Detection of Edges from Projections

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***Abstract*—In a number of applications of computerized tomography, the ultimate goal is to detect and characterize objects within a cross section. Detection of edges of different contrast regions yields the required information. This paper addresses the problem of detecting edges from projection data. It has been shown that the class of linear edge detection operators used on images can be used for detection of edges *directly* from projection data. This not only reduces the computational burden but also avoids getting into difficulties of postprocessing a reconstructed image. This is accomplished by a convolution backprojection operation. For example, with the Marr–Hildreth edge detection operator, the filtering function that is to be used on the projection data is the Radon transform of the Laplacian of the 2-D Gaussian function which is combined with the reconstruction filter. Simulation results showing the efficacy of the proposed method and a comparison with edges detected from the reconstructed image are presented.**

I. Introduction

DETECTION of edges from projection data arises in a number of computerized tomography (CT) applications, e.g., medical imaging and nondestructive testing. Object contours need to be determined even in the reconstruction of cross sections in positron emission tomography (PET) [1], [2]. In medical applications detection and outlining of boundaries of organs and tumors are required, in addition to reconstruction for diagnostic interpretations of CT scans. The ultimate objective in other applications of CT such as nondestructive testing [3], oceanography [4], and plant physiological studies [5] is to extract certain specific information about the cross section that is imaged, rather than obtaining a high-resolution image. Hence, in these CT applications high-quality imaging is not necessary to achieve the final goal of detecting and characterizing objects within the cross section.

Two approaches are possible for detecting edges in the above applications. The first approach is a two step procedure, where the cross section is first reconstructed from the projections. This is then followed by an edge detection operation on the reconstructed image. This approach is followed by Belanger *et al.* [6] and by Selfridge and Prewitt [7] for automating the analysis of CT scan reconstruction. The disadvantages of this approach are, first, that a good number of projections are required to obtain a fair quality of the reconstructed image on which edge detection has to be performed and, second, that postprocessing a reconstructed image is often found difficult because even when the noise in the projection (measurements) is white, the noise in the reconstructed image is nonwhite [8]. The necessity of using an appropriate 2-D whitening filter on the reconstructed image in the context of detection of objects is pointed out by Hanson [9]. In order to avoid these problems, in the second approach the edges are obtained *directly* from the projection data rather than detecting it from the reconstructed image [10]–[13]. Localization and identification of boundaries of high contrast objects (such as bone and metallic surgical clips) directly from projection data is considered by Glover and Pelec [10]. Bergstrom *et al.* [13 consider backprojecting points of maximum slopes in the projection, which corresponds to boundaries of activity region, into the image plane, thus obtaining an outline of the shape and size of the reconstruction region. However, as pointed out in [1], a prerequisite for this method (though fulfilled in head studies), is that the object have a convex shape. Further, the problem of finding the maximum of the derivative is, on one hand sensitive to noise, thus resulting in several maxima outside the object, while, on the other, the activity in brain tissue and lesions produces maxima inside the skull. Rossi and Willsky [11]–[12] utilize a finite parametric model for the object and use statistical detection and estimation procedures to estimate the parameters directly from projection data. Only single-object location and geometry estimation are considered in [11], and the problems which arise in the case of multiple objects are outlined in [12]. The conceptual framework in [11] and [12] is extended to the 3-D case by Bresler and Macovski [13].

Certain applications of modeling and filtering in the Radon space have recently been investigated by Srinivasa *et al.* [14]–[15]. In this paper, by using the convolution property of the Radon transform, it is shown that the class of linear edge detection operators can be used for detection of edges *directly* from the projection data. This not only reduces the computational burden but also avoids the difficulty of postprocessing a reconstructed image. The Marr–Hildreth operator is simply chosen as an example to demonstrate the use of one such operator in illustrating the new approach. However, any other linear edge detection operator could as well be used. The paper is organized as follows. Section II gives a brief description of a class of linear operators, in particular the Marr–Hildreth operator [16], used in the detection of edges in images. This section also shows the use of this class of linear edge detection operators in detecting the edges *directly* from projection data. The performance of this method is demonstrated by computer simulations in Section III. Edges detected from the reconstructed image are also provided for comparison. Section IV concludes the paper.

