Optimal Power Allocation for Multiaccess Fading Channels with Minimum Rate Guarantees

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ABSTRACT

Future wireless networks will have to support multimedia services which possess inherent quality of service (QoS) requirements. A minimum rate guarantees is often an important QOS requirement. We propose dynamic power and rate allocation strategies that counter the effect of channel variations while guaranteeing a minimum rate to each user.

I. INTRODUCTION

Wireless systems of the future will support multiple traffic classes with diverse quality of service (QoS) requirements. Applications, such as interactive telephony are sensitive to delay, but can tolerate some loss, whereas for store and forward applications the requirements are quite the opposite. We consider the uplink of a single "cell" wireless system comprising M mobile stations communicating with a base station (BS). Each user requires two service classes: the constant bit rate (CBR) service where a source is allowed to send at a negotiated rate at all times (suitable for telephony), and the available bit rate (ABR) service where a source can transmit at a time varying rate, using feedback information from the receiver, based on the channel conditions (suitable for Internet applications). The probability of error is an important QoS measure for ABR service class. We derive a dynamic rate and power allocation policy that maximizes the average rate available to ABR traffic with near zero error probability while guaranteeing a prenegotiated rate to CBR traffic by countering the effect of channel variations seen on a multiaccess fading channel.

We can view the above problem from a different perspective. Since the wireless channel is inherently time-varying, one has to employ a rate or power allocation policy based on the channel state (as in CDMA power control literature [3]) in order to guarantee a minimum rate and at the same time utilizing the available resources optimally. In a recent paper [1], Hanly and Tse have considered the resource allocation problem in a multiaccess channel with the objective of minimizing the throughput capacity. In a sequel to this paper [2], they went on to discuss the capacity region when users need delay guarantees. They considered the framework in which delay guarantees are provided by each user transmitting at fixed guaranteed rate. This is a very restrictive assumption and often leads to the wastage of transmission opportunities. Since at times when the channel state is good, the BS could allow a rate in excess of the minimum required while restricting to the minimum rate when the channel is bad. In the CBR/ABR setting introduced earlier, the rate in excess of the minimum can be provided to each users ABR traffic. Thus the problem stated earlier can be Seen as a combination of the two problems considered by Hanly and Tse.

Another viewpoint is to maximize the revenue earned by the service provider while maintaining the desired QoS. Suppose each user pays the service provider (BS) an additional amount say μ per unit average rate provided, in excess of the minimum required. Thus objective would be to maximize the revenue of the service provider while satisfying each user's QoS requirement and the power constraint. We will consider mutual information in a block and a minimum is guaranteed over each block. For a reasonable sized block, this gets translated into the bit rate available at a higher layer by using appropriate coding-decoding techniques. The minimum rate guarantee provides a bound on the tail distribution of the transmission delay [4].

In this paper, we are not utilizing the queue state information while allocating resources; this could help improve the revenue and it is under consideration in a subsequent work. Since the maximum allowed long run rate averaged over the fading states is termed as throughput capacity, the problem of maximizing the revenue could also be looked as maximizing a weighted averaged throughput of the system while having a minimum rate guarantee. One could even obtain the throughput capacity region by varying the weights.

The rest of this is paper is organized as follows. Section II describes the system model and provides some background material required subsequently. This is followed by problem formulation. Section IV provides the analytical results for the formulated problem. In the same section, we give examples of single user and two user cases. We explicitly characterize the rate allocation and power allocation policies for the two examples considered. Section V summarizes the result obtained. Toward the end of this paper we have an appendix providing the proofs of some lemmas stated in the main body of the paper.

