Modeling and Analysis of Air Campaign Resource Allocation: A Spatio–Temporal Decomposition Approach

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***Abstract*—In this paper, we address the modeling and analysis issues associated with a generic theater level campaign where two adversaries pit their military resources against each other over a sequence of multiple engagements. In particular, we consider the scenario of an air raid campaign where one adversary uses suppression of enemy air defense (SEAD) aircraft and bombers (BMBs) against the other adversary's invading ground troops (GTs) that are defended by their mobile air defense (AD) units. The original problem is decomposed into a temporal and a spatial resource allocation problem. The temporal resource allocation problem is formulated and solved in a game-theoretical framework as a multiple resource interaction problem with linear attrition functions. The spatial resource allocation problem is posed as a risk minimization problem in which the optimal corridor of ingress and optimal movement of the GTs and AD units are decided by the adversaries. These two solutions are integrated using an aggregation/deaggregation approach to evaluate resource strengths and distribute losses. Several simulation experiments were carried out to demonstrate the main ideas.**

***Index Terms*—Air campaign modeling, applied game theory, military campaigns, resource allocation, resource interaction models.**

I. Introduction

In large-scale theater level military operations, air campaigns are frequently used as the preferred mode of invasion of enemy territory [1]–[3]. They are also effective measures of defense against enemy invasion into friendly territory. Advancements in computing infrastructure, communication systems, and weapon systems technology have opened up significantly novel possibilities in the design and development of automated theater level air campaign management systems. This has led to renewed interest by researchers in recent years in the general area of automated battle management of which air campaigns are an important component [4]–[11]. These papers indicate that automation of air campaign planning operations offers several avenues to apply theoretical and conceptual developments arising from game theory, operations research, artificial intelligence, and decision theory.

A typical air campaign involves several classes of flight vehicles, each having its own operational role, utility, and capability. For instance, an air campaign may consist of vehicles for surveillance and reconnaissance, strike aircraft for detection and suppression of enemy air defenses (also known as SEAD aircraft), fighter aircraft for aerial air defense (AD), and bombers (BMBs) for destroying ground-based targets of value to the enemy. In its generic form, a viable model for an air campaign consists of two adversaries and several resources of various types belonging to each of the adversaries. One of the adversaries carries out the air campaign in order to thwart some objective of the other adversary who, in turn, defends itself against the air campaign while trying to achieve its goals. This process results in interactions among the adversaries' resources. These interactions occur several times over a prolonged time period and the results of these interactions ultimately decide the outcome of the campaign. The types of resources used depends on the objectives of the adversaries. Some possible objectives are reaching a target in the enemy territory and destroying it, preventing enemy forces from reaching a destination with sufficient strength, movement of resources to ensure survivability, etc.

An air campaign against enemy forces has all the elements of a two player game in which each player takes certain decisions regarding the utilization of available resources in each mission, the route to be selected for attack, the movement of various resources, and other relevant aspects of a military mission. Although campaign modeling *per se* is a game-theoretical problem, given its large size and attendant complexity, formulations of campaigns as a game have not been successful in terms of obtaining optimal solutions. Conceptual and computational difficulties abound in these models. As a result, there are hardly any efforts available in the literature that address the problem of air campaign planning in its totality. Of course, considerable literature does exist on individual problems that arise as a part of the decision making process in air campaign planning.

The problem addressed in our paper is that of an air campaign in which the friendly (BLUE) forces attempt to thwart the invasion of its territory by the enemy (RED) forces. BLUE has two types of resources at its disposal—SEAD aircraft and BMBs. RED has AD units and ground troops (GTs). The SEAD aircraft detect and destroy enemy ADs and create a safe corridor for the BMBs to penetrate into territory defended by the RED AD units and destroy the invading GTs. The campaign is carried out in the

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form of several SEAD/BMB missions, each lasting for perhaps half a day or even a complete day, while the RED forces plan the movement of their AD units in order to afford maximum protection to their invading GTs. Although this problem appears to be specific to a certain type of air campaign application, it has many of the major components of a generic air campaign and has been used in the literature to illustrate various aspects of air campaign planning, modeling, analysis, and simulation [11].

It turns out that even a simply stated problem like this, when formally expressed in a game-theoretical framework, poses intractable difficulties in its solution. This is perhaps the main reason that this type of air campaign planning problem, although of great importance to military strategists, has seldom been addressed in its totality either in the game-theory literature or in the control theory literature. Some of the complications that arise in attempting to solve this problem are due to several apparently unrelated objectives that each adversary has at different stages of the game. For instance, the BLUE forces have to decide the optimum route of ingression into the enemy territory and the number of SEAD aircraft and BMBs that should be used in each mission. The RED forces, on the other hand, have to decide on how to move its AD units and GT units, and also how many of the AD units to be used for defense purposes. This imposes an asymmetry among the adversary's decision variables.

In the early days of development of game theory and its application to analysis and design of military strategy, researchers had indeed addressed problems that have some similarity with the problem addressed in our paper. A brief review of these papers follows.

One of the earliest papers that addresses the air campaign problem is the tactical air war game formulated by Berkovitz and Dresher [12]. This paper was a generalization of an earlier paper by Fulkerson and Johnson [13] and was a seminal work that demonstrated the application of game theory to realistic warfare modeling. In this paper the two adversaries (RED and BLUE) are evenly matched in terms of their resources, thus imposing a certain symmetry in their resource types, capabilities, and decision variables. Specifically, both players have several aircraft in their arsenal. These aircraft have to be assigned different roles of counter-air, air defense, and ground support. This tactical air war game is assumed to consist of a sequence of several missions, each of which consists of simultaneous counter-air, air defense, and close-support operations undertaken by each adversary to achieve an objective represented by a payoff function which is described as the difference in the number of planes allocated for ground support. This is based on the assumption that the number of aircraft allocated for ground support is, in some sense, responsible for the advancement that the ground forces of the adversaries achieve in the battlefield. Solution is sought in terms of the optimal partitioning of the aircraft resources in each mission by both adversaries. It turns out that for a large number of stages both players need to use mixed strategies. In a later paper, Berkovitz and Dresher [14] extended this model to the situation when the players have two types of aircraft (BMBs and fighters) that could be partitioned among the three different tasks. The assumption of resource symmetry was maintained. The optimal strategies were shown to be mixed for both players.

Another classical paper that addressed a problem that has some similarity to the resource allocation problem addressed in our paper was by Blackwell [15] on multicomponent attrition games where the two players have several resources each. The game is defined in normal form where each player selects an action from a finite set of actions. Each action pair leads to an attrition to each resource of the players. The game is played several times until at least one of the resources is reduced to zero. The game is defined through the attrition matrices associated with each resource. The strategy of each player is defined in terms of a probability distribution on its action set. In spite of its simplicity, Blackwell's multicomponent attrition game is difficult to solve. Even when the attrition matrices are constant, the game proves to be intractable [16]. Blackwell therefore solves a somewhat different problem in which he investigates the asymptotic behavior of the optimal probability distribution on the discrete action space (that is, optimal mixed strategies) as the initial resource levels of the two players are increased infinitely while keeping the relative resource levels fixed. This transforms the game to an infinitely repeated game with no limitation on the attrition to resources. The solution to this game is obtained in terms of conditions on the initial resource levels of the two players such that probability of win of one of the players asymptotically approaches one. Certain deep results in approachability-excludability theory in vector games, based on the generalization of the classical Minimax theorem to games with vector payoffs [17], are used to arrive at these results. While this is an interesting result, its actual interpretation in the context of a practical warfare situation appears to be difficult mainly due to the assumption of the initial resource levels being infinitely large.

There have been a few other attempts to solve similar problems relevant to military campaigns. One such attempt is by Bracken and McGill [18], where a mathematical programming approach is taken to formulate the problem of aircraft sortie allocation for a two stage game. The formulation of the game and the proposed algorithm depends strongly on the convexity of the payoff function. Another recent effort in this direction is by Griggs *et al.* [4], who develop an air mission planning algorithm using decision analytic and mixed integer programming techniques.

Our model differs from the tactical air war model in several ways. For instance, we recognize the fact that in the modern scenario the roles and capabilities of military aircraft are highly specialized and so aircraft cannot be treated as a single resource that can be partitioned and used for several different tasks. Therefore, we consider different types of aircraft as different resources. Also, GTs and AD units are considered as separate resources. Furthermore, we model the air campaign planning problem as an asymmetric game where one of the adversaries carries out an air campaign against the other adversary who mounts a ground invasion escorted by AD units. Finally, we explicitly consider the spatial dimension of the problem in terms of movement of the GTs and mobile AD units, as well as the creation of the SEAD/BMB ingress corridors and distribution of active and passive AD units. This spatial aspect of an air campaign has been completely suppressed in the tactical air war game. Contributions that consider only the temporal resource allocation problem, but with the type of models described here are [19] and [20]. Another related attempt at the temporal resource allocation problem based on a discretization of the state and action spaces is reported in Krichman *et al.* [22], demonstrating the need for mixed strategy solutions for multiple stages. A recent paper by Cruz *et al.* [23] describes a discrete time air operation model, where the solution is proposed in terms of a single-stage Nash strategy.

Blackwell's multicomponent attrition game has certain similarities with our model in terms of the resource attrition process. However, there are some important differences too. Our multiple resource interaction game is a continuous kernel game with the actions being functions of the available resource levels, while in Blackwell's multicomponent attrition game, the action space is discrete and no specific relation between the actions and the available resource strengths is assumed. In our model the attrition matrices may not be constant over the many plays of the game as they are functions of the available resource levels. The interaction dynamics in our model is specified through a well-defined sequence of interactions whereas in Blackwell's model the interaction is assumed to be simultaneous. Since the air campaign problem must explicitly model the sequence in which the resources interact, the multiple resource interaction model is closer to reality than Blackwell's multicomponent attrition model.

In this paper, we retain the game-theoretical premise underlying an air campaign while taking into account both its temporal and spatial dimensions. We develop a simple but realistic multiple resource interaction model that addresses several important issues arising in SEAD-assisted air campaigns. The basic premise behind this approach is that any military campaign is basically a sequence of interactions between different resources of the adversaries. Furthermore, these interactions take place in two different dimensions: 1) spatial and 2) temporal. In the spatial dimension, the adversaries decide on the spatial distribution of resources in the battlefield and the movement and route planning of these resources in order to achieve their goal. In the temporal dimension, the adversaries decide on the strength of their resources to be used at different times during the campaign. We make an assumption (which is somewhat heuristic, but it does have a logical premise) that the overall air campaign problem can be decoupled in these dimensions and the resultant problems can be addressed separately. This is the spatio–temporal approach taken in this paper. Solutions to these problems are then integrated to obtain an overall decision making capability for the campaign. This last is not very straightforward and there is no guarantee of an optimal solution in the global sense. However, this approach does seem to yield a potentially useful set of solutions that take into account both the spatial and temporal aspects of the overall problem. It should be noted that the rigorous theoretical results of the temporal resource allocation problem has been reported in Ghose *et al.* [19], [20]. In this paper, we present just the relevant aspects of the more detailed results available in the above papers, and use them in solving the overall air campaign problem. Some preliminary results on the spatio–temporal resource allocation problem is also available in [21].

The paper is organized as follows. In Section II we present the details of the model representing the air campaign and identify the temporal and spatial aspects of the overall problem. In Section III we solve the spatial resource allocation problems to obtain optimal ingress corridor and optimal GT and AD movement strategies. In Section IV we use the aggregated model to solve the temporal resource allocation by both adversaries. In Section V we present the aggregation and deaggregation technique used to integrate the temporal and spatial solutions. In Section VI several simulation studies are presented to illustrate the main concepts behind our approach. Section VII concludes the paper.

