

## Design of Six Sigma Supply Chains

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**Abstract**— Variability reduction and business process synchronization are acknowledged as key to achieving sharp and timely deliveries in supply chain networks. In this paper, we introduce a new notion, which we call *six sigma supply chains* to describe and quantify supply chains with sharp and timely deliveries, and develop an innovative approach for designing such networks. We show that design of six sigma supply chains can be formulated as a mathematical programming problem, opening up a rich, new framework for studying supply chain design optimization problems. To show the efficacy of the notion and the design methodology, we focus on a design optimization problem, which we call as the Inventory Optimization (IOPT) problem. We formulate and solve the IOPT problem for a four stage, make-to-order liquid petroleum gas supply chain. The solution of the problem offers rich insights into inventory - service level tradeoffs in supply chain networks and proves the potential of the new approach presented in this paper.

### I. INTRODUCTION

Lead times of individual business processes and the variabilities in the lead times are key determinants of end-to-end delivery performance in supply chain networks. When the number of resources, operations, and organizations in a supply chain increases, variability destroys synchronization among the individual processes, leading to poor delivery performance. On the other hand, by reducing variability all along the supply chain in an intelligent way, proper synchronization can be achieved among the constituent processes. This motivates us to explore variability reduction as a means to achieving outstanding delivery performance. We approach this problem in an innovative way by looking at a striking analogy from mechanical design tolerancing.

Variability reduction is a key idea in the statistical tolerancing approaches that are widely used in mechanical design tolerancing [1]. A complex supply chain network is much like a complex electro-mechanical assembly. Each individual business process in a given supply chain process is analogous to an individual subassembly. Minimizing defective or out-of-date deliveries in supply chains can therefore be viewed as minimizing tolerancing defects in electro-mechanical assemblies. This analogy provides the motivation and foundation for this paper.

#### A. Contributions

The contribution of this paper is two fold. First, we introduce the notion of *six sigma supply chains*. We show that the design of six sigma supply chains can be expressed

in a natural way as a mathematical programming problem. This provides an appealing framework for studying a rich variety of design optimization and tactical decision making problems in the supply chain context.

The second part of the paper proves the potential of the proposed methodology by focusing on a specific design optimization problem which we call the Inventory Optimization (IOPT) problem. We investigate this problem with the specific objective of tying up design of six sigma supply chains with supply chain inventory optimization. Given a multistage supply chain network, the IOPT problem seeks to find optimal allocation of lead time variabilities and inventories to individual stages, so as to achieve required levels of delivery performance in a cost-effective way. The study uses a representative liquid petroleum gas (LPG) supply chain network, with four stages: supplier, inbound logistics, manufacturer, and outbound logistics. The results obtained are extremely useful for a supply chain asset manager to quantitatively assess inventory-service level trade offs. For example, a supply chain manager for the LPG supply chain will be able to determine the optimal number of LPG trucks to keep at the regional depot and the optimal way of choosing logistics providers, so as to ensure six sigma delivery of LPG trucks to destinations.

In our view, the concepts and approach developed in this paper provide a framework in which a rich variety of supply chain design and tactical decision problems can be addressed.

#### B. Relevant Work

Lead time compression in supply chains is the subject of several recent papers, see for example, Narahari, Viswanadham, and Rajarshi [2]. Statistical design tolerancing is a mature subject in the design community. The key ideas in statistical design tolerancing which provide the core inputs to this paper are: (1) theory of process capability indices [3]; (2) tolerance analysis and tolerance synthesis techniques [4], [5]; (3) Motorola six sigma program [6]; and (4) design for tolerancing [1], [7].

Inventory optimization in supply chains is the topic of numerous papers in the past decade. Important ones of relevance here are on multiechelon supply chains [8]. Recent work by Schwartz and Weng [9] is particularly relevant here. This paper discusses the joint effect of lead time variability and demand uncertainty, as well as the

effect of "fair-shares" allocation, on safety stocks in a four-link JIT supply chain. The paper by Garg, Narahari, and Viswanadham [10] contains some of the preliminary ideas of this current paper.