II. SYSTEM MODEL

There are M mobile stations communicating with the base station (BS). The BS is assumed to have multiuser detection capability. We assume a slotted system, that the channel state does not change over a slot and that the transmitters and the BS can track the channel. The rate requirement vector per slot for the CBR sources is $\rho = [\rho_1, \rho_2, \ldots, \rho_M]$. We assume infinite backlog of ABR traffic at each transmitter. The channel coding for ABR traffic is done in a way similar to the one suggested in [3] (Refer appendix for details) while CBR traffic is encoded at a fixed code rate: The i^{th} user has a long run average transmitter power constraint of $\bar{P}_i$. Define $\bar{P} = [\bar{P}_1, \bar{P}_2, \cdots, \bar{P}_M]$. Given an average power constraint, Tse and Hanly [1] obtained a delay Limited capacity region identifying the rates at which each user can transmit at all times within some tolerance in terms of probability of error. We assume that ρ belong to the delay Limited capacity region, otherwise the problem is infeasible.

We consider the maximization of a weighted throughput for the ABR traffic. The weights μ_i could be though1 of as an extra nonnegative amount paid by the i^{th} user per unit long run average rate provided in addition to ρ_i. Different values for the extra mount per unit average rate is justifiable since one user could be more quality conscious than another, and hence ready to pay more. The channel gain process is assumed to be stationary and ergodic. In a particular slot k, the channel state of the i^{th} user is $h_i[k]$. Let the channel state vector in k^{th} slot be $h[k] = [h_1[k], h_2[k], \cdots, h_M[k]]$. Given the channel state $h[k]$, let the rate allocation vector be $R(h[k]) = [R_1(h[k]), R_2(h[k]), \cdots, R_M(h[k])]$ and the power allocation vector be $\mathcal{P}(h[k]) = [\mathcal{P}_1(h[k]), \mathcal{P}_2(h[k]), \cdots, \mathcal{P}_M(h[k])]$ where $R(h[k]) \geq \rho$ for all channel states h and for all k. Then the weighted average rate available for ABR traffic or the average revenue for the service provider is given by

$$\sum_{i=1}^{M} \mu_i \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (R_i(h[k]) - \rho_i).$$

Since the rate vector $R(h[k])$ is a function of stationary and ergodic process $h[k]$, $R(h[k])$ is a stationary and ergodic process. Let H denote the random vector representing the channel state vector h. Then the weighted revenue from ABR traffic is,

$$\sum_{i=1}^{M} \mu_i (E_H[R_i(H)] - \rho_i)$$

The average power vector is $E_H[\mathcal{P}(H)]$ where the averaging is done over all channel state vectors. Define the average rate vector $R = E_H[R(H)]$.

III. The Revenue Optimization Problem

A. Preliminaries

In this subsection we will state some well known results. Let the receiver ambient noise power be σ^2. Define $S \subset \{1, 2, \cdots, M\}$. Given $R \in \mathcal{R}^{+M}$, define $R(S) = \sum_{i\in S} R_i$. The capacity region of an additive white Gaussian noise channel for any given h and power vector P is given by

$$C_g(h, P) = \Big\{ R \in \mathcal{R}^{+M} : R(S) \leq \frac{1}{2}\log\Big(1 + \frac{\sum_{j\in S} h_j P_j}{\sigma^2}\Big) \text{ for every } S \subset \{1, \cdots, M\} \Big\}.$$

Given a rate vector r and h, the set of received power vectors that can support r is

$$Q(h, r) = \{q : \exists P s.t. q_i = h_i P_i, r \in C_g(h, P)\}.$$

If the transmitter and receiver can track the channel, the power allocation can be changed with channel state. The capacity region for any power allocation policy $\mathcal{P}(\cdot)$ is given by

$$C_f(\mathcal{P}) = \Big\{ R : R(S) \leq E_H\Big[\frac{1}{2}\log\Big(1 + \frac{\sum_{j\in S} H_j \mathcal{P}_j(H)}{\sigma^2}\Big)\Big] \; forall S \subset \{1, \cdots, M\} \Big\}.$$

The following result is proved in [1].

Lemma 1: $C_f(\mathcal{P})$ and $C_g(h, P)$ are polymatroids. $Q(h, r)$ is a contra-polymatroid.