II. Problem Formulation

A. *Air Campaign Against Enemy Invasion*

The problem we address concerns the invasion of RED GTs into the BLUE territory. The RED GTs are protected by several RED AD units. The BLUE forces, on the other hand, try to destroy as much of the GT strength as possible while trying to avoid the AD units. To do this the BLUE forces employ two types of resources. The first constitutes several SEAD units that are certain types of flight vehicles equipped with sophisticated sensor systems that detect the presence of AD units by latching on to their emitted signals and then destroys them using anti-radiation missiles [24], [25]. The SEAD units are used to create a safe corridor for the BMB aircraft, which are the second type of resources used by the BLUE forces, to penetrate enemy territory. The BMBs are used to destroy primarily the GTs.

The campaign is assumed to take place on a battlefield modeled as a gameboard consisting of a collection of several hexagonal sectors, each having a maximum of six neighboring sectors. A corridor is defined as a string of sectors from a starting point to an end point. Normally, the starting point would be the location of a SEAD unit or a BMB unit and the end point would be the location of the target (which in this case would be a GT).

The operational objectives of the various resources are as follows:

1) GTs attempts to advance until it reaches a well-defined BLUE border;
2) ADs attempt to destroy SEADs and BMBs to protect themselves and the GT;
3) SEADs precede BMBs to sanitize a corridor of AD threats;
4) BMBs fly through the sanitized corridor to target GTs.

The air campaign is modeled as a multistage campaign where each stage, comprising of a mission, is of about half a day duration. At the beginning of each stage, both players need to make the following decisions:

BLUE Decisions: i) Determination of the ingress corridor and ii) selection of the number of BMBs and SEADs to be used in a given mission.

RED Decisions: i) The total number or strength of active AD units; ii) identification of active and passive ADs; iii) movement of AD units; and iv) movement of GT units.

The possible states that a unit of a given resource can assume are as follows:

AD (Active, Passive): Active AD units are those that have their radars in the tracking mode. Therefore, they not only pose a risk to the BLUE aircraft but are also detectable by the SEADs. Passive AD units are those that have their radars turned off and hence are undetectable. They can also be moved from one sector to another during a campaign stage. Since their radars are turned off they have a low probability of being detected by SEADs.

SEAD (Active, Passive): Active SEADs are those that are used in the current stage or mission of the campaign. Passive SEADs are kept in reserve for use in later stages of the campaign.

BMB (Active, Passive): Active BMB are used in the current stage or mission of the campaign and passive BMB are kept in reserve.

The air campaign takes place on a gameboard having a single border that separates the BLUE and the RED territories. BLUE SEAD and BMB base is located somewhere in the BLUE territory. The RED forces have resources of GTs and AD units that are located somewhere in the RED territory.

Some of the major assumptions are listed below.

1) The air campaign is played out in several stages.
2) Each stage is of sufficiently long duration to enable one SEAD/BMB mission to be completed and to enable the RED forces to move its GTs and ADs.
3) A single mission in the BLUE air campaign consists of a single ingression by SEADs followed by the BMBs.
4) The SEADs engage ADs on its ingress corridor while BMBs may engage both ADs and GTs.
5) The ADs have the choice of remaining hidden (passive) or they may have their radars in the track mode (active).
6) When ADs are active, they may be assumed to remain stationary and when they are passive they are undetected and can be moved to adjacent sectors.

Note that the last assumption may mean that the ADs will not be able to keep up with the GTs on a long route. Therefore, a more realistic assumption would be to allow ADs to move alongside the GTs and expend them selectively. That is, ADs that become active may remain active until they are destroyed while the passive ones continue to move with the GTs until they are called upon to become active. Alternatively, one may move both active and passive ADs with the passive ADs remaining undetected. Some of these alternatives will be examined in the simulation study.

B. Notations and the Spatio–Temporal Model

The gameboard is denoted by $\mathcal{G} = \{g_1, \ldots, g_m\}$ where each g_i identifies a sector in the gameboard. The set of sectors adjacent to a sector g_i is denoted by $N(g_i)$. In the following, we use u to denote the decision variables under the control of the BLUE player and v to denote the decision variables under the control of the RED player. The superscripts generally identify the type of decision variables or player resources, while the subscript k denotes the stage of the campaign.

A SEAD/BMB corridor in $\mathcal{G}$ at the kth stage is denoted by $u_k^c = \{c_1, \ldots, c_{r_k}\}$ where each $c_i \in \mathcal{G}$ and $c_{i+1} \in N(c_i)$. Furthermore, c_{r_k} is a sector containing the GT unit and c_1 is the sector where the SEAD and BMB bases are located. In its most general form r_k could either be a fixed integer (which would imply that each SEAD/BMB mission must be of a certain fixed length in terms of number of sectors), or it could be bounded above by $r_k \leq r_{\max}$ (which would imply a constraint on the SEAD/BMB capability in terms of fuel or endurance), or it could be an arbitrarily large integer (implying that no such constraint exists on the SEAD/BMB capability). Note that u_k^c is one of the decision variables of the BLUE forces in the kth stage of the game.

Let $g_k^t \in \mathcal{G}$ denote the location of the GT units on the gameboard in stage k.

Let v_k^t denote the movement of the GT unit from its current location to one of the neighboring sectors. Thus, v_k^t is a decision variable of the RED forces at the kth stage of the game.

Let $\mathcal{D} = \{d_1, \ldots, d_s\}$ denote the collection of AD units.

At a given stage k, let the location of AD units on the gameboard be given by $E_k = \{e_k^{d_1}, \ldots, e_k^{d_s}\}$ with $e_k^{d_i} \in \mathcal{G}$ implying that the AD unit d_i is located in the sector $e_k^{d_i} \in \mathcal{G}$ at stage k of the game.

Let $v_k^{m_i}$ define the movement of the AD unit d_i from its current location to one of its neighboring sectors. Thus, $v_k^m = (v_k^{m_1}, \ldots, v_k^{m_s})$ is the decision variable of the RED forces at the kth stage of the game.

Let $v_k^s = (v_k^{s_1}, \ldots, v_k^{s_s})$ define the status vector for the AD units, where $v_k^{s_i}$ denotes the status (active or passive) of the AD unit d_i in stage k. We may take $v_k^{s_i} = 1$ if the AD unit is active and $v_k^{s_i} = 0$ if it is passive.

Let S_k^g be the GT strength, S_k^s be the SEAD strength, and S_k^b be the BMB strength available at the beginning of the stage k. Let $S_k^d = (S_k^{d_1}, \ldots, S_k^{d_s})$ be the AD strength vector where $S_k^{d_i}$ is the strength of the AD unit d_i. The strength of SEAD, BMBs, and GTs can be directly modeled as numbers or capabilities of the specific resources. The strength of the AD units, however, requires a somewhat more elaborate treatment in the form of an aggregation process. We will discuss this aspect later.

The state of the game at each stage k is completely defined by $g_k^t, E_k, S_k^s, S_k^b, S_k^g$, and S_k^d.

The control variables of the players at stage k are as follows.

The RED controls are v_k^t (GT movement), v_k^m (AD movement), v_k^s (AD status, active or passive), and $v_k^d = (v_k^{d_1}, \ldots, v_k^{d_s})$ with $v_k^{d_i} \in [0, S_k^{d_i}]$ (AD strength used). Note that the status control variable v_k^s and the AD strength control variable v_k^d can be merged by assuming that when the AD strength used is zero, its status is passive.

The BLUE controls are u_k^c (ingress corridor), $u_k^s \in [0, S_k^s]$ (SEAD strength used), and $u_k^b \in [0, S_k^b]$ (BMB strength used).

The loss functions are defined as follows. Let L^g denote the loss function for GT strength, L^d denote the loss function for AD strength, L^s denote the loss function for SEAD strength, and L^b denote the loss function for BMB strength.

The state equations would be given by

$$g_{k+1}^t = \tilde{G}\left(g_k^t, v_k^t\right) \tag{1}$$

$$E_{k+1} = \tilde{E}\left(E_k, v_k^m\right) \tag{2}$$

$$S_{k+1}^s = S_k^s - L^s\left(u_k^c, u_k^s, E_k, v_k^d, v_k^s\right) \tag{3}$$

$$S_{k+1}^b = S_k^b - L^b\left(u_k^c, u_k^s, u_k^b, E_k, v_k^d, v_k^s\right) \tag{4}$$

$$S_{k+1}^g = S_k^g - L^g\left(u_k^c, u_k^s, u_k^b, E_k, v_k^d, v_k^s, S_k^g\right) \tag{5}$$

$$S_{k+1}^d = S_k^d - L^d\left(u_k^c, u_k^s, u_k^b, E_k, v_k^d, v_k^s\right) \tag{6}$$

where $k = 1, \ldots, N$ denote stages in the game, with N being the total number of stages. In the above, the movement of the GT and AD units are defined through (1) and (2), whereas (3)–(6) define the variation in the resource strength.

Attritions on SEAD, BMB, and GT strengths are defined through the loss functions that depend on various parameters, control, and states of the system. For instance, the loss in GT strength is modeled through the function L^g and is a function of the surviving BMB strength that reaches the GT position. The surviving BMB strength is itself a function of the surviving AD strength after the SEADs have passed through the corridor, and the BMB strength (u_k^b) used by the BLUE. Finally, the surviving AD strength is a function of the SEAD strength used (u_k^s), the AD strength used (v_k^d), AD unit placement (E_k) and status (v_k^s), and the ingress corridor (u_k^c). The other loss functions are similarly defined.

The strength of each AD unit, unlike the other resource strengths, depends on the spatial dimension of the problem. In particular, it depends on the corridor and the location of the AD unit. Consider Fig. 1, where the envelope around the AD unit defines its zone of influence, or the region in which the AD unit generates risk for the BLUE aircraft (SEAD or BMB). It could be considered as the kill probability or a quantity which is a function of the kill probability, terrain features, etc. We will call it risk and define a risk index for each sector within the zone of influence of the AD unit. The total strength of the AD unit d_i at the stage k is denoted as $T_k^{d_i}$ and is defined as the sum of the risk indices associated with the sectors within the zone of influence of the AD unit. However, this total strength is not the AD strength since not all of it can be used for interacting with SEADs or BMBs. The strength of the AD unit d_i on the given corridor is defined as the sum of the risk indices on the sectors that form the corridor and are within the influence zone of the AD unit, and is denoted as $S_k^{d_i}$.

The game as formulated here has a significant spatial dimension in the sense that the actual locations of the GTs and ADs determine the losses that the SEAD and BMB missions suffer and hence determine their effectiveness. The spatial dimension of this problem concerns the movement and location of RED GTs and ADs and the selection of the string of sectors that defines an ingress corridor for the BLUE SEADs, and gives rise to the spatial resource allocation problem. The state equations that correspond to the spatial dimension are (1) and (2). The temporal dimension of the game is concerned with the decision on how many of the SEAD and BMB of the BLUE forces and how many of the ADs of the RED forces should participate in a mission. This gives rise to the temporal resource allocation problem. The state equations that correspond to the temporal dimension are (3)–(6).

III. Spatial Resource Allocation

The spatial resource allocation problem basically addresses the problem of creating an optimal corridor of ingress for the BLUE SEADs and BMBs. It also addresses the problem of the movement of the GTs and the ADs of the RED forces.

