Multi-access Poisson Traffic Communication with Random Coding, Independent Decoding and Unequal Powers

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Abstract — We model and analyse the stability of the communication scheme. We obtain an arrival rate stability limit that is identical for all SNR distributions considered.

I. Introduction

Multi-access random-coded communication, of messages that arrive in a Poisson process to an infinite transmitter population, and that achieves any desired value for the random-coding upper bound on mean message error probability by determining message signal durations appropriately, has been considered in [1]. Independent equal-power bandpass-white Gaussian message signals have been assumed, each of which occupies the entire channel bandwidth and is transmitted in parallel starting at the respective message arrival time, on an AWGN channel.

We consider the case of unequal powers, represented by i.i.d. received-message-signal to noise ratios (SNRs) for the respective message signals. We assume that the SNRs are known at the receiver, and that messages are decoded independent of one another but using knowledge of SNRs of overlapping signals. We model and analyse the stability of the scheme, and study mean signal durations.

II. The communication scheme

The signal duration D_m for a message m is determined from the equation

$$\int_{A_m}^{A_m+D_m} W\rho \ln\left(1+\frac{\Gamma_m}{(1+\rho)(1+\sum_{n\in N(t),n\neq m}\Gamma_n)}\right)dt = -\ln P_e+\rho\ln M$$

where ρ is a parameter of the scheme with value fixed in $(0,1]$, P_e is the tolerable error probability, M is the message alphabet size , W is the two-sided bandwidth of the equivalent baseband channel, A_m is the arrival time of message m, $N(t)$ is the set of messages with signals overlapping at time t, and Γ_n is the SNR of overlapping message n. The random coding bound on the mean error probability in decoding message m from the signal received over the duration D_m is then P_e [1]. Thus, the integrand in the equation and $-\ln P_e+\rho\ln M$ can be respectively considered as the service rate at time t and the service requirement, of message m.

III. Stability Analysis

We construct a Markov-chain processor-sharing queueing model, that assumes that the random variable SNR has a finite number I of possible values that are all positive and finite, and uses an Erlang approximation for service requirement. Then, the value ϕ^* of total service rate in the limit of large number of messages is $\frac{W\rho}{1+\rho}$. We prove the following theorem, which yields the arrival rate stability limit as $\frac{W\rho}{(1+\rho)(-\ln P_e+\rho\ln M)}$ that is identical for all SNR distributions considered.

Theorem: Messages of class i, $1\leq i\leq I$, arrive to a processor sharing queue in independent Poisson processes of respective rates λ_i, with independent service requirements of Erlang-L_i distribution and mean S_i. A service rate function ϕ_i specifies the service rate of each message of class i in state α of the queue as $\phi_i(\alpha)$. With number $n(\alpha)$ of messages in the queue and sum $\phi(\alpha)$ of the service rates of these messages, assume that $\phi(\alpha)\xrightarrow{n(\alpha)\to\infty}\phi^*$. Then, the Markov chain representing the queue (a) is positive recurrent and yields finite stationary mean for the total residual service requirement (and hence for the number of messages) if $\sum_{i=1}^{I}\lambda_i S_i<\phi^*$, and (b) is transient if $\sum_{i=1}^{I}\lambda_i S_i>\phi^*$.

We prove the theorem by obtaining appropriate drift conditions [2] for suitably defined Lyapunov functions of the state of the discrete-time chain that is embedded at transitions of the original continuous-time chain.

IV. Mean Signal Durations

We solve for the stationary distribution of the four-state Markov-chain that models the loss system consisting of two transmitters that may overlap in transmission, each with its respective SNR value. We observe instances, for which transmitted messages have sufficiently large equal fixed throughputs at the the two SNR values, in which mean signal duration is less when one SNR is set to certain values less than, rather than equal to, that of the other SNR. We observe the same behaviour in simulations with two-valued SNR in an infinite transmitter population.

References

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