The throughput capacity region for the multiaccess fading channel when transmitters and receiver can track the channel is given by

$$C(\bar{P}) = \bigcup_{\mathcal{P}\in F} C_f(\mathcal{P})$$

where F is set of feasible power control policies satisfying the average power constraint

$$F = \{\mathcal{P} : E_H[\mathcal{P}_i(H)] \leq \bar{P}_i \text{ forall } i\}.$$

B. Our Formulation

The problem addressed in this paper is to find power and rate allocation policies which maximize the weighted throughput while satisfying the minimum rate vector and the average power constraints. Thus the average rate vector R should belong to $C(\bar{P})$ and in each channel state the rate vector should be greater than equal to the minimum rate required ρ. Thus for each channel state h, we have to find an optimal $(R^*(h), \mathcal{P}^*(h))$ that solves the following optimization problem.

$$\max_{(R(h),\mathcal{P}(h),\forall h)} \sum_{i=1}^{M} \mu_i (R_i - \rho_i),$$
$$\text{s.t. } R \in C(\bar{P}) \text{ and } R(h) \geq \rho, \text{ for all } h.$$

Equivalently,

$$\max_{(R(h),\mathcal{P}(h))} \mu \cdot R \text{ s.t. } R \in C(\bar{P}) \text{ and } R(h) \geq \rho, \; \forall h \qquad (1)$$

where $\mu . R = \sum_{i=1}^{M} \mu_i R_i$. The following lemma is a modification of lemma 3.10 in [1]. The proof of the lemma is in the zppendix.

Lemma 2: Let $(R^*(h), \mathcal{P}^*(h))$ be the solution of the following optimization problem

$$\max_{(R(h),\mathcal{P}(h))} \mu \cdot R \text{ s.t. } R \in C(\bar{P}) \text{ and } R(h) \geq \rho, \; \forall h$$

for some positive $\mu \in \mathcal{R}_+^M$. For a given μ, $(R^*(h), \mathcal{P}^*(h))$ is the solution of above problem if and only if there exists $\lambda \in \mathcal{R}_+^M$, rate allocation policy $R(\cdot)$ and power allocation policy $\mathcal{P}(\cdot)$ such that for every joint fading state h, $(R(h), \mathcal{P}(h))$ is a solution to the optimization problem

$$\max_{(r,p)} \mu \cdot r - \lambda \cdot p \text{ s.t. } r \in C_g(h,p) \text{and } r \geq \rho, \qquad (2)$$

$$E_H[R_i(H)] = \mathbf{Rf},\ E_H[\mathcal{P}_i(H)] = \bar{P}_i,\ \forall i$$

where $\bar{P}_i$ is the power constraint and R_i^* is the optimal average rate for user i.

IV. Analysis

Given the channel state h and the Lagrange multiplier λ, using the Lemma 2, the optimization problem (1) can be written as,

$$\max_{(r,p)} \mu \cdot r - \lambda \cdot p \text{ s.t. } r \in C_g(h,p) \text{ and } r \geq \rho \qquad (3)$$

The above optimization problem is equivalent to,

$$\max_{(r,q)} \mu \cdot r - \frac{\lambda}{h} \cdot q \text{ s.t. } q \in Q(h,r) \text{ and } r \geq \rho \qquad (4)$$

where by slight abuse of notation, $\frac{\lambda}{h}$ denotes the row vector $(\frac{\lambda_1}{h_1}, \cdots, \frac{\lambda_M}{h_M})$. Without loss of generality, we can assume that $\frac{\lambda_1}{h_1} \geq \frac{\lambda_2}{h_2} \geq \cdots \geq \frac{\lambda_M}{h_M}$. Then for any given rate vector $\underline{r}$, the minimum value of $\frac{\lambda}{h} \cdot q$ subject to the above said contra: polymatroid constraint $Q(h,r)$ is given by

$$\sum_{i=1}^{M} \frac{\lambda_i}{h_i} \{ f(\sum_{k=1}^{i} r_k) - f(\sum_{k=1}^{i-1} r_k) \}$$

where the function $f(x) := \sigma^2(e^{2cx} - 1)$ and for convenience we have defined a constant $c = \ln(2)$.