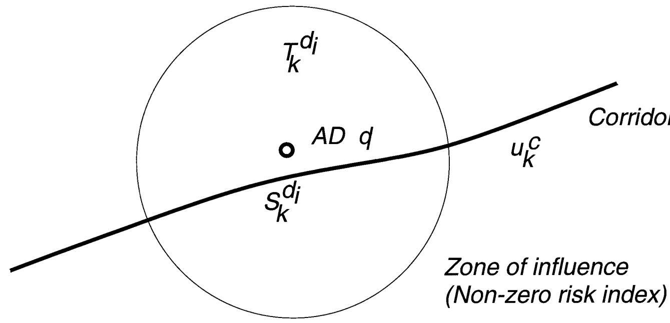

Fig. 1. AD strength model.

A. Ingress Corridor

A placement E_k of ADs imposes a risk profile on the gameboard for BLUE aircraft. It quantifies the risk or danger to a BLUE aircraft in passing through a sector. This risk is a function of the locations and capabilities of ADs used by RED forces. It may also depend on the kill probability of the AD unit, the terrain, and the type of aircraft. An optimal corridor would be one with the minimum risk associated with it. The concept of risk profile has been earlier used in AD applications quite frequently (e.g., see [26]). In the following discussion, we omit the stage index "k."

Let the risk on a sector g_i on the gameboard, due to the location of the AD unit d_j in the sector e^{d_j} (given by placement E), be denoted by $r_{ij}\,|_E$. Then, the risk profile on the gameboard due to d_j in e^{d_j} is given by

$$r_j\,|_E = \{r_{1j}\,|_E, \ldots, r_{mj}\,|_E\} = \{r_{1j}, \ldots, r_{mj}\}\,|_E. \tag{7}$$

Then the risk profile due to the placement E of all the ADs is given by

$$\mathcal{R}(E) = \{\mathcal{R}(E, g_1), \ldots, \mathcal{R}(E, g_m)\} \tag{8}$$

where $\mathcal{R}(E, g_i)$, the risk at sector g_i due to the AD placement E, is defined as

$$\mathcal{R}(E, g_i) = \sum_{d_j \in \mathcal{D}} r_{ij}\,|_E. \tag{9}$$

The risk $\rho(E, u^c)$ on corridor u^c, due to an AD placement E is defined as

$$\rho(E, u^c) = \sum_{c_i \in u^c} \mathcal{R}(E, c_i). \tag{10}$$

For a given AD placement E, the corridor creation problem may be formulated as

$$\min_{u^c} \rho(E, u^c). \tag{11}$$

Dijkstra's shortest path algorithm [27] can be used to compute the minimum risk corridor.

The above solution is acceptable if we assume the AD units to be stationary. On the other hand, if the AD units are capable of movement (that is, they exercise control through v^m), then we may formulate the problem as

$$\min_{u^c} \max_{v^m} \rho(\tilde{E}(E, v^m), u^c) \tag{12}$$

where $\tilde{E}(E, v^m)$ is defined as the placement obtained by applying the control v^m to the current placement of AD units. This can be viewed as a game where the two players try to control the risk profile to optimize the risk ρ on the corridor. The BLUE forces do this by controlling the determination of the sectors that constitute the corridor, and the RED forces do this by controlling the placement of the AD units.

This latter equation [(12)] is a zero-sum game in normal form and is unlikely to have a solution in pure strategies. A saddle point in mixed strategies, however, would exist since the control sets of the players are discrete and finite (since there are a finite number of possible placements of AD unit given the constraint on their movement and there are a finite number of loop-less paths from the SEAD/BMB base to the GTs), and so the game, in principle, can be represented as a normal form zero-sum matrix game, where one player tries to minimize the risk and the other player tries to maximize it. A mixed strategy, however, would be difficult to implement, or even interpret, when the game is played only a limited number of times and the support set for the mixed strategies is large. A more reasonable solution to this problem would be a Minimax (or security) strategy.

From the point of view of the BLUE forces, a Minimax solution can be obtained as follows. Suppose we have a collection $\hat{U}^c$ of a finite number of corridors $u^{c_1}, \ldots, u^{c_r}$ that are feasible for the BLUE forces, then the problem can be formulated as

$$\min_{u^{c_i} \in \hat{U}^c} \left\{ \max_{v^m} \rho \left(\tilde{E}(E, v^m), u^{c_i} \right) \right\}. \tag{13}$$

This problem can be solved by obtaining, for every $u^{c_i} \in \hat{U}^c$, the quantity

$$H(u^{c_i}) = \max_{v^m} \rho \left(\tilde{E}(E, v^m), u^{c_i} \right) \tag{14}$$

and then computing

$$\min_{u^{c_i} \in \hat{U}^c} H\left(u^{c_i}\right). \tag{15}$$

This is easy if the set $\hat{U}^c$ has a finite (and small) number of elements. If we consider all possible corridors on the gameboard, then it would yield a finite but inordinately large number of corridors. To limit this number, we first solve the problem posed in (11) with E as the current placement. The minimum risk corridor so obtained is called the *nominal corridor* and is denoted by u^c_{nom}. Now, we compute $H(u^c_{\text{nom}})$ from (14). Obviously, any corridor u^c for which $\rho(E, u^c) \geq H(u^c_{\text{nom}})$ need not be considered as a member of $\hat{U}^c$. Therefore, we need to find $\hat{U}^c$ such that

$$\hat{U}^c = \left\{ u^c \,|\, \rho\left(E, u^c_{\text{nom}}\right) \leq \rho(E, u^c) \leq H\left(u^c_{\text{nom}}\right) \right\}. \tag{16}$$

To implement this, we need an algorithm that computes corridors of increasing cost. For example, an algorithm for solving the K-shortest path problem on a graph [28] would serve this purpose quite well. Each new corridor u^c_{new} obtained by this process, if it satisfies $H(u^c_{\text{new}}) < H(u^c_{\text{nom}})$, can be used as the nominal corridor for computing (16). This process is guaranteed to terminate since there are only a finite number of loopless corridors possible. When the algorithm terminates, the elements of $\hat{U}^c$ are the Minimax solution to the problem in (12) and give the optimal corridors for the BLUE forces.

For the RED forces, a problem to obtain optimal movement of the AD units can be formulated as

$$\max_{v^m} \{ \min_{u^c} \rho(\tilde{E}(E, v^m), u^c) \} \tag{17}$$

and solved by computing the minimum risk corridor for every possible movement of ADs.

B. Movement of GTs

The GTs have the objective of getting to the BLUE border with minimum losses. Thus, the route taken by the GTs should be such that the strength of BMB units must undergo the maximum possible attrition as it attacks each sector through which the GT units pass. The objective is to replace the GT movement problem with the problem of creating a minimum risk path on the gameboard with appropriately chosen risk profile. Based on this requirement, the problem is solved using the following procedure.

For the current AD placement E the risk profile $\mathcal{R}(E)$ on the gameboard can be determined as before. We denote the cost to the GTs, if they are located in sector g_i, to be $L^{g_i}(E)$, which is defined as the losses inflicted on the GTs by the BLUE BMBs that survive interaction with ADs if BLUE decides to attack sector g_i with optimal SEAD and BMB resource strength (which is obtained from the solution of the temporal resource allocation problem discussed in the next section). This computation would take into account the losses suffered by the SEAD and BMBs as they fly through the minimum risk path from the SEAD/BMB base to the sector g_i. Assuming that the GTs are located in g_i, let u^{c,g_i} denote the minimum risk corridor to g_i for BLUE obtained by solving (11). This information (that is, the risk profile $\mathcal{R}(E)$ and the corridor u^{c,g_i}) can be used to obtain the aggregated AD strength on the corridor.

Since the available SEAD resource strength S^s and BMB resource strength S^b are also known, the solution of a temporal resource allocation problem (as explained in the next section) would yield the optimal values of AD, SEAD, and BMB strengths.

The amount of BMB strength that survive the mission to g_i can then be computed by taking into account the loss functions given in the state equation (4). Finally, (5) can be used to obtain the losses that the GTs suffer if they are located in g_i and are attacked by BLUE SEAD/BMBs through the corridor u^{c,g_i}.

Repeating this procedure for every sector g_i, we obtain a loss profile for the GTs as

$$\mathcal{L}^g = \{L^{g_1}(E), \ldots, L^{g_m}(E)\}. \tag{18}$$

This loss profile serves as the risk profile for GTs and is used to find the minimum cost path (for the GTs) from the actual current location of the GTs to the BLUE border, using Dijkstra's shortest path algorithm. The first step in this path is considered as the GT movement for the RED forces in the current stage.

C. Movement of AD Units

The objective of AD unit movement is to maximize the risk to the BLUE aircraft flying through an ingress corridor. The procedure to determine AD movement is given below.

We first determine the optimal BLUE ingress corridor u^c between the BLUE base and the GT location g^t, assuming that all the ADs are active. Note that the current strength of the ADs (which would have suffered losses in the engagement in the previous stage) is used to create this risk profile.

Given the current placement E of ADs, let $\hat{E}$ be any other placement, obtained by moving some subset of the ADs into neighboring sectors. The set of feasible placements $\mathcal{E}$ is defined by the constraint on the GT movement control variable v^m. Furthermore, the subset of ADs selected for movement is also crucial and may depend on the status of the ADs.

We choose a $\hat{E} \in \mathcal{E}$ by solving

$$\max_{\hat{E}\in\mathcal{E}} \rho(\hat{E}, u^c). \tag{19}$$

In the above, it is assumed that the ingress corridor is fixed and the ADs move to maximize the risk on the corridor. A disadvantage of this procedure (as we will see in the simulation) is that the corridor anticipated by the RED forces may be different from the corridor actually used by the BLUE forces. This can be rectified by imposing constraints either on the movement of the ADs or on the selection of the subset of ADs that are eligible to move. This is somewhat heuristic and a more rational, and also computationally more intensive, way to accomplish this would be to adopt the procedure given in (17).

IV. Temporal Resource Allocation

The temporal resource allocation problem basically addresses the problem of allocation of BMB, SEAD, and AD resources by the two players over several missions.

A. Problem Formulation

The scenario for the SEAD air campaign is shown in Fig. 2 which shows the ground troop position "GT" and the SEAD/BMB base. The corridor through which the SEAD/BMB fly is also shown along with the ADs that have influence on the corridor. The shaded area shows the lethal zone of the AD units. In the temporal resource allocation problem we dispense with the spatial dimensions of the overall problem and assume that the air campaign takes place on a single corridor defended by ADs of the RED forces from SEADs and BMBs of the BLUE forces that fly from one end of the corridor (where the SEAD and BMB stations are located) to the other end of the corridor (where the target GT is located). The ingress corridor u_k^c is assumed to be known (actually it is obtained from the solution of the spatial resource allocation problem described in the previous section). We also assume that in the state equations (3)–(6) g_k^t and E_k, in addition to u_k^c, are known. Another (somewhat strong) assumption made is that the AD strengths are aggregated into a single state variable representing the total AD strength denoted by S_k^a. Thus, the interaction of the SEADs and BMBs with the ADs are represented by a single interaction each and the attrition arising from each of these interactions is distributed among the constituent ADs using a deaggregation technique. The aggregation and deaggregation process, described in the next section, is closely related to the spatial dimensions of the problem which determines the corridor of operation and which, in turn, defines the effectiveness of ADs against SEADs and BMBs through loss/attrition functions.