Manuscript received April 11, 1989; revised August 26, 1991.

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IEEE Log Number 9104849.

II. EDGE DETECTION

A. Edge Detection in Images

A number of methods exist for detecting edges in images. Some of them are general in the sense that they can be used independently of the application, while others are specific for a particular application and make use of *apriori* information. The class of linear operators used for detecting edges in an image, which is of interest in the present study, has three essential operations: filtering, differentiation, and the subsequent detection of features such as peaks or zero crossings. The filtering or smoothing operation serves two purposes: it reduces the effect of noise on the detection of intensity changes and sets the resolution or scale at which intensity changes are to be detected. The second operation, differentiation, accentuates intensity changes and transforms the image into a representation from which properties of these changes can be extracted more easily. An intensity change along a particular orientation in the image gives rise to a peak in the first directional derivative of intensity measured perpendicular to the change, or a zero crossing (ZC) in the second derivative. If directional operators are utilized, they should be applied for a number of directions in order to detect intensity changes at different orientations in the image. From a computational point of view, it would be efficient to apply a single nondirectional operator.

The Marr-Hildreth operator, also known as the $\nabla^2 G$ operator, has provision for observing intensity changes at different scales and is one example of the class of linear operators used for detecting edges in images. It uses the Gaussian function for filtering and the Laplacian for differentiation and finally detects edges by detecting the zero crossings. The Gaussian function has the desired characteristic of being optimally localized in both the space and the frequency domain. Convolving an image $f(x, y)$ with the Gaussian function $g(x, y)$ effectively wipes out all structures at scales much smaller than the space constant σ of the Gaussian. The importance of Gaussian filtering and its similarities with filtering functions used in other edge detection operators are pointed out in [16]. The simplest nondirectional linear differential operator is the Laplacian. The first two operations in the Marr–Hildreth operator are combined into a single operation since both are linear. A single convolution of the image allows the detection of intensity changes at all orientations, for a given scale. Hence, for the detection of edges using a $\nabla^2 G$ operator, the required output is

$$\tilde{f}''(x,y) = \left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g(x,y)\right] ** f(x,y) \quad (1)$$

For the circularly symmetric 2-D Gaussian function,

$$g(r) = \frac{1}{2\pi\sigma^2} \exp^{-r^2/2\sigma^2}, \quad (2)$$

where $r^2 = x^2 + y^2$, and σ is the standard deviation of the Gaussian function. The term in the square brackets in (1) evaluates to

$$g''_\sigma(r) = \left(\frac{-1}{\pi\sigma^4}\right)\left(1 - \frac{r^2}{2\sigma^2}\right) \exp^{-r^2/2\sigma^2}. \quad (3)$$

It will now be shown that the class of linear operators used for detecting edges in images can be used to detect edges directly from projection data.

B. Edge Detection Directly from Projection Data

The theoretical basis for edge detection directly from projection data is the fact that two-dimensional filtering can be implemented using the Radon transform. The projection $p_\theta(t)$, i.e., the Radon transform of $f(x, y)$, is defined as

$$\begin{aligned} p_\theta(t) &= \mathcal{R}[f(x,y)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - t)\,dx\,dy. \end{aligned} \quad (4)$$

$f(x, y)$ can be recovered from the projections by first filtering $p_\theta(t)$ by a reconstruction filter, $r(t)$, whose frequency response is $|\omega|$ and then backprojecting the filtered projection, $p'_\theta(t)$, [17]. The backprojection operator is defined as

$$f(x,y) = \int_0^\pi p'_\theta(x\cos\theta + y\sin\theta)\,d\theta, \quad (5)$$

and

$$p'_\theta(t) = p_\theta(t) * r(t). \quad (6)$$

The central slice theorem provides a fundamental relationship between the 2-D Fourier transform of $f(x, y)$ and the 1-D Fourier transform of $p_\theta(t)$. As a consequence, the Radon transform of the 2-D convolution of two functions, $h(x, y)$ and $f(x, y)$, is equal to the 1-D convolution of the Radon transforms of the individual functions [17]. The filtered backprojection operation can be used to get the convolved 2-D output. In the present problem the projections are available and information on the edges of the underlying image is of interest. The key step then is to convert the 2-D convolution step required in detecting edges in images to 1-D convolutions in the Radon space. Using the convolution property of the Radon transform,