Next we take care of the inequality constraint $r \geq \rho$ by asserting that there exists a Lagrange multiplier $w \in \mathcal{R}_+^M$ such that the optimization problem (1) is equivalent to

$$\min_{\underline{r}} \Big\{ \sum_{i=1}^{M} -\mu_i r_i + \textstyle\sum_{i=1}^{M} \frac{\lambda_i}{h_i} \{ f(\sum_{k=1}^{i} r_k) - f(\sum_{k=1}^{i-1} r_k) \} - w_i r_i \Big\}.$$

Rewriting the above equation in a convenient form we get,

$$\min_{\underline{r}} \Big\{ (-w_i - \mu_i) r_i + \frac{\lambda_M}{h_M} f(\sum_{k=1}^{M} r_k) + \sum_{k=1}^{M-1} \left(\frac{\lambda_k}{h_k} - \frac{\lambda_{k+1}}{h_{k+1}} \right) f(\sum_{m=1}^{k} r_m) \Big\}.$$

Let $\mathbf{I}$ denote the set $\{1, 2, \cdots, M\}$. Differentiating with respect to r_i and equating to zero we get,

$$w_i = -\mu_i + \frac{\lambda_M}{h_M} f_i'(\sum_{k=1}^{M} r_k) + \sum_{k=i}^{M-1} \left(\frac{\lambda_k}{h_k} - \frac{\lambda_{k+1}}{h_{k+1}} \right) f_i'(\sum_{m=1}^{k} r_m),\ i \in I. \qquad (5)$$

The solution to the above problem that satisfies the Kuhn-Tucker (KT) conditions is as follows.

Theorem 1: Given h, let $(u_1, u_2, \cdots, u_n)$ solves for r the following system of equations

$$\sum_{k=i}^{M} \left(\frac{\lambda_k'}{h_k} - \frac{\lambda_{k+1}'}{h_{k+1}} \right) e^{2c \sum_{m=1}^{k} r_m} = \mu_i \text{ for } i \in I \qquad (6)$$

where $\frac{\lambda_{M+1}}{h_{M+1}}$ is zero and $\lambda_i' = 2c\sigma^2 \lambda_i$ for $i \in \mathbf{I}$.
Let $u_i \leq \rho_i$ for all $i \in \mathcal{J} \subset \mathbf{I}$. Then let $(z_1, z_2, \cdots, z_n)$ be the solution for r but only for a subset of the system of equations (6), i.e., for all $i \notin \mathcal{J}$ and $z_k = \rho_k$ for all $k \in \mathcal{J}$. The rate allocation policy is $r_i = z_i$ for all $i \in \mathbf{I}$. The power allocation policy is $p_i = \frac{1}{h_i} \{ f(\sum_{k=1}^{i} r_k) - f(\sum_{k=1}^{i-1} r_k) \}$.
Remark: When $\mathcal{J} = \mathbf{I}$ we call the channel state to be bad and thus users transmit at the minimum desired rate ρ. Whereas if $\mathcal{J} \subset \mathbf{I}$, the channel is bad for some users while good for others Therefore the users who are in bad state transmits at a rate ρ_i but others can transmit at rates higher than ρ_i. If the channel is good for everyone, i.e., $\mathcal{J}$ is the empty set then the users transmits at rates u which is greater than ρ. The λ_i, $i \in \mathbf{I}$, can be obtained using the average power constraint of each user.

We state the following lemma giving the structural result

Lemma 3: Given h, the user with largest value of μ and largest channel gain h gets the highest transmission rate.