## II. SIX SIGMA SUPPLY CHAINS

We define a six sigma supply chain as a network of supply chain elements which, given the customer specified window and the target delivery date, results in a delivery probability (DP) of at most 3.4 ppm (i.e. at most 3.4 defective deliveries in one million opportunities). All triples  $(C_p, C_{pk}, C_{pm})$  that guarantee an actual yield of at least 3.4 ppm (or  $DP=6\sigma$ ) would correspond to a six sigma supply chain. The indices  $C_p$  and  $C_{pm}$  completely determine the delivery probability and we need the index  $C_{pm}$  to specify how concentrated the deliveries are around the target delivery date. For this reason, we call the index  $C_{pm}$  as delivery sharpness (DS) [11]. It is important to note that in order to achieve  $DP=6\sigma$ , the delivery sharpness needs to assume appropriately high values. In a given setting, however, there may be a need for extremely sharp deliveries (highly accurate deliveries) implying that the  $C_{pm}$  index is required to be very high. This can be specified as an additional requirement of the designer.

### A. Design of Six Sigma Supply Chains

A major design objective in supply chain networks is to deliver finished products to the customers within a time as close to the target delivery date as possible, with as few defective deliveries as possible at the minimum cost. To give an idea of how the design problem can be formulated, let us consider a supply chain with  $n$  business processes such that each of them contributes to the order-to-delivery cycle of customer desired products. Let  $X_i$  be the cycle time of process  $i$ . It is realistic to assume that each  $X_i$  is a continuous random variable with mean  $\mu_i$  and standard deviation  $\sigma_i$ . The order-to-delivery time  $Y$  can then be considered as a deterministic function of  $X_i$ 's:

$$Y = f(X_1, \dots, X_n)$$

If we assume that the cost of delivering the products depends only on the first two moments of these random variables, the total cost of the process can be described as:

$$Z = g(\mu_1, \sigma_1, \dots, \mu_n, \sigma_n)$$

where  $g$  is some deterministic function.

The customer specifies a lower specification limit  $L$ , an upper specification limit  $U$ , and a target value  $\tau$  for this order-to-delivery lead time. With respect to this customer specification, we are required to choose the parameters of  $X_1, \dots, X_n$  so as to minimize the total cost involved in reaching the products to the customers, achieving a six sigma level of delivery performance.

Thus the design problem can be stated as the following mathematical programming problem:

Minimize  $Z = g(\mu_1, \sigma_1, \dots, \mu_n, \sigma_n)$   
subject to

$$\begin{aligned} \text{DS for order-to-delivery time} &\geq C_{pm}^* \\ \text{DP for order-to-delivery time} &\geq 6\sigma \\ \mu_i &> 0 \quad \forall i \\ \sigma_i &> 0 \quad \forall i \end{aligned}$$

where  $C_{pm}^*$  is a required lower bound on delivery sharpness. The objective function  $Z$  of this formulation captures the total cost involved in taking the product to the customer, going through the individual business processes. We have assumed that this cost is determined by the first two moments of lead times of the individual business processes. One can define  $Z$  in a more general way if necessary. The decision variables in this formulation are means and/or standard deviations of individual processes. The constraints of this formulation guarantee a minimum level of delivery sharpness ( $C_{pm}^*$  is the minimum level of delivery sharpness required) and at least a six sigma level of delivery probability.

Depending on the nature of the objective function and decision variables chosen, the six sigma supply chain design problem assumes interesting forms. We consider some problems below under two categories: (1) generic design problems and (2) concrete design problems.

#### Generic Design Problems:

- Optimal allocation of process means
- Optimal allocation of process variances
- Optimal allocation of customer windows

#### Concrete Design Problems:

- Due date setting
- Choice of customers
- Inventory allocation
- Capacity planning
- Vendor selection
- Choice of logistics modes, logistics providers
- Choice of manufacturing control policies

These problems can arise at any level of the hierarchical design. Thus in order to develop a complete suite for designing a complex supply chain network for six sigma delivery performance through the hierarchical design scheme, we need to address all such sub problems beforehand. In the next section, we consider one such sub-problem, optimal allocation of inventory in a multistage six sigma supply chain, and develop a methodology for this problem.