$$\mathcal{R}[h(x,y) ** f(x,y)] = \mathcal{R}[h(x,y)] * \mathcal{R}[f(x,y)], \quad (7)$$

a new filtering function for detecting edges directly from projection data is obtained. For the example of the Marr–Hildreth operator considered, the 1-D filtering function $e_\sigma(t)$, which is the Radon transform of the Laplacian of the Gaussian function, is obtained. Hence,

$$e_\sigma(t) = \mathcal{R}[\nabla^2 g(x,y)]$$

evaluates to

$$e_\sigma(t) = \left(\frac{1}{\sqrt{2\pi}\,\sigma^5}\right)(t^2 - \sigma^2)\exp^{-t^2/2\sigma^2}. \quad (8)$$

This edge detection filter function is combined with the reconstruction filter, $r(t)$, to obtain a new filtering function, $e'_\sigma(t)$:

$$e'_\sigma(t) = e_\sigma(t) * r(t). \quad (9)$$

Given the projections $p_\theta(t)$, 1-D convolutions are performed with $e'_\sigma(t)$ for various view angles to obtain the filtered projections,

$$p''_\theta(t) = p_\theta(t) * e'_\sigma(t). \tag{10}$$

The edges are located by finding the zero crossings in $\tilde{f}''(x,y)$, which is done by performing the backprojection operation (5) on $p''_\theta(t)$. The procedure for detecting edges directly from projection data can be summarized in the following steps:

1) Obtain the filtered projections $p''_\theta(t)$ using the filtering function $e'_\sigma(t)$ given in (9) for each view angle θ. Fast convolution using FFT can be utilized for this purpose.
2) Backproject the filtered projections $p''_\theta(t)$ to obtain $\tilde{f}''(x,y)$. When this operation is implemented on sampled data, has to be performed digitally, a finite difference approximation of (5) is used. Hence,

$$\tilde{f}''(x_i, y_j) = \frac{\pi}{J} \sum_{k=1}^{J} p''_{\theta_k}(x_i \cos\theta_k + y_j \sin\theta_k) \tag{11}$$

   where J is the number of projections. The evaluation of (11) requires interpolation of projections [17]. Alternatively, a weighted backprojection can be used. Each projection value is weighted by the length of intersection of the line with the pixel, and the sum of all such weighted contributions from all line integrals will give the value of a pixel in the resulting image, which will be referred to as the layergram.
3) Detect zero crossings in the resulting layergram. A binary image is first obtained by setting all negative pixels in the layergram to 0 and positive pixels to 1. Next, an edge is marked if there is at least one neighbor for which the candidate pixel value differs. Four nearest neighbors are used for this purpose. In order to reduce spurious edges, information about the magnitude of the zero crossing can be used when converting the layergram to a binary image.

C. Remarks

The reduction in computation is evident in the procedure described above owing to the following. In the two-step procedure of reconstruction of the image followed by edge detection, 1-D filtering of the projection data (for reconstruction) and 2-D filtering of the reconstructed image (for edge detection) are required. However, in the method presented in this paper, the 2-D filtering operation required for edge detection is converted to a 1-D filtering operation in the Radon space. Further, the two filtering operations, one for reconstruction (using $r(t)$) and another for edge detection (using $e_\sigma(t)$), are combined into a single filtering operation (using $e'_\sigma(t)$) and $\tilde{f}''(x,y)$ is obtained directly. By integrating the two filtering operations into one, the intermediate steps of reconstruction of the image and further preprocessing necessary before performing an edge detection (mentioned in Section I) are avoided. It should be pointed out that the final operation of detection of edges, like detection of peaks or zero crossings, is done on the image (layergram).