As we discussed in the introduction, these policies can be looked as maximizing a weighted throughput of the system. The boundary of the throughput capacity region can be obtained by varying μ such that $\sum_{i=1}^{M} \mu_i = 1$ and for each μ, taking the average of allocated rates over all channel states. Now let us assume that we are interested in sum throughput, i.e., no bias to a particular user. This is the system throughput. We can take $\mu_i = 1$ for $i \in \mathbf{I}$. Define $f'(z) = \frac{d}{dz} f(x)$. Then Equation **5** becomes

$$w_i = -1 + \frac{\lambda_M}{h_M} f'(\sum_{k=1}^{M} r_k) + \sum_{k=i}^{M-1} \left(\frac{\lambda_k}{h_k} - \frac{\lambda_{k+1}}{h_{k+1}} \right) f'(\sum_{m=1}^{k} r_m).$$

For $j > i$, we have

$$w_i - w_j = \sum_{k=i}^{j-1} \left(\frac{\lambda_k}{h_k} - \frac{\lambda_{k+1}}{h_{k+1}} \right) f'(\sum_{m=1}^{k} r_m). \qquad (7)$$

Thus for $j > i$, $w_i - w_j \geq 0$ implying that w_i is a non-increasing function of i. We intend to find out which of the inequality constraints are inactive, i.e., $w_i = 0$. Fitly let $w_M = 0$ and $w_i > 0, i = \{1, \cdots, M-1\}$. This means $r_i = \rho_i$, for $i = \{1, \cdots, M-1\}$ (KT condition). Thus

$$r_M = \frac{1}{2c} \ln \left(\frac{h_M}{\lambda_M'} \right) - \sum_{i=1}^{M-1} \rho_i$$

where $\lambda_i' = 2c\sigma^2\lambda_i$.

Now let us consider the case $w_M = 0, w_{M-1} = 0$ and $w_i > 0, i = \{1, \cdot\cdot, M-2\}$. This means $r_i = \rho_i$, for $i = \{1, \cdot\cdot\cdot, M-2\}$ (KT Condition). Thus Equation 7 implies that $\frac{\lambda_{M-1}}{h_{M-1}} = \frac{\lambda_M}{h_M}$. This gives a non-unique solution that satisfies

$$r_M + r_{M-1} = \frac{1}{2c}\ln\left(\frac{h_M}{\lambda_M'}\right) - \sum_{i=1}^{M-2}\rho_i$$

Since the users have a joint channel state distribution with a continuous density, $\frac{\lambda_{M-1}}{h_{M-1}} = \frac{\lambda_M}{h_M}$ happens only with probability zero. Hence the only solution is

$$r_i = \rho_i,\ i \in I - \{M\} \text{ and } r_M = \frac{1}{2c}\ln\left(\frac{h_M}{\lambda_M'}\right) - \sum_{i=1}^{M-1}\rho_i$$

Moreover the value of $r_M > \rho_M$. The solution seems to be quite intuitive since the system throughput can only be maximized while satisfying each user's minimum rate when in any channel state only one user with the best channel gain transmits at a rate in excess of its minimum rate.

A. *Example for two users*

Given the channel state (h_1, h_2), Define

$$g_1 = \frac{\lambda_1' 2^{2\rho_1}}{\mu_1 - \mu_2 + \mu_2 2^{-2\rho_2}},\ g_2 = \frac{\lambda_2'}{\mu_2}2^{2(\rho_1+\rho_2)}$$

$$a_1 = \frac{1}{2}\log\left(\frac{\mu_1}{\frac{\lambda_2'}{h_2}(2^{2\rho_2}-1) + \frac{\lambda_1'}{h_1}}\right),$$

$$a_2 = \frac{1}{2}\log\left(\frac{\mu_2 - \mu_1}{\frac{\lambda_1'}{h_1} - \frac{\lambda_2'}{h_2}}\right);$$

If $h_1 \le g_1$ and $h_2 \le g_2$ then $r_1 = \rho_1$ and $r_2 = \rho_2$. If $h_1 \le g_1$ but $h_2 > g_2$ then $r_1 = \rho_1$ and $r_2 = \frac{1}{2}\log(\frac{h_2\mu_2}{\lambda_2'}) - \rho_1$. If $h_2 \le g_2$ but $h_1 > g_1$ then $r_1 = a_1$ and $r_2 = \rho_2$; else $r_1 = a_2$ and $r_2 = \frac{1}{2}\log(\frac{h_2\mu_2}{\lambda_2'}) - r_1$. It is easy to check that the minimum rates are guaranteed.