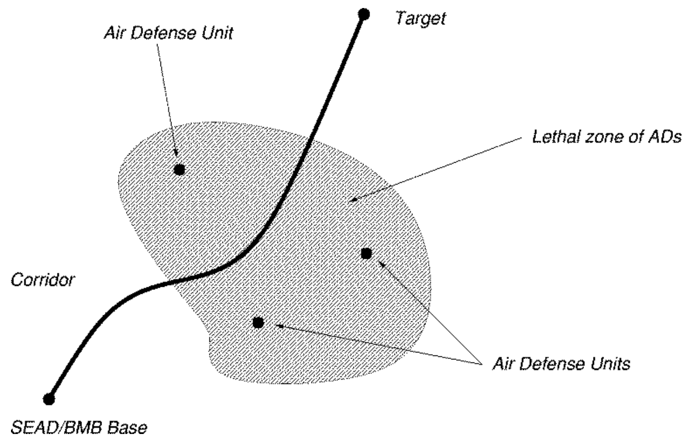

Fig. 2. SEAD-assisted air campaign scenario.

A stage in a game is defined as a single sortie in which SEADs and BMBs participate. At any given stage k of the game, the BLUE forces have an available SEAD strength of S_k^s and a BMB strength of S_k^b. Similarly, the RED forces have an available AD strength of S_k^a. The target has a strength (or value) of S_k^g. The quantities S_k^s, S_k^b, S_k^a, and S_k^g are known to the players at the beginning of a stage. Note that S_k^d in (6) is now replaced with the aggregated AD strength S_k^a.

The solution to the temporal resource allocation problem is an optimal decision by both players regarding the amount of resource to be used in each stage of the game. More specifically, the BLUE forces have to decide on the amount (that is, number or strength) of SEADs and BMBs to be used in each sortie or stage and the amount of these resources to be kept in reserve for use in later stages. Similarly, the RED forces have to decide on the amount of AD strength to be used to defend the corridor at each stage of the game and the amount to be kept in reserve for use in later stages. It is assumed that each adversary has a finite level or strength in each resource. The objective of the BLUE forces is to minimize the effect of the surviving GT strength over a specified number of stages while the objective of the RED forces is to maximize this effect.

At any given stage k of the game, the BLUE forces partition S_k^s and S_k^b as

$$S_k^s = u_k^s + r_k^s, \quad S_k^b = u_k^b + r_k^b \tag{20}$$

where $u_k^s \in [0, S_k^s]$ and $u_k^b \in [0, S_k^b]$ are used by the BLUE forces in the campaign at the kth stage and $r_k^s = S_k^s - u_k^s$ and $r_k^b = S_k^b - u_k^b$ are kept in reserve or "rest" for later use. Thus, the decision that the BLUE forces need to take at the beginning of each stage is how much of the SEAD and BMB force strengths should be used for the campaign at that stage and how much of these strengths are to be kept in reserve.

Similarly, at stage k, the RED forces have the option of keeping some of its ADs "hidden" (or passive) while the rest can be switched on (or made active) to track and engage SEADs and BMBs. Thus, the RED forces partition its AD strength as

$$S_k^a = v_k^a + r_k^a \tag{21}$$

where $v_k^a \in [0, S_k^a]$ is the AD strength used to engage SEADs and BMBs and $r_k^a = S_k^a - v_k^a$ is the AD strength kept in reserve for later use. Note that v_k^a represents the aggregated AD strength used according to the individual AD strength vector v_k^d and the AD status vector v_k^s. Thus, the decision variables of BLUE forces at the beginning of stage k in the temporal resource allocation game is (u_k^s, u_k^b) and for the RED forces it is v_k^a.

The sequence of interaction between the resources of the two players and the damages suffered by the resources due to these interactions with the adversary's resources in a given stage k are as follows:

First, the SEADs fly along a designated corridor and engage ADs located on it. The ADs and the SEADs inflict damage on each other. Let s_k^1 denote the surviving SEAD strength and a_k^1 denote the surviving AD strength

$$s_k^1 = \max\{0, u_k^s - L_a^s(v_k^a, u_k^s)\} \quad (22)$$
$$a_k^1 = \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} \quad (23)$$

where $L_a^s(\cdot,\cdot)$ defines the damage that the SEAD strength suffers when it is confronted with AD strength, and $L_s^a(\cdot,\cdot)$ defines the damage that the AD strength suffers in its interaction with SEAD strength. Note that L_a^s is derived from L^s in (3) for a specific corridor u_k^c and placement of AD units E_k. Furthermore, the variables v_k^s and v_k^d are subsumed into the decision variable v_k^a. A similar interpretation for L_s^a also holds except for the fact that this function represents only that part of the attrition that ADs suffer due to interaction with SEADs.

Next, the BMBs fly through the corridor and are engaged by ADs defending the corridor

$$\begin{aligned} b_k^2 &= \text{Surviving BMB strength} = \max\left\{0, u_k^b - L_a^b\left(a_k^1, u_k^b\right)\right\} \\ &= \max\left\{0, u_k^b - L_a^b\left(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b\right)\right\} \end{aligned} \quad (24)$$

$$\begin{aligned} a_k^2 &= \text{Surviving AD strength} = \max\left\{0, a_k^1 - L_b^a\left(a_k^1, u_k^b\right)\right\} \\ &= \max\{0, \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} \\ &\quad - L_b^a\left(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b\right)\} \end{aligned} \quad (25)$$

where $L_a^b(\cdot,\cdot)$ defines the damage that the ADs inflict on the BMBs and $L_b^a(\cdot,\cdot)$ defines the damage that the BMBs inflict on the ADs. The functions L_a^b and L_b^a have an interpretation similar to L_a^s and L_s^a above, except that L_b^a represents only that part of the attrition that ADs suffer due to interaction with BMBs. The functions L_s^a and L_b^a together represent the function L^d in the state equation (6).

Finally, the BMBs engage the GTs at the end of the corridor

$$\begin{aligned} g_k^3 &= \text{Surviving GT strength} \\ &= \max\left\{0, S_k^g - L_b^g\left(b_k^2, S_k^g\right)\right\} \\ &= \max\left\{0, S_k^g - L_b^g\left(\max\left\{0, u_k^b\right.\right.\right. \\ &\quad \left.\left.\left. - L_a^b\left(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b\right)\right\}, S_k^g\right)\right\} \end{aligned} \quad (26)$$

where $L_b^g(\cdot,\cdot)$ is the damage that BMBs inflict on GTs and has a similar interpretation.

At the next stage $k+1$, the two players have the following resource strengths available:

$$\begin{aligned} S_{k+1}^s &= r_k^s + s_k^1, \quad S_{k+1}^b = r_k^b + b_k^2 \\ S_{k+1}^a &= r_k^a + a_k^2, \quad S_{k+1}^g = g_k^3. \end{aligned} \quad (27)$$

The complete state equations corresponding to the above sequence of resource interactions are as follows:

$$S_{k+1}^s = \max\{0, u_k^s - L_a^s(v_k^a, u_k^s)\} + (S_k^s - u_k^s) \quad (28)$$
$$\begin{aligned} S_{k+1}^b &= \max\left\{0, u_k^b - L_a^b\left(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b\right)\right\} \\ &\quad + \left(S_k^b - u_k^b\right) \end{aligned} \quad (29)$$
$$\begin{aligned} S_{k+1}^a &= \max\{0, \max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\} - L_b^a(\max\{0, v_k^a \\ &\quad - L_s^a(v_k^a, u_k^s)\}, u_k^b)\} + (S_k^a - v_k^a) \end{aligned} \quad (30)$$
$$\begin{aligned} S_{k+1}^g &= \max\left\{0, S_k^g - L_b^g\left(\max\left\{0, u_k^b\right.\right.\right. \\ &\quad \left.\left.\left. - L_a^b\left(\max\{0, v_k^a - L_s^a(v_k^a, u_k^s)\}, u_k^b\right)\right\}, S_k^g\right)\right\} \end{aligned} \quad (31)$$

with the controls of the two players denoted as $u_k = (u_k^s, u_k^b)$ with $u_k^s \in [0, S_k^s]$ and $u_k^b \in [0, S_k^b]$; and $v_k = v_k^a$ with $v_k^a \in [0, S_k^a]$.

A resource interaction table that summarizes the above sequence of interactions is shown in Fig. 3. In this game, we define the payoff to be the cumulative damage caused by the surviving GTs at each stage. This could be represented as the sum of the surviving GT strengths at each stage. This is the payoff that the RED forces maximize and the BLUE forces minimize. The payoff at the end of the designated N stages is

$$J = \sum_{k=1}^{N} S_{k+1}^g. \quad (32)$$

Considering a simplified model where the loss to a given resource is assumed to be a function of only the adversary's interacting resource strength (that is, the strength of the adversary's resource that interacts with the given resource). Moreover, the losses are "linear" in the sense that the loss to a player's given resource is proportional to the adversary's resource strength with which the given resource interacts, but within the bounds of resource availability. Let

$$\begin{aligned} &L_a^s(v_k^a, u_k^s) = l_a^s(v_k^a) = \alpha v_k^a, \quad L_s^a(v_k^a, u_k^s) = l_s^a(u_k^s) = \beta u_k^s \\ &L_a^b\left(a_k^1, u_k^b\right) = l_a^b\left(a_k^1\right) = \gamma a_k^1, \quad L_b^a\left(a_k^1, u_k^b\right) = l_b^a\left(u_k^b\right) = \eta u_k^b \\ &L_b^g\left(b_k^2, S_k^g\right) = l_b^g(b_k^2) = \theta b_k^2 \end{aligned} \quad (33)$$

where the function $l_y^x(\cdot)$: $R \to R$ denotes the loss in strength suffered by resource x when it interacts with resource y of the adversary, and $\alpha, \beta, \gamma, \eta$, and θ are nonnegative scalars. The first equation means that α SEAD strength is destroyed by one unit of AD strength. The other loss parameters have a similar interpretation.

The corresponding state equations are

$$S_{k+1}^s = \max\{0, u_k^s - \alpha v_k^a\} + (S_k^s - u_k^s) \quad (34)$$
$$\begin{aligned} S_{k+1}^b &= \max\left\{0, u_k^b - \gamma(\max\{0, v_k^a - \beta u_k^s\})\right\} \\ &\quad + \left(S_k^b - u_k^b\right) \end{aligned} \quad (35)$$
$$\begin{aligned} S_{k+1}^a &= \max\left\{0, \max\{0, v_k^a - \beta u_k^s\} - \eta u_k^b\right\} \\ &\quad + (S_k^a - v_k^a) \end{aligned} \quad (36)$$
$$\begin{aligned} S_{k+1}^g &= \max\left\{0, S_k^g - \theta\left(\max\left\{0, u_k^b\right.\right.\right. \\ &\quad \left.\left.\left. -\gamma(\max\{0, v_k^a - \beta u_k^s\})\right)\right)\right\} \end{aligned} \quad (37)$$

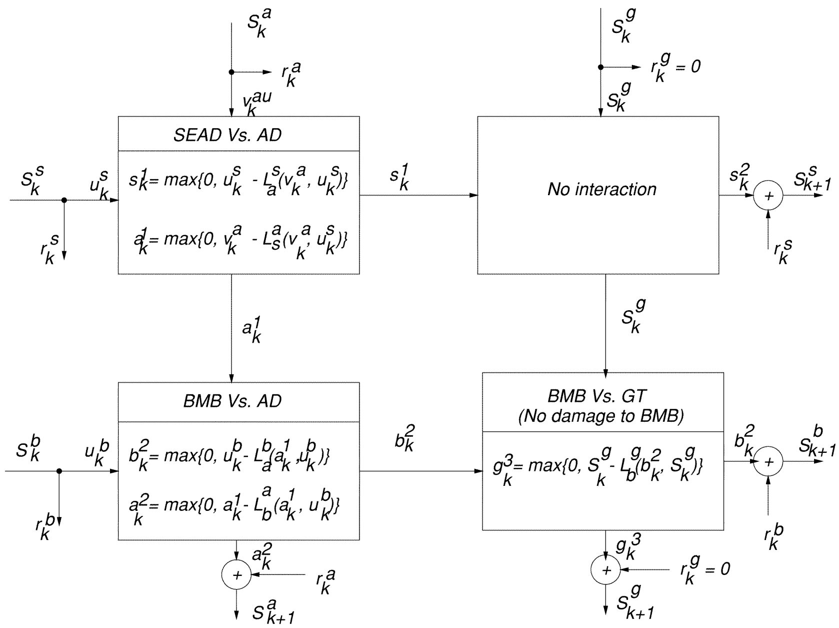

Fig. 3. Resource interaction table for stage k for SEAD-assisted air campaign.

with the control variables constrained as in the general state equations.