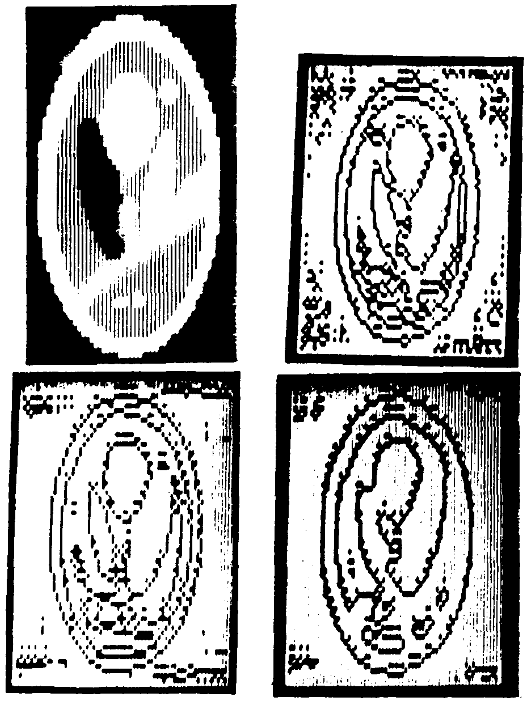

Fig. 1. ***Top left,* head phantom. Edges detected directly from the projections of the head phantom. The projections are equispaced in [0°, 180°). *Top right,* 45 projections with $\sigma = 1.0$. *Bottom left,* 180 projections with $\sigma = 0.7$. *Bottom right,* 180 projections with $\sigma = 1.4$.**

From the computational point of view it is interesting to note the similarities between the method presented here and the procedure of Rossi and Willsky [11] used for localizing objects directly from projection data. Both the methods involve backprojecting filtered projections. The differences are, however, in the filtering function used and in the interpretation of the resulting image. In the present method as well as in [11], the resulting image obtained after a convolution backprojection operation is not interpreted as a reconstructed image, since the convolving kernel is not chosen with direct reconstruction in mind. The image which is obtained in [11] by a matched filtering operation is a log likelihood function. Here, the filtering function is the Radon transform of a linear edge detection operator used in image edge detection problems. Peak detection is required in [11], while in the present method, using the Marr–Hildreth operator, detection of zero crossings is required. Extensive use of *a priori* information is made in [11] for the case of single-object localization. The procedure give here is useful for the case of multiple objects. ***A priori*** information can be used to remove false edge points and to combine the outputs obtained at different resolutions (σ).

III. Simulation Studies

The Shepp and Logan phantom [18] of size (64 x 64) used in the following simulation studies is shown in Fig. 1. Projection data equispaced over [0°, 180°) are computed as

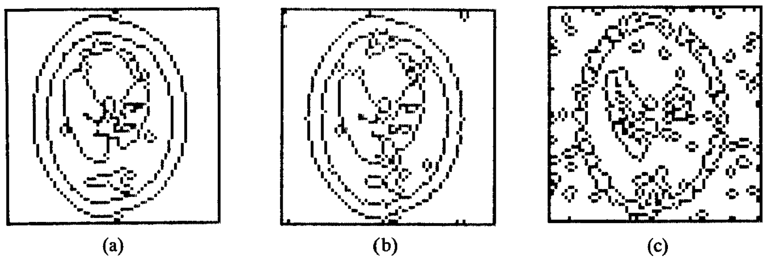

Fig. 2. Edges detected directly from projection data. The projections are equispaced in $[0°, 180°)$. The magnitudes of the zero crossings are used to discard spurious edges. (a) 180 projections; (b) 45 projections; (c) 10 projections.