## III. INVENTORY OPTIMIZATION IN A MULTISTAGE SUPPLY CHAIN

In this section, we describe a representative supply chain example for liquid petroleum gas (LPG), with four

stages: supplier (refinery), inbound logistics, manufacturer (regional depot for LPG), and outbound logistics [9]. We formulate the six sigma design problem, based on the concepts developed in earlier sections, for this supply chain. Then we show how one can allocate variabilities to lead times of individual stages so as to achieve six sigma delivery performance. We also show how to compute the optimal inventory to be maintained at the regional depot to support six sigma delivery performance.

**A. A Four Stage Supply Chain Model with Demand and Lead Time Uncertainty**

1) *Model Description:* Consider  $N$  geographically dispersed distribution centers (DCs) supplying retailer demand for some product as shown in Figure 1. The product belongs to a category which does not make it profitable for the distribution center to maintain any inventory. An immediate example is a distributor who supplies trucks laden with bottled Liquid-Petroleum-Gas (LPG) cylinders (call these as LPG trucks or finished product now onward) to retail outlets and industrial customers. In a situation like this, as soon as a demand for a LPG truck arrives at any DC, the DC immediately places an order for one unit of product (in this case, an LPG truck) to a major regional depot (RD). The RD maintains an inventory of LPG trucks and after receiving the order, if on-hand inventory of LPG trucks is positive then an LPG truck is sent to the DC via outbound logistics. On the other hand, if on-hand inventory is zero, the order gets backordered at the RD. At the RD, the processing involves unloading LPG from LPG tankers into LPG reservoirs, filling the LPG into cylinders, bottling the cylinders and finally loading the cylinders onto trucks.

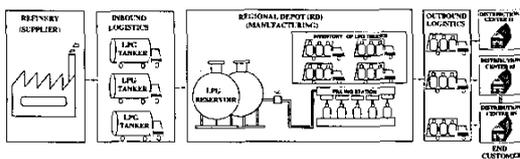


Fig. 1. A four link linear supply chain model

The inventory at RD is replenished as follows. The RD starts with on-hand inventory  $R$  and every time an order is received, it places an order to the supplier for one LPG tanker (called as semi-finished product now onward) which is sufficient to produce one LPG truck. In this case, the supplier corresponds to a refinery which will produce LPG tankers. In the literature such a replenishment model is known as the  $\langle Q, R \rangle$  model [12] with  $Q = 1$ . In such a model, the inventory position (on-hand plus on-order minus backorders) is always constant and is equal to  $R$ .

The LPG supply chain network is a typical example of a multi-echelon supply chain network. The four stages could

be described as (descriptions in parentheses corresponds to the LPG example): (1) Procurement or Supplier (refinery); (2) Inbound Logistics (transportation of LPG tankers from refinery to RD); (3) Manufacturing (RD); (4) Outbound Logistics (customer order processing and transportation of LPG trucks from RD to a DC).

2) *System Parameters:* This section presents the notation used for various system parameters.

*Lead Time Parameters:*

- $X_1 \sim N(\mu_1, \sigma_1^2)$  = Procurement lead time
- $X_2 \sim N(\mu_2, \sigma_2^2)$  = Inbound logistics lead time
- $X_3 \sim N(\mu_3, \sigma_3^2)$  = Manufacturing lead time
- $X_4 \sim N(\mu_4, \sigma_4^2)$  = Outbound logistics lead time
- $L_m$  = Time between placement of an order by manufacturer and receipt of item at the manufacturer
- $L_f$  = Time between placement of an order by manufacturer and completion of processing of the item at the manufacturer
- $L_c$  = End-to-end lead time of customer's order
- $\bar{L}_c$  = An upper bound on  $L_c$
- $(\mu_m, \sigma_m^2)$  = Mean and variance of  $L_m$
- $(\mu_f, \sigma_f^2)$  = Mean and variance of  $L_f$
- $(\mu_c, \sigma_c^2)$  = Mean and variance of  $L_c$
- $(\bar{\mu}_c, \bar{\sigma}_c^2)$  = Mean and variance of  $\bar{L}_c$