B. The Single-Stage Game

Consider the payoff at the kth stage

$$J_k(u_k, v_k) = S_{k+1}^g \tag{38}$$

with the controls $u_k = (u_k^s, u_k^b) \in U_k = [0, S_k^s] \times [0, S_k^b]$ and $v_k = (v_k^a) \in V_k = [0, S_k^a]$. Suppose we want to solve the game only for the kth stage treating it as a single-stage game. That is

$$\min_{(u_k^s, u_k^b) \in [0,S_k^s]\times[0,S_k^b]} \max_{v_k^a \in [0,S_k^a]} \left[\max\left\{0, S_k^g - \theta\left(\max\left\{0, u_k^b - \gamma\left(\max\left\{0, v_k^a - \beta u_k^s\right\}\right)\right\}\right)\right\}\right]. \tag{39}$$

The payoff function (38) does not satisfy the standard convexity-concavity properties normally used for proving the existence of saddle point in pure strategies [29]. However, the payoff function does satisfy the property that it is a monotonically decreasing function of u_k^s and u_k^b and a monotonically increasing function of v_k^a (we define a function $f\colon R \to R$ to be monotonically increasing (monotonically decreasing) if $f(x) \geq f(y) (f(x) \leq f(y))$ whenever $x > y$). This property can be used along with Fan's Minimax theorem [30] to prove that a saddle point in pure strategies exists for the kth stage of the game, treated as a single-stage game, with the payoff as given in (38). The detailed proofs of these assertions are given in [19] and [20]. Note that the saddle point may not be unique and multiple saddle points may exist. The interchangeability property of saddle points ensure that the payoff for all saddle point strategies is the same [29]. It is possible to characterize the saddle point strategies of the two players using the standard saddle point property according to which, if (u_k^*, v_k^*) is a saddle point strategy pair, then

$$J_k(u_k^*, v_k) \leq J_k(u_k^*, v_k^*) \leq J_k(u_k, v_k^*), \quad \text{for all } u_k \in U_k, v_k \in V_k. \tag{40}$$

It is possible to solve this game to obtain the saddle point strategies for the two players in closed form. The solution procedure is based on the fact that only one of the following three situations can arise:

1) RED has sufficient AD resources to destroy all the BMBs before they reach the GTs, even when BLUE uses all its available SEAD and BMB strength;
2) BLUE has enough SEAD and BMB resources to ensure that enough BMBs survive after interaction with ADs to completely destroy the GTs, even when RED uses all its available AD strength;
3) even when the two players use the maximum resources available, the GTs are neither completely destroyed nor do they survive intact.

Based on these observations and the monotonicity property of the payoff function and loss functions [19], [20], the solution of the single-stage game, in terms of the saddle point strategies of the two players, can be obtained as follows:

1) If $S_k^a \geq \beta S_k^s$ and $\gamma(S_k^a - \beta S_k^s) \geq S_k^b$, then the saddle point strategies are given as

$$v_k^* = v_k^{a*} \in \left\{v_k^a \in V_k : \gamma\left(v_k^a - \beta S_k^s\right) \geq S_k^b\right\} \tag{41}$$
$$u_k^* = \left(u_k^{s*}, u_k^{b*}\right) \in \left\{[0, S_k^s] \times \left[0, S_k^b\right]\right\} \tag{42}$$

and the value of the game is S_k^g.

2) If $\beta S_k^s \geq S_k^a$ and $\theta S_k^b \geq S_k^g$; Or if $\beta S_k^s < S_k^a$ and $\theta(S_k^b - \gamma(S_k^a - \beta S_k^s)) \geq S_k^g$; then the saddle point strategies are given as

$$v_k^* = v_k^{a*} \in [0, S_k^a] \tag{43}$$
$$u_k^* = \left(u_k^{s*}, u_k^{b*}\right) \in \left\{\left(u_k^s, u_k^b\right) \in U_k : \beta u_k^s \geq S_k^a; \theta u_k^b \geq S_k^g\right\} \bigcup \left\{\left(u_k^s, u_k^b\right) \in U_k : \beta u_k^s < S_k^a; \theta\left(u_k^b - \gamma\left(S_k^a - \beta u_k^s\right)\right) \geq S_k^g\right\} \tag{44}$$

and the value of the game is zero.

3) If neither of the conditions in 1) and 2) above holds, then the saddle point strategies are given as

$$v_k^{a*} = S_k^a. \tag{45}$$

If $S_k^a \geq \beta S_k^s$ then

$$\left(u_k^{s*}, u_k^{b*}\right) = \left(S_k^s, S_k^b\right) \tag{46}$$

else (that is, if $S_k^a < \beta S_k^s$)

$$\left(u_k^{s*}, u_k^{b*}\right) \in [\hat{u}_k^s, S_k^s] \times \left[S_k^b\right] \tag{47}$$

where $\hat{u}_k^s$ is such that $S_k^a = \beta \hat{u}_k^s$. The value of the game is $\max\{0, S_k^g - \theta(\max\{0, S_k^b - \gamma(\max\{0, S_k^a - \beta S_k^s\})\})\}$.

Note that the saddle point, except in one of the cases in 3), is not unique in general. However, by the interchangeability property of saddle point strategies in zero-sum games, each pair of strategies selected from the above sets is a saddle point strategy pair and yields the same payoff which is the value of the game. This multiplicity of saddle point strategies, in fact, gives rise to a related problem of selection of saddle point strategies by the players. This aspect has been discussed in detail in [20].

If the condition in 1) above holds then we say that RED has a "winning" (denoted by "W") strategy. Essentially, if 1) holds then RED can ensure the destruction of the BMBs completely before they reach the GTs, irrespective of the resources used by BLUE. Consequently, all the GTs survive. If 1) does not hold then we say that RED has a "nonwinning" (denoted by NW) strategy implying that BLUE can destroy some nonzero amount of GTs irrespective of the resources used by RED. Similarly, if the condition in 2) holds then BLUE is said to have a winning strategy since BLUE would be able to destroy the GTs completely, irrespective of the AD strength used by RED. If 2) does not hold then BLUE has a nonwinning strategy and some nonzero GTs will survive irrespective of the resources used by BLUE. If neither the conditions in 1) nor the conditions in 2) are satisfied then both players have nonwinning strategy. Obviously, as mentioned earlier, both players cannot have winning strategy at any given stage.

C. Multistage Game

With the above results in place, we examine the multistage game. The termination of the game occurs if either:

1) all GTs get destroyed before the last stage N is reached, or
2) The last stage N is reached.

From the above results, the game terminates before stage N if BLUE has a winning strategy, and uses it, at any stage.

In the multistage game, the player's controls are defined as

$$u = (u_1, \ldots, u_N) \tag{48}$$

$$v = (v_1, \ldots, v_N) \tag{49}$$

where N is the number of stages; $u_k = (u_k^s, u_k^b)$, and $v_k = (v_k^a)$; with $u_k^s \in [0, S_k^s], u_k^b \in [0, S_k^b], v_k^a \in [0, S_k^a]$; for all $k = 1, \ldots, N$.

In the multistage game, we would like to find saddle point strategies to achieve

$$\min_{u=(u_1,\ldots,u_N)} \max_{v=(v_1,\ldots,v_N)} \sum_{k=1}^{N} S_{k+1}^g. \tag{50}$$

Unfortunately, there is no guarantee of a saddle point in pure strategies existing unless we impose further conditions on the payoff kernel at each stage. In the multistage game define the payoff kernel of the game at the stage k as

$$J_k(S_k, u_k, v_k) + \mathcal{V}_{k+1}(f(S_k, u_k, v_k)) \tag{51}$$

where $\mathcal{V}_k(S_k)$ is the value of the game at stage k, obtained when players play optimally, and $S_{k+1} = f(S_k, u_k, v_k)$ represents the state equations (34)–(37). The variable $S_k = (S_k^s, S_k^b, S_k^a, S_k^g)$ represents the resource strengths. The optimal payoff is given by

$$\mathcal{V}_k(S_k) = \min_{u_k} \max_{v_k} [J_k(S_k, u_k, v_k) + \mathcal{V}_{k+1}(f(S_k, u_k, v_k))]. \tag{52}$$

If a saddle point exists, then the solution of the above problem gives the optimal strategies of the players at the kth stage of the multistage game. The optimal payoff of the multistage game is then given by $\mathcal{V}_1(S_1)$.

It turns out that if $J_k(S_k, u_k, v_k) + \mathcal{V}_{k+1}(f(S_k, u_k, v_k))$ satisfies the property that

$$\begin{aligned} &J_k\left(S_k, \left(S_k^s, S_k^b\right), v_k^a\right) + \mathcal{V}_{k+1}\left(f\left(S_k, (S_k^s, S_k^b), v_k^a\right)\right) \\ &\quad \leq J_k\left(S_k, \left(u_k^s, u_k^b\right), v_k^a\right) + \mathcal{V}_{k+1}\left(f\left(S_k, (u_k^s, u_k^b), v_k^a\right)\right) \\ &\qquad \text{for all } u_k \text{ and a fixed } v_k \end{aligned} \tag{53}$$

$$\begin{aligned} &J_k\left(S_k, \left(u_k^s, u_k^b\right), S_k^a\right) + \mathcal{V}_{k+1}\left(f\left(S_k, (u_k^s, u_k^b), S_k^a\right)\right) \\ &\quad \geq J_k\left(S_k, \left(u_k^s, u_k^b\right), v_k^a\right) + \mathcal{V}_{k+1}\left(f\left(S_k, (u_k^s, u_k^b), v_k^a\right)\right) \\ &\qquad \text{for all } v_k \text{ and a fixed } u_k \end{aligned} \tag{54}$$

then the multistage game has a saddle point in pure strategies at each stage k.

The details of the proof of the above assertion is given in [20], which basically uses the idea that the conditions (53) and (54) ensure that for each player there exists a choice of resource levels that satisfy the conditions of Fan's Minimax theorem [30].

If the optimal pure strategies for the players at each stage are $u_1^*, \ldots, u_N^*$ and $v_1^*, \ldots, v_N^*$, then the optimal pure strategies for the multistage game are defined as $u^* = (u_1^*, \ldots, u_N^*)$ and $v^* = (v_1^*, \ldots, v_N^*)$.

It can be shown that if the conditions given in (53) and (54) hold then a saddle point pure strategy for the multistage game is a stationary strategy given by the optimal solution of the single-stage game. This assertion is based on the observation that if at a given stage k both players have only NW solutions then any deviation from the single-stage saddle point solution would result in higher surviving resource strengths of the other player. The cumulative surviving GT strength will accordingly decrease or increase at the end of the game. If BLUE has a W solution in a given stage and deviates from it (that is, uses a NW solution), then the payoff in that stage is nonzero, thus increasing the total payoff. Similarly, if RED has a W solution and deviates from it in that stage (and uses a NW solution), then the surviving GT strength decreases, thus reducing the payoff in that stage. In subsequent stages the payoff may decrease further or remain the same. In any case, the deviation by RED decreases the cumulative payoff at the end of the game.

