavelength conversion, for a given wavelength.

B. K -shortest path Heuristic Algorithm approach

We executed the above algorithm on the NSFNET network with the value of $K = 5$, for the connection matrix shown in table I. The greater the value of K , more will be number of connections realised, because there are more alternative paths available for wavelength assignment. The results based on the type of arrangement of lightpaths are captured in figures 3,4,5 along with the results of ILP.

Comparing ascending order of lightpath arrangements in figure 3, we find that in terms of performance no conversion, degree 2 and degree 3 conversion gives better performance in that order. This is because in the case of no conversion, when short lightpaths are assigned wavelengths, then for the longer lightpaths because of wavelength continuity constraint we may not be able to find an identical wavelength. But there is increase in number of connections for the same wavelength for degree 2 and degree 3 because wavelength conversion is allowed. Comparing descending order of lightpath arrangements in figure 4, we find that in terms of performance degree 2, degree 3 and no conversion gives better performance in that order. This is because in the case of degree 2 conversion, initially the algorithm assigns wavelengths to lightpaths without wavelength conversion and then does the conversion when blocking occurs. In no conversion case the number of connections realised is less than degree 2, because lightpaths gets blocked due to non availability of identical wavelength in all links of the lightpath. The number of connections realised for the degree 3 conversion case is in between no conversion and degree 2 conversion because, initially the lightpaths are assigned identical wavelengths in all links of the lightpath, then when blocking occurs the wavelength can undergo degree 2 or degree 3 conversion in order to assign wavelength to the lightpath. Since the wavelength has the chance of undergoing higher degree(3) of conversion, more number of lightpaths are blocked at the later stage, which results in reduction in total number of connections realised, when compared to degree 2 conversion. Comparing random order of lightpath arrangements in figure 5, we find that in terms of performance degree 3, degree 2 and no conversion gives better performance in that order.

In all the types of conversion, ascending order of lightpath arrangement initially gives better results than the descending order, and random order because the shortest lightpaths are assigned wavelengths first and then when we try to assign wavelength to the longer lightpaths, we do not get wavelengths to color

the lightpath.

We infer the following from the results:

(a). Regardless of type of arrangement of lightpaths, the performance is good in the following order: degree 3 conversion, degree 2 conversion and then no conversion. (b). Regardless of type of conversion, the performance is good in the following order: random, descending and ascending order of lightpath arrangements.

The heuristic algorithm for execution takes less than a minute on SUN ultra 1 workstation.

IV. CONCLUSION

For solving the Max-RWA problem on random mesh networks, two algorithms were presented. One is computation intensive integer linear program and other is less computation intensive K -shortest path heuristic algorithm. The number of connections realised by the heuristic algorithm is less than that of the ILP based algorithm and vary when the lightpaths are arranged in ascending, descending and random order. The results show that use of wavelength converters reduces the wavelength blocking at switching nodes.

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Fig. 2. NSFNET

0	1	3	1	1	1	3	0	2	0	1	2	0	3
0	0	0	2	2	2	1	1	1	2	1	0	1	3
3	2	0	3	0	1	2	3	1	3	1	2	2	0
3	1	0	0	1	1	2	3	2	2	1	2	1	3
1	3	0	2	0	1	0	2	0	3	0	1	1	3
1	2	1	3	2	0	1	3	3	1	0	1	1	2
2	2	3	1	3	3	0	0	3	1	2	0	3	3
3	1	2	3	1	0	1	0	0	3	2	0	3	0
3	0	1	3	3	3	1	0	0	2	1	1	1	0
0	0	0	1	2	0	2	0	1	0	1	0	0	3
1	0	0	2	0	3	0	1	0	3	0	3	1	3
2	3	1	1	3	2	3	2	2	2	2	0	1	3
2	0	1	2	0	1	2	0	3	0	2	1	0	3
1	1	0	2	1	0	1	3	0	1	2	1	3	0

TABLE I

SESSION MATRIX FOR NSFNET (268 CONNECTIONS)

F	LP bound deg of conv			LP rounded result deg of conv		
	1	2	3	1	2	3
10	198	198	198	190	196	196
11	208	208	208	200	204	208
12	218	218	218	207	211	212
13	228	228	228	213	216	221
14	238	238	238	226	233	234
15	248	248	248	234	243	247
16	258	258	258	240	251	256
17	263	263	263	250	257	260
18	267	267	267	255	259	262
19	268	268	268	263	268	268
20	268	268	268	265	268	268
21	268	268	268	267	268	268
22	268	268	268	267	268	268
23	268	268	268	268	268	268

TABLE II

RESULT OBTAINED FOR NSFNET BY ILP FORMULATION.

Fig. 3. Results for NSFNET by heuristic approach, for no conversion, limited conversion of degree 2/degree 3 when lightpaths are arranged in ascending order. Total number of lightpaths is 268.

Fig. 4. Results for NSFNET by heuristic approach, for no conversion, limited conversion of degree 2/degree 3 when lightpaths are arranged in descending order. Total number of lightpaths is 268.

Fig. 5. Results for NSFNET by heuristic approach, for no conversion, limited conversion of degree 2/degree 3 when lightpaths are arranged in random order. Total number of lightpaths is 268.