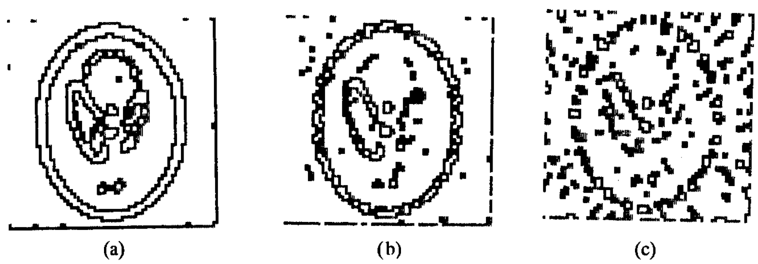

Fig. 3. Edges detected after reconstructing the image from projection data. The projections are equispaced in $[0°, 180°)$. The magnitudes of the zero crossings are used to discard spurious edges. (a) 180 projections; (b) 45 projections; (c) 10 projections.

weighted summation of pixel values. The length of intersection of the ray with the pixel is used as the weight. The procedure for detection of edges directly from projection data described above is used to obtain the required information. The resulting zero crossings description (edge images), without using any threshold for removing the transitions of small slopes, are shown in Fig. 1. These results are obtained by varying the number of projections and the value of σ. The major outlines of the ellipses in the head phantom are brought out rather fairly even when only 45 projections are used (top row). Increasing the number of projections yields better results (bottom row). The importance of the value of σ in detecting edges at different scale levels can also be seen. High values of σ result in edges which are coarse, while lower values of σ result in edges which are fine. It should be pointed out that a better description of the boundaries can be obtained by judiciously combining the outputs obtained at different σ's. However, there are spurious edges as well. In order to reduce the spurious edges, level crossings can be detected instead of zero crossings in the layergram. The magnitudes of the zero crossings are often employed to discard spurious edges. In the rest of the results shown in Figs. 2 and 3, this information is used when converting the layergram into a binary image. The value of σ is fixed and the number of projections used is varied. The edges detected directly from projection data using 180, 45, and 10 projections are shown in Fig. 2. For comparison purposes, the corresponding results of edges detected on reconstructed images are shown in Fig. 3. The image is first reconstructed from the projection data using the filtered backprojection method. Edges are then detected using the Marr–Hildreth operator on the reconstructed image. By comparing the results in Figs. 2 and 3, it can be seen that the proposed method of detecting edges directly from projection data is satisfactory.

IV. Discussions and Conclusions

This paper has addressed the problem of detecting edges in an image from its projections, which is required in a number of applications. The emphasis on detecting object boundaries directly from projections arises from the fact that it not only reduces the computational burden but also avoids getting into the difficulties of postprocessing a reconstructed image. A new procedure for detecting edges directly from projections, without the necessity of reconstructing the image, is given. Utilizing the convolution property of the Radon transform, it is shown that linear operators used on images for edge detection can be used directly on the projection data. It turns out that the resulting operation required is a convolution backprojection. The similarity with another procedure for localizing objects directly from projection data is pointed out.

In the present work, only the nondirectional edge detection operator of Marr and Hildreth is considered. Simulation results are presented showing the performance of this method. A number of outputs with different values of σ are required when the $\nabla^2 G$ operator is used for detecting edges in images. The problem of sorting out the relevant changes at each resolution and combining them into a representation that can be used more effectively is difficult. The design of robust methods for detecting zero crossings and the combination of their descriptions from different operator sizes (σ) remain unsolved in edge detection problems.

Linear directional edge detection operators could also be used using the framework presented in this paper. The only difference with the use of such operators is that the filtering function would be different for each angle. Further, this approach could be easily extended to detecting edges in 3-D images.

Another interesting problem is signal reconstruction from their zero crossings. The zero crossing description of a $\nabla^2 G$ image contains very rich information. The positions of zero crossings correspond to the location of intensity changes in the image, while the slope of a zero crossing is related to the contrast and width of the intensity change [19]. Linear interpolation may be used for localizing edges for subpixel accuracy [20]. Relating the zero crossings in an image to that of its projections is an interesting problem and seems to be useful in projection-based computer vision systems.

A computational approach to detecting edges in images is considered by Canny [21]. One can explore the possibility of extending this framework to obtain edges directly from projection data.

There is a growing interest in using the Radon space representation of images in computer vision and graphics [22]. Special-purpose architectures are also being designed to compute projections of an image. The new technique can be used for detecting edges in an image in projection-based computer vision systems.

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