*Demand Process Parameters:*

- $\lambda_i$  = Order arrival rate from  $i^{\text{th}}$  customer (item/year)
- $\lambda = \sum_i^N \lambda_i$  = Poisson arrival rate of orders at the manufacturer
- $R$  = Inventory level at the manufacturing node
- $Q = 1$  = Reorder quantity of the manufacturer
- $M_o$  = Stockout probability at the manufacturing node
- $E$  = Average number of backorders per unit time at the manufacturing node (item/time)
- $B$  = Expected number of backorders with the manufacturer at arbitrary time  $t$  (item)
- $D$  = Expected number of onhand inventory with the manufacturer at arbitrary time  $t$  (item)
- $\psi_m(x)$  = Steady state probability that the manufacturer has net inventory equal to  $x$
- $p(x; \lambda t) = \frac{\exp(-\lambda t)(\lambda t)^x}{x!}$
- $P(r; \lambda t) = \sum_{x=r}^{\infty} p(x; \lambda t)$

*Cost Parameters:*

- $\mathcal{X}_1 = f_1(\mu_1, \sigma_1) =$  Procurement cost (\$/item)  
 $\mathcal{X}_2 = f_2(\mu_2, \sigma_2) =$  Inbound logistics cost (\$/item)  
 $\mathcal{X}_3 = f_3(\mu_3, \sigma_3) =$  Manufacturing cost (\$/item)  
 $\mathcal{X}_4 = f_4(\mu_4, \sigma_4) =$  Outbound logistics cost (\$/item)  
 $A =$  Order placing cost for manufacturer (\$/order)  
 $\Pi =$  Fixed part of backorder cost (\$/item)  
 $\hat{\Pi} =$  Variable part of backorder cost (\$/item-time)  
 $I =$  Inventory carrying cost (\$/time-\$ invested)  
 $C =$  Cost of raw material (\$/item)  
 $C_m =$  Capital tied up with each item ready to be shipped via outbound logistics (\$/item)

*Delivery Quality Parameters:*

- $C_p, C_{pk}, C_{pm} =$  Supply chain process capability indices for end-to-end lead time of customer order  
 $(\tau, T) =$  Delivery window specified by customer  
 $U = \tau + T =$  Upper limit of delivery window  
 $L = \tau - T =$  Lower limit of delivery window  
 $b = |\tau - \mu_c| =$  Bias for  $L_c$   
 $\bar{b} = |\tau - \bar{\mu}_c| =$  Bias for  $\bar{L}_c$   
 $d = \min(U - \mu_c, \mu_c - L)$   
 $\bar{d} = \min(U - \bar{\mu}_c, \bar{\mu}_c - L)$

*B. System Analysis*

1) *Lead Time Analysis of Delivery Process:* In this section we present some results concerning the dynamics of flow of material in the chain triggered by an end customer order as well as manufacturer order and compute the related end-to-end lead times. The proofs are provided in the detailed report [11]. The theorem is based on the work of Tackács [12].

*Lemma 2.1:* An upper bound on end-to-end lead time ( $L_c$ ) experienced by an end customer is

$$\bar{L}_c = X_4 + M_o (X_1 + X_2 + X_3)$$

where  $M_o$  is the stockout probability at the manufacturer.

*Theorem 2.1:* Let the  $\langle Q, R \rangle$  policy with  $Q = 1$  be followed for controlling the inventory of a given item at a single location where the demand is Poisson distributed with rate  $\lambda$ , and let the replenishment lead times be nonnegative independent random variables (i.e., orders can cross) with density  $g(t)$  and mean  $\mu$ . The steady state probability of having net inventory (on hand inventory minus backorders)  $x$  by such a system can be given by:

$$\psi(x) = \frac{\exp(-\lambda\mu)(\lambda\mu)^x}{x!}$$

In other words, the state probabilities are independent of the nature of the replenishment lead time distribution if the lead times are nonnegative and independent.