B. *Single user example*

If $h_1 \le 2^{2\rho_1}\frac{\lambda_1'}{\mu_1}$ then $r_1 = \rho_1$; else $r_1 = \frac{1}{2}\log(\frac{\mu_1 h_1}{\lambda_1'}) > \rho_1$. The power allocation policy when the channel is good is given by $\sigma^2(\frac{\mu_1}{\lambda_1'} - \frac{1}{h_1})^+$. This policy has the well known water filling form.

V. CONCLUSION

We have obtained optimal rate and power allocation policy that maximize the ABR traffic throughput while providing rate guarantees to the CBR traffic subject to the transmitter power constraints. The allocated rate is largest for the user with best channel gain and the largest weight. But when the weights are equal, the user with best channel gain transmits the ABR data as well as the CBR data while others transmit the CBR data only. The policy for the general case can be obtained using a simple algorithm.

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APPENDIX

Channel Coding for ABR traffic: We quantize the channel state to finitely many values say N. There corresponds a rate vector to each channel state. Thus we split each users ABR data into N parallel data streams and encode each stream at the corresponding rate. Given a channel state say j, the encoded data from stream j is transmitted. Decoding is a reverse operation of demultiplexing the received data onto different streams and then decoding each individual stream.

Proof of lemma 2 The optimization problem is to find $R^*(h)$ and $P^*(h)$ which maximizes the following linear functional defined on the space of all functions $(R(h), P(h))$,

$$\mu \cdot E_H[R(H)]$$

subject to

$$E_H[R(H)] \in C(\bar{P}) \text{ and } R(h) \ge \rho,\ \forall h$$

Since the constraint set is convex and the assumption that ρ belongs to the delay'limited capacity region, there exists a Lagrange multiplier λ such that the above problem is equivalent to

$$\max_{(R(h),P(h))} E_H[\mu \cdot R(H) - \lambda \cdot P(H)]$$
$$\text{s.t. } E_H[R(H)] \in C_f(P(h)) \text{ and } R(h) \ge \rho,\ \forall h$$

Consider the dual of $C_f(P)$, say $D_f(R)$ which is a contra-polymatroid. Thus given any rate function $R(\cdot)$ within the feasible set, the minimum value of the functional $E_H[\lambda \cdot P(H)]$ is

$$E_H[\sum_{i=1}^{M}\frac{\lambda_{\pi_H(i)}}{H_{\pi_H(i)}}\{2^{2\sum_{k=1}^{i}R_{\pi_H(k)}(H)} - 2^{2\sum_{k=1}^{i-1}R_{\pi_H(k)}(H)}\}]$$

where the ordering $\pi(\cdot)$ is function of the channel gain h. Now the problem can be written as

$$\max_{R(\cdot)\ge\rho} E_H\Big[\mu \cdot R_H - \sum_{i=1}^{M}\frac{\lambda_{\pi_H(i)}}{H_{\pi_H(i)}}\{2^{2\sum_{k=1}^{i}R_{\pi_H(k)}(H)} - 2^{2\sum_{k=1}^{i-1}R_{\pi_H(k)}(H)}\}\Big]$$

Thus given any channel gain vector h, the problem is same as,

$$\max_{r \geq \rho} \left\{ \mu \cdot r - \sum_{i=1}^{M} \frac{\lambda_{\pi(i)}}{h_{\pi(i)}} \{ 2^{2 \sum_{k=1}^{i} r_{\pi(k)}} - 2^{2 \sum_{k=1}^{i-1} r_{\pi(k)}} \} \right\}$$

Since $Q(h,r)$ is a contra-polymatroid, we get

$$\max_{r \geq \rho, q} \mu \cdot r - \frac{\lambda}{h} q$$

$$\text{s.t. } q \in Q(h,r)$$

But $C_g(h,p)$ is the dual of $Q(h,r)$, hence we get

$$\max_{r,p} \{ \mu \cdot r - \lambda \cdot p \}$$

$$\text{s.t. } r \in C_g(h,p) \text{ and } r \geq \rho$$

Hence proved.