The conditions given in (53) and (54) are not easy to verify for a game with a large number of stages. However, for games with smaller number of stages it is possible to verify these conditions computationally. If these condition do not hold then the optimal strategies of the players would be mixed behavior strategies. Even if they are pure strategies they may no longer be stationary [20].

For the linear model adopted in this paper the optimal strategies for the players at each stage can be further simplified as follows:

If $(S_k^a) \in \mathcal{M}$ then $v_k^{a*} = S_k^a$.
If $(S_k^a) \notin \mathcal{M}$, then $v_k^{a*} \in \{[0, S_k^a] \backslash \mathcal{M}\}$.
If $(S_k^s, S_k^b) \in \mathcal{N}$, then $u_k^{b*} = S_k^b; u_k^{s*} = S_k^s$ if $S_k^a/\beta > S_k^s$ and $u_k^{s*} \in [S_k^a/\beta, S_k^s]$ otherwise.
If $(S_k^s, S_k^b) \notin \mathcal{N}$, then $(u_j^{s*}, u_k^{b*}) \in \{\{[0, S_k^s] \times [0, S_k^b]\} \backslash \mathcal{N}\}$.

In the above, the sets $\mathcal{N}$ and $\mathcal{M}$ are defined as

$$\mathcal{N}_1 = \left\{(x^{us}, x^{ub}) : x^{us} \geq \frac{S_k^a}{\beta}; x^{ub} < \frac{S_k^g}{\theta}\right\} \tag{55}$$

$$\mathcal{N}_2 = \left\{(x^{us}, x^{ub}) : x^{us} < \frac{S_k^a}{\beta}\right.$$
$$\left. x^{ub} < \gamma(S_k^a - \beta x^{us}) + \frac{S_k^g}{\theta}\right\} \tag{56}$$

$$\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 \tag{57}$$

$$\mathcal{M} = \left\{y^{va} : y^{va} < S_k^b/\gamma + \beta S_k^s\right\} \tag{58}$$

where x^{us}, x^{ub}, and y^{va} are variables that correspond to the SEAD, BMB, and AD resource strengths, respectively. The sets $\mathcal{M}$ and $\mathcal{N}$ are such that the BLUE player will not be able to destroy RED GTs completely if it confines its allocation to the set $\mathcal{N}$. Similarly, the RED player will not be able to protect its GTs completely (so that they remain undamaged) if it confines its allocation to the set $\mathcal{M}$. A schematic representation of the optimal allocation for BLUE is given in Fig. 4. The optimal allocations are in the shaded region shown in the figure if the available resource strengths are such that the point (S_k^s, S_k^b) does not lie in the interior of $\mathcal{N}$. In which case, any allocation in the shaded region is optimal and will destroy the GTs completely. These are the "winning" solutions. Otherwise, if the point lies in the interior of $\mathcal{N}$, then

1) if this point lies on the left of the line $x^{us} = S_k^a/\beta$ then the optimal allocation is (S_k^s, S_k^b);
2) if it lies on the right side of this line then the optimal allocation is $u_k^{s*} \in [S_k^a/\beta, S_k^s]$ and $u_k^{b*} = S_k^b$.

These solutions are the "nonwinning" solutions. Similarly, if S_k^a lies in the interior of $\mathcal{M}$ then S_k^a is the optimal allocation, and is a "nonwinning" solution. Otherwise, the optimal allocation would be any point in $[S_k^b/\gamma + \beta S_k^s, S_k^a]$ and is called "winning". Each such winning allocation would destroy the BLUE BMBs completely so that no damage would be inflicted on the GTs.

Although, depending on the available resource levels, the game admits multiple saddle points in pure strategies, it is logical for players to avoid using excessive resources. This implies that the RED forces will use

$$v_k^{a*} = \min\left\{S_k^b/\gamma + \beta S_k^s, S_k^a\right\} \tag{59}$$

and the BLUE forces will select a Pareto minimum point from its solution set given above. The Pareto minimum set is shown

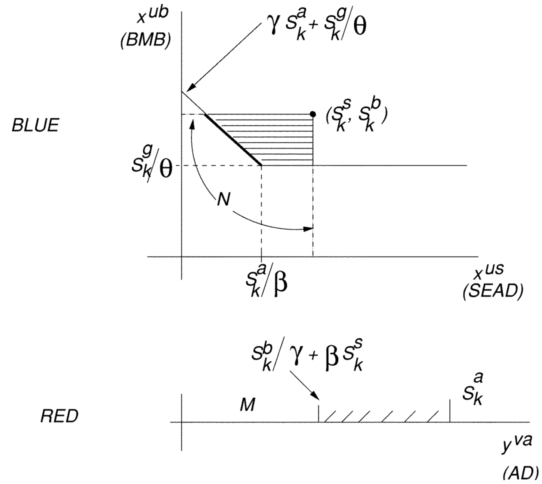

Fig. 4. Optimal resource allocation and the Pareto minimum set.

in Fig. 4 as the bold line when the available resources are not in the interior of $\mathcal{N}$. When the resource level is in the interior of $\mathcal{N}$ then: 1) if $S_k^s \leq S_k^a/\beta$ then $u_k^* = (S_k^s, S_k^a)$; 2) if $S_k^s > S_k^a/\beta$ then $u_k^* = (S_k^a/\beta, S_k^a)$.

V. Integration of Spatial and Temporal Solutions

The solutions to the spatial and temporal resource allocation problems are integrated through a aggregation/deaggregation process described below.

A. Aggregation and Deaggregation

In the following, we suppress "k," denoting the kth stage, from the variables for simplicity. The aggregation process essentially generates the risk profile and computes the aggregated AD strength S^a. The deaggregation process distributes the aggregated losses to the AD strength among the constituent ADs and determines the status of the AD units by distributing the AD strength used (v^a) among the ADs.

For the aggregation process, we use a vector F that defines the effectiveness of an AD unit against an aircraft in a sector near the AD location. That is, the strength of an AD unit is F_0 on a sector where this AD unit is located, F_1 on a sector one step away from the AD location, and so on. In general, the strength of an AD unit is F_i on a sector located i steps away from the AD location. For computational simplicity we assume that only a finite number of F_i's are nonzero. We do not specify the exact nature of F_i's except that it may be related to several factors that define the effectiveness of an AD against an aircraft. For example, a simple model for F_i could be a function of a probability of an aircraft being destroyed while passing through a sector located i steps away from the AD location.

Assuming that the risk to the BLUE aircraft on a sector is also a function of the effectiveness of individual AD units, the risk profile $\mathcal{R}(E)$ can be computed. For example, if we assume that the risks are also the same as the AD effectiveness, and can be added linearly, then it is simply the summation of the risks at each sector. Therefore

$$r_{ij}|_E = \sum_{d_j \in E} \sigma_{d_j} F_{\delta\left(g_i, e^{d_j}\right)} \tag{60}$$

where $\delta(g_i, e^{d_j})$ is the distance in number of sectors between the sector g_i and the sector e^{d_j} where the AD unit is located. The variable σ_{d_j} is the strength reduction factor associated with the AD denoted as d_j. This is equal to one at the start of the game and gradually reduces as the ADs suffer attrition in subsequent stages. We will describe later how this quantity is computed [see (68)].

To compute the aggregated AD strength S^a under the assumption of linear additivity of risks, we let

$$S^{d_j} = \sigma_{d_j} \sum_i n^i(e^{d_j}) F_i \tag{61}$$

which is the strength of the AD unit d_j on the corridor. The integer $n^i(e^{d_j})$ is the number of sectors that are on the corridor and are exactly i steps away from the sector e^{d_j}. The total available AD strength can be calculated by summing all the individual AD strengths

$$S^a = \sum_{j=1}^{s} S^{d_j}. \tag{62}$$

Knowing the available resource strengths S^a, S^b, S^s, and S^g, we solve the temporal resource allocation problem and obtain the optimal amounts v^a, u^s, and u^b allocated by the two players at the current stage. The allocation v^a has to be obtained by deciding on the identity and number of ADs to be made active during this stage. The increments in allocated AD strength achieved by activating additional AD units are discrete. Therefore, we select a subset $\hat{\mathcal{D}} = \{d_{q_i}\}_{i=1}^{\bar{m}}$ of $\mathcal{D} = \{d_1, \ldots, d_s\}$ such that we have

$$S^a(\hat{\mathcal{D}}) = \sum_{d_i \in \hat{\mathcal{D}}} S^{d_i} = \bar{v^a} \geq v^a. \tag{63}$$

The elements of the status vector v^s then take values 1 or 0 according to $v^{s_i} = 1$ if $d_i \in \hat{\mathcal{D}}$ (active ADs) and $v^{s_i} = 0$ if $d_i \notin \hat{\mathcal{D}}$ (passive ADs).

A straightforward way to determine $\hat{\mathcal{D}}$ is to sort the AD strengths in descending order and take the minimum number $\bar{m}$ of the largest elements that sum up to a quantity $\bar{v^a} \geq v^a$.

For deaggregation, we use a vector H that defines the effectiveness of a SEAD against an AD. The effectiveness of a SEAD i sectors away from an AD is assumed to be given by H_i. This factor has a similar interpretation as the effectiveness factors F_i defined above.

We compute the vulnerability of the AD unit d_j to the damage inflicted by the aircraft flying through the corridor as

$$h_j = \sum_i n^i(e^{d_j}) H_i. \tag{64}$$

Let the aggregated damage to ADs, inflicted by the SEADs, be denoted by L_s^a. This damage can be distributed among the active AD units proportionally to their vulnerabilities as follows: Let $h_{q_1}, \ldots, h_{q_{\bar{m}}}$ be the vulnerability indices of the active ADs. Then, we assume that the damage sustained by the q_jth active AD is

$$\Delta S^{d_{q_j}} = \left(\frac{h_{q_j}}{\sum_{i=1}^{\bar{m}} h_{q_i}} \right) L_s^a. \tag{65}$$

The strength of the q_jth active AD after its interaction with SEADs is

$$S^{d_{q_j}} - \Delta S^{d_{q_j}}. \tag{66}$$

Its strength reduces by a factor of

$$\hat{\sigma}_{d_{q_j}} = \frac{S^{d_{q_j}} - \Delta S^{d_{q_j}}}{S^{d_{q_j}}} \tag{67}$$

and the new strength reduction factor (that is, the strength reduction factor at the next stage) for this AD becomes

$$\sigma^+_{d_{q_j}} = \hat{\sigma}_{d_{q_j}} \sigma_{d_{q_j}}. \tag{68}$$

A similar distribution of attrition is defined when the ADs interact with BMBs. We omit the details.

VI. Simulation Experiments

A. The Simulation Algorithm

The algorithm used for simulating the decision making process in an air campaign is as follows.

1) BLUE creates a ingress corridor assuming (a) AD locations on the gameboard are known, and (b) all ADs are active.
2) Compute the AD strength using the aggregation process given in Section V-A.
3) Solve the temporal resource allocation problem for both BLUE and RED. The solution to the temporal resource allocation problem is used to obtain the number of SEADs and BMBs to be employed for the current stage of the campaign. It is also used to specify the number and location of ADs to be made active using the deaggregation procedure given in Section V-A.
4) The engagement is simulated with the interactions taking place sequentially as given in Fig. 3.
5) Distribute damages (or losses) to the ADs using the deaggregation process given in Section V-A.
6) Solve the GT movement problem for RED using the AD strength remaining after the engagement.
7) Create the RED anticipated corridor and solve the AD movement problem.

The algorithm is shown in Fig. 5. Each block represents a decision process by one or both players. The engagement block mainly represents the application of the temporal resource allocation solution and computation of the outcome of the sequential interactions between aggregated resource strengths. Hence, this is not a "true" simulation of the several resource interactions constituting the air campaign, but rather it is a simulation of the aggregated multiple resource interaction model representing the air campaign.

The information dependence of each block in stage $k + 1$ is shown in Fig. 5. The thick dark lines represent the decision-making sequence used in the algorithm. The thin dark lines indicate the information flow in the algorithm. More explicitly, the information requirement of each decision block in the simulation is as follows.

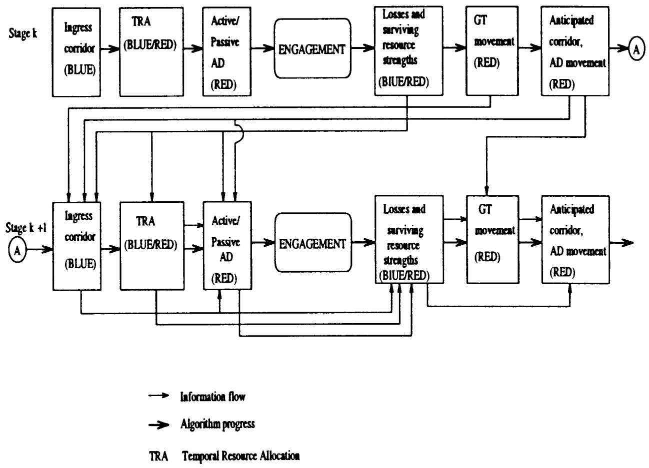

Fig. 5. Block diagram for the air campaign simulation algorithm.

1) *Computation of the ingress corridor:* Surviving resource strengths from the previous stage (actually, it only requires the strengths of the ADs), the current location of the GT (which is obtained from the GT movement decision block of the previous stage), and the current AD location (which is obtained from the AD movement decision block).
2) *Temporal resource allocation:* Current ingress corridor and the aggregated strengths of the surviving resources.
3) *Determination of AD status:* Solution of the temporal resource allocation problem, the current ingress corridor, the current location of the ADs, and the surviving AD resource strengths from the previous stage.
4) *Computation of surviving resource strengths:* Current ingress corridor, the temporal resource allocation by players, and the identification of the active and passive ADs by RED.
5) *GT movement:* Surviving resource strengths and current AD location.
6) *AD movement:* New resource strengths and current GT location.

Note that this information flow pattern gives a certain informational advantage to the BLUE forces in the sense that at a given stage the AD locations and GT location are known to the BLUE forces while planning the corridor. For an informational pattern where this information is not available to the BLUE forces, the optimal corridor creation decision block should be replaced by the game-theoretical version given in (12) or the BLUE security strategy obtained from (13)–(16). Similarly, when BLUE has the information advantage (as in this case), RED could also use a security strategy as given by (17). In fact, in the last simulation experiment we examine this situation. Finally, other types of information availability to the players can be simulated by appropriately modifying the information flow into the decision blocks in Fig. 5.

B. Assumptions

The major assumptions are the same as described in Section II. The specific numerical values used in the simulation are given here. In the simulation studies presented in this section, we assume a 7×7 hexagonal gameboard, representing the battlefield. The BLUE territorial border is the bottom most row of the gameboard. At the beginning of the air campaign, the following resource levels are available to the two adversaries:

BLUE: 20 SEADs and 10 BMBs;

RED: 25 GT units and 4 AD units.

The location of the BLUE SEAD/BMB bases are in one of the bottom most sectors comprising the BLUE border. This is the sector from which all the BLUE ingress corridors originate (see Figs. 6–10). The GT and AD units are represented by the symbols shown in Fig. 6, and their initial locations are shown in the Stage 1 configuration in all of the figures.

The effectiveness of ADs against the BLUE aircraft (both SEAD and BMB) is as follows:

$$F_0 = 1.5, F_1 = 4, F_2 = 2.5.$$

Note that these numbers reflect the assumption that an AD is less effective in the sector where it is located, mainly because most surface to air missiles do not have boost phase control (that is, they are hard to control for the first few seconds after launch).

The vulnerability of the ADs to the SEADs (or the effectiveness of the SEADs against ADs) is defined as

$$H_0 = 0.7, H_1 = 0.5, H_2 = 0.3, H_3 = 0.1.$$

The loss functions (to be used for the temporal resource allocation decisions and calculation of attrition) are represented through the following coefficients:

$$\alpha = 0.2, \beta = 0.3, \gamma = 0.1, \eta = 0, \theta = 1.$$

The ingress corridor creation algorithm assumes a low arbitrary path cost (or sector to sector transition cost) of about 0.1 to allow the shortest path algorithm to avoid generating corridors with loops or corridors that are counter-intuitively long and winding. In practice, this cost could reflect a penalty on the time, the fuel expended, or risk due to prolonged exposure to other RED lethal resources.

C. Simulation Results

We carry out five simulations with identical initial configuration of resources in which the GTs are allowed to move one sector in each stage. In the first four simulations, RED predicts the BLUE ingress corridor given the current placement of AD units in every stage, and then moves the AD units to maximize risk on this predicted corridor. The simulations differ on the constraint placed on AD movement. We experiment with several modes of AD movement.

In Fig. 6, which shows the first simulation study, both active and passive ADs are allowed to move a maximum of two sectors at each stage. The obvious disadvantage of this plan is that the ADs can move too close (that is, overcommit the available AD resources) to the ingress corridor anticipated by RED, giving BLUE an opportunity to plan the ingress corridor so that it bypasses the new AD locations. As a result, ADs do not inflict much damage on the BLUE SEADs and BMBs and hence fail to protect the GTs effectively.

Note that the available AD strength is computed based upon the effectiveness of the ADs against BLUE aircraft and is defined as the resultant risk imposed on the ingress corridor. Thus, the AD strength is corridor dependent. In Stage 1, the ingress

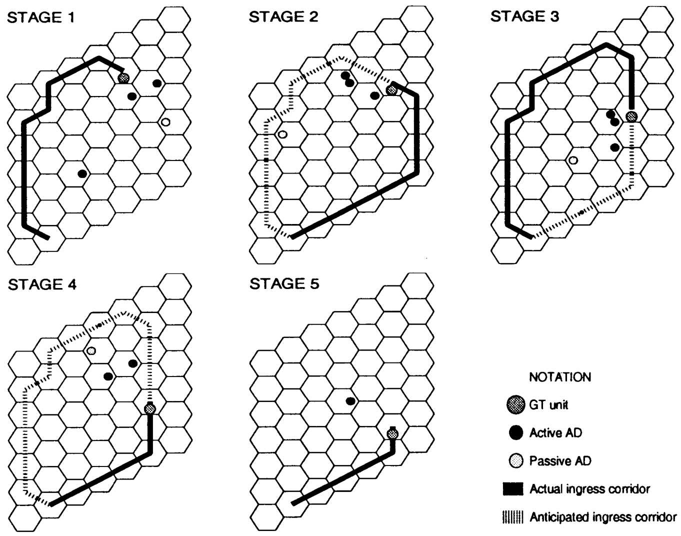

Fig. 6. Two-step AD movement. Both active and passive ADs are eligible. *Stage 1:* $S^g = 25, S^a = v^a = 23, S^s = u^s = 20, S^b = u^b = 10$. *Stage 2:* $S^g = 16.7, S^a = v^a = 12.8, S^s = u^s = 15.4, S^b = u^b = 8.3$. *Stage 3:* $S^g = 9.2, S^a = v^a = 5.6, S^s = u^s = 12.8, S^b = u^b = 7.5$. *Stage 4:* $S^g = 1.9, S^a = v^a = 3, S^s = 11.7, S^b = 7.3; u^s = 9.9, u^b = 1.9$. *Stage 5:* $S^g = 0$ (destroyed), $S^a = 2.5, S^s = 11.1, S^b = 7.2$.

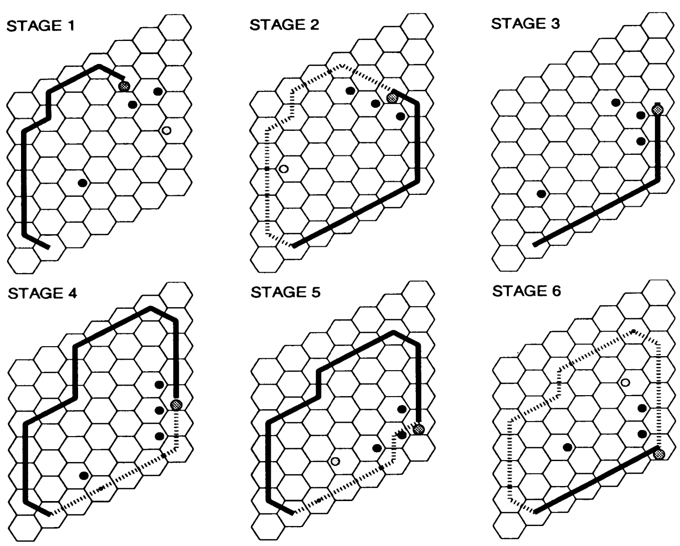

Fig. 7. One-step AD movement. Both active and passive ADs are eligible. *Stage 1:* $S^g = 25, S^a = v^a = 23, S^s = u^s = 20, S^b = u^b = 10$. *Stage 2:* $S^g = 16.7, S^a = 23.2, S^s = 15.4, S^b = 8.3$. *Stage 3:* $S^g = 10.26, S^a = 25.47, S^s = 10.7, S^b = 6.4$. *Stage 4:* $S^g = 6, S^a = 17.9, S^s = 5.6, S^b = 4.2$. *Stage 5:* $S^g = 3.5, S^a = 9.5, S^s = 2.1, S^b = 2.6$. *Stage 6:* $S^g = 1.75, S^a = 11.2, S^s = 0.2, S^b = 1.71$.

corridor is shown by the thick black line. The active ADs are close to the corridor (that is, within two sectors of the corridor). The single passive AD does not have any influence on the ingress corridor. At the end of Stage 1 (and at the beginning of Stage 2) all the resources have suffered some attrition during the interactions in Stage 1. Note that the BLUE resources have suffered less damage than the RED resources. At the beginning of Stage 2, the GTs and ADs have moved. The ADs are moved by the RED forces by first computing an anticipated corridor (shown by the shaded path) and then moving the ADs so as to impose the maximum possible risk on this path. It turns out that

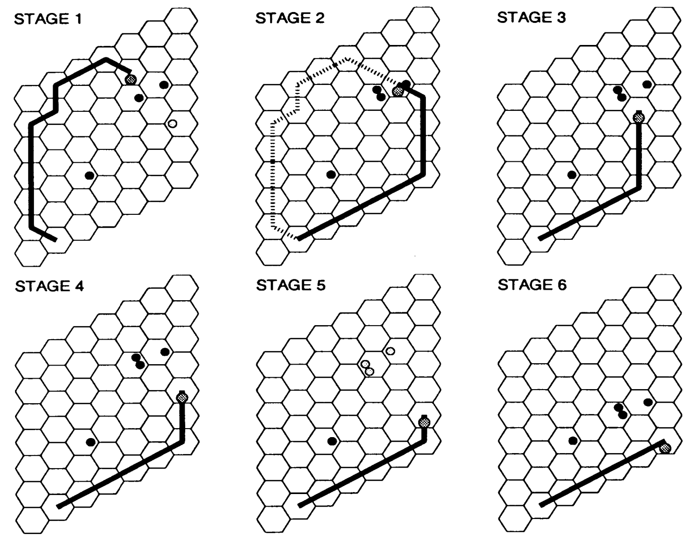

Fig. 8. Two-step AD movement. Only passive ADs are eligible. *Stage 1:* $S^g = 25, S^a = v^a = 23, S^s = u^s = 20, S^b = u^b = 10$. *Stage 2:* $S^g = 16.7, S^a = v^a = 27.1, S^s = u^s = 15.4, S^b = u^b = 8.3$. *Stage 3:* $S^g = 10.7, S^a = v^a = 18, S^s = u^s = 10, S^b = u^b = 6$. *Stage 4:* $S^g = 6.1, S^a = v^a = 8.3, S^s = u^s = 6.3, S^b = u^b = 4.5$. *Stage 5:* $S^g = 2.2, S^a = v^a = 3.2, S^s = u^s = 4.7, S^b = 3.9; u^b = 2.4$. *Stage 6:* $S^g = 0$ (destroyed), $S^a = 10, S^s = 4, S^b = 3.7$.

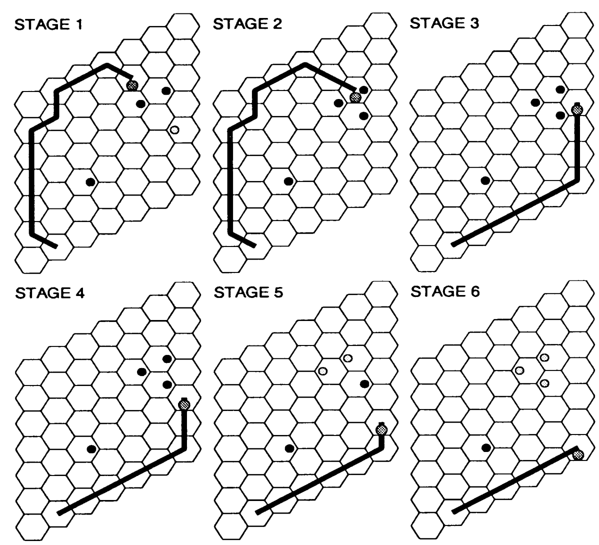

Fig. 9. One-step AD movement. Only passive ADs are eligible. *Stage 1:* $S^g = 25, S^a = v^a = 23, S^s = u^s = 20, S^b = u^b = 10$. *Stage 2:* $S^g = 16.7, S^a = v^a = 27.7, S^s = u^s = 15.4, S^b = u^b = 8.3$. *Stage 3:* $S^g = 10.7, S^a = v^a = 19.6, S^s = u^s = 9.9, S^b = u^b = 6$. *Stage 4:* $S^g = 6.4, S^a = v^a = 10.4, S^s = u^s = 6, S^b = u^b = 4.3$. *Stage 5:* $S^g = 2.9, S^a = v^a = 4.4, S^s = u^s = 3.9, S^b = 3.5; u^b = 3.2$. *Stage 6:* $S^g = 0$ (destroyed), $S^a = 2.5, S^s = u^s = 3, S^b = 3.1$.

the shaded path (or the anticipated corridor) is similar to the actual ingress corridor in the previous stage. This is expected since the anticipated corridor is computed using the AD configuration in the previous stage and, although the AD strengths have undergone attrition in Stage 1, they impose a similar relative risk map on the gameboard. Because of the over-commitment by RED, the BLUE ingress corridor is quite different from the RED anticipated corridor and so the BLUE forces are able to inflict more damage than the RED forces. This process continues through Stage 4 when all the GTs are destroyed. At Stage 3 one AD

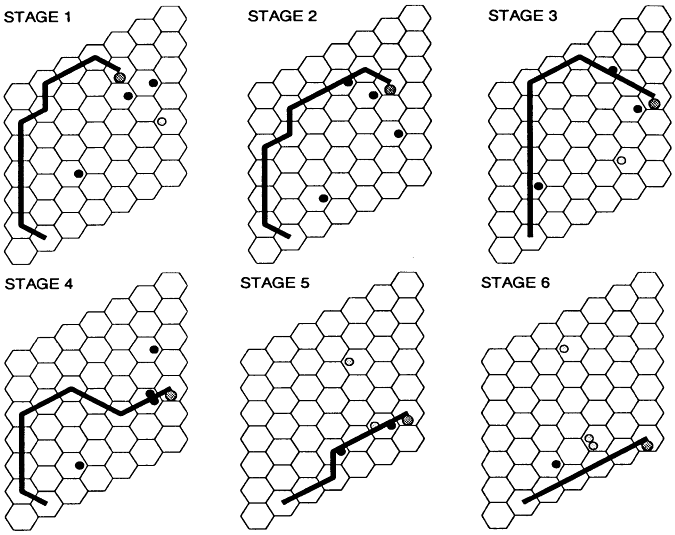

Fig. 10. One-step AD movement. Both active and passive ADs are eligible. Security strategy for RED. *Stage 1:* $S^g = 25, S^a = v^a = 23, S^s = u^s = 20, S^b = u^b = 10$. *Stage 2:* $S^g = 16.7, S^a = v^a = 31.5, S^s = u^s = 15.4, S^b = u^b = 8.3$. *Stage 3:* $S^g = 11, S^a = v^a = 25, S^s = u^s = 9.1, S^b = u^b = 5.6$. *Stage 4:* $S^g = 7.7, S^a = v^a = 25.5, S^s = u^s = 4.1, S^b = u^b = 3.4$. *Stage 5:* $S^g = 6.7, S^a = 20.7, S^s = u^s = 0$ (destroyed), $S^b = u^b = 1; v^a = 9.7$. *Stage 6:* $S^g = 6.7, S^a = 20.1, S^s = u^s = 0$ (destroyed), $S^b = 0$ (destroyed).

unit gets completely destroyed and after Stage 4 only one AD unit remains. Actually, Stage 5 is redundant in this simulation since the GTs are destroyed in Stage 4. However, it illustrates the possible GT and AD movements and creation of ingress corridor if the GTs were still there. This simulation shows that due to over-commitment by the RED forces, and the informational advantage of the BLUE forces, BLUE takes advantage of the situation and creates corridors that bypass the AD units.

In Fig. 6, the available resource strengths and used resource strengths given in the caption are those that are computed by the solution of the temporal resource allocation problem. Note that, in a general case, the actual deaggregation process would allocate strengths and losses that are slightly different from those prescribed by the optimal solution due to the discrete nature of the AD strength allocation.

In the above simulation study, the ineffectiveness of RED strategy was largely due to the over-commitment of the AD units. In the next simulation study we try to control this aspect by limiting the extent of AD movement per stage. In Fig. 7 both passive and active ADs are allowed to move a maximum of only one step per stage. It turns out that the ADs still over-commit on the anticipated corridor, but the ADs are utilized more efficiently than in the previous case.

The game goes up to six stages and, in the end, the GTs reach the BLUE border with a small surviving strength. The defense by the RED ADs seems to be quite effective and so a large amount of SEAD and BMB resources are destroyed. However, one of the deficiencies of the RED strategy is that the corridor anticipated by RED is almost always different from the actual corridor used by BLUE (except in Stage 3). In spite of this, the RED defense is effective because the ADs move along with GTs and provide adequate protection. Note that the AD strength available increases slightly between Stage 5 and Stage 6. This happens because of a more favorable configuration of ADs on the gameboard with respect to the ingress corridor. Also, note that none of the ADs get destroyed completely in this simulation.

To balance the mismatch between the anticipated corridor and the actual corridor, in the next simulation study, we allow only the passive ADs to move at each stage by two steps. This constraint is fairly realistic since an active AD would have its radars in the track mode to engage BLUE aircraft. It would be impossible for it to move while in the active mode. The simulation results are shown in Fig. 8.

Except in one stage (Stage 2), in all other stages the anticipated corridor by the RED is the same as the actual corridor. However, a major drawback of this approach is that the active ADs get left behind since they are stationary. They move only when their status changes from active to passive. Therefore, even though none of the ADs gets destroyed completely in this simulation study, their effectiveness is reduced due to the fact that they get left behind and have no influence on the ingress corridor. Also, all the GTs get destroyed before reaching the BLUE border.

The next study is similar to the above but with the RED passive ADs being allowed to move only one step. The results are shown in Fig. 9. The anticipated corridors and the actual ingress corridors are the same in each stage. However, in almost all the stages, all ADs are on and so they remain stationary and get left behind. Therefore, their influence on the corridor diminishes in later stages. All GTs also get destroyed.

In the next simulation, we approach the AD movement decisions from a game-theoretical viewpoint. Consider a game, in which the RED player's move is to chose a feasible placement of ADs, while the BLUE player picks an ingress corridor. As explained in Sections III-A and III-C, the solution of this game in mixed strategies exists (since the set of possible AD placements and loopless corridors on the gameboard is finite), although these are difficult to compute and interpret. However, using security strategies for either (or both) players appears more meaningful, while still being somewhat computation-intensive.

We consider the case when the RED player makes the move first, and BLUE can plan the ingress corridor given the new AD placement. This is the same information pattern that we have assumed in the previous four simulation studies. However, the simulation differs in the sense that RED selects its AD movements using (17). The best new placement of ADs is computed by obtaining the minimum risk corridor for every possible movement of the ADs, and then maximizing over those minimum risks. This provides the most effective use of ADs.

The results are shown in Fig. 10. The anticipated corridor and the actual ingress corridor is the same at each stage. The RED forces are able to destroy the SEADs and BMBs completely and reach the BLUE border with some GT strength.

A deficiency of the above simulation studies is that they do not actually model the air campaign engagement but rather they use the aggregated version of the air campaign, created for solving the temporal resource allocation problem, as the simulation platform. A more realistic simulation platform with the individual un-aggregated interactions between BLUE and RED resources would have given a more realistic set of results. However, this type of elaborate simulation is beyond the scope of this paper. The studies presented above are adequate for illustrative purposes. Their main intention is to show how the air campaign planning decisions (for both adversaries) can be derived from game-theoretical formulations by decomposing the problem into its temporal and spatial dimensions, and used in actual air campaign decison making by both adversaries. The results of the simulation studies also show that starting from the same initial spatial and temporal configuration of resources, the outcome of the game changes significantly depending on the information pattern imposed on the air campaign. A related important issue concerns the strategies adopted by the players. Game-theoretical strategies are shown to be superior, although more computation-intensive, to arbitrary strategies.

VII. Concluding Remarks

In this paper, we presented a game-theoretical formulation of an air campaign incorporating both the spatial and temporal dimensions in the model. The solutions to these two resource allocation problems are obtained separately and integrated through an aggregation/deaggregation technique. The air campaign problem addressed has certain generic elements that are applicable to other theater-level campaign scenarios. Several examples are presented to show the effect of different strategies on the outcome of the game. The approach shows how air campaign decisions (both for offense and defense) can be derived from game-theoretical solution concepts applied to a multiple resource allocation model of the air campaign. The simulations illustrate several interesting solutions that arise due to the way that players use their information based on game-theoretical ideas. Further work in this direction would involve derivation of strategies for a more realistic multiple interaction model that does not aggregate the interactions into single interaction blocks, development of computational techniques to obtain temporal resource allocation strategies for different payoff functions and objectives, and incorporation of the spatial decision making process into the temporal resource allocation decisions directly.

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