Using the results in [12], we can derive expressions for the stockout probability ( $M_o$ ), the average number of backorders per unit time ( $E$ ), the expected number of backorders at any random instant ( $B$ ), and the expected number of onhand inventory at any random instant ( $D$ ). These expressions are listed below.

$$M_o = P(R; \lambda \mu_f) = \sum_{k=R}^{\infty} \frac{\exp(-\lambda \mu_f)(\lambda \mu_f)^k}{k!}$$

$$= 1 - \exp(-\lambda \mu_f) \sum_{k=0}^{R-1} \frac{(\lambda \mu_f)^k}{k!} \quad (1)$$

$$E = \lambda M_o \quad (2)$$

$$B = \lambda \mu_f P(R-1; \lambda \mu_f) - R M_o$$

$$= M_o \left( \lambda \mu_f - R \right) + \frac{\exp(-\lambda \mu_f)(\lambda \mu_f)^R}{(R-1)!} \quad (3)$$

$$D = R - \lambda \mu_f + B \quad (4)$$

The expression for  $M_o$  serves in deriving an important conclusion about upper bound on end-to-end lead time for customer (i.e.  $\bar{L}_c$ ) which is described (without proof) in the form of Lemma 2.2.

*Lemma 2.2:* For a fixed value of  $R$ ,  $\lambda$ , and  $\mu_f$ , the upper bound on end-to-end lead time experienced by an end customer (i.e.  $\bar{L}_c$ ) is a normal random variable with mean  $\bar{\mu}_c$  and variance  $\bar{\sigma}_c^2$  given as follows:

$$\bar{\mu}_c = \mu_4 + M_o (\mu_1 + \mu_2 + \mu_3) \quad (5)$$

$$\bar{\sigma}_c^2 = \sigma_4^2 + M_o^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \quad (6)$$

where  $M_o$  is given by Equation (1).

*C. Formulation of IOPT*

The objective of the study here is to find out how variability should be allocated to the lead times of the individual stages and what should be the optimal value of inventory level  $R$ , such that specified levels of DP and DS are achieved in the steady state condition for the customer lead time, in a cost effective manner. We call this problem as the *Inventory Optimization (IOPT)* problem in six sigma supply chains.

1) *Constraints :*

Observe that  $\bar{L}_c$  is an upper bound on end customer lead time  $L_c$ , so if we specify the constraints which assure to attain the specified levels of DP and DS for  $\bar{L}_c$ , it will automatically imply that  $L_c$  attains the same or even better levels of DP and DS than specified. These constraints can be written down as follows.

$$\text{DS for } \bar{L}_c \geq C_{pm}^* \quad (7)$$

$$\text{DP for } \bar{L}_c \geq \theta \sigma \quad (8)$$

To express these constraints in terms of decision variables  $\sigma_i$ 's, consider the Lemma 2.2 which provides the relation of variance  $\bar{\sigma}_c^2$  with variances of individual stages, for a given value of  $R$  (note that  $\lambda$  and  $\mu_f$  are anyway known here). Thus, for a given value of  $R$ ,  $\bar{\sigma}_c^2$  can be expressed in terms of  $C_p$  and  $C_{pk}$  of  $L_c$  in the following manner:

$$\bar{\sigma}_c^2 = \frac{T^2}{9C_p^2} = \frac{\bar{d}^2}{9C_{pk}^2} \quad (9)$$

where  $T$ , the tolerance of customer delivery window, is a known parameter in the IOPT problem and  $\bar{d}$  is given as follows.

$$\bar{d} = \min(U - \bar{\mu}_c, \bar{\mu}_c - L) \quad (10)$$

Substituting the value of  $\bar{\mu}_c$ , from Lemma 2.2 in the above relation, we get

$$\bar{d} = \min \left( \left[ U - \mu_4 - M_o \sum_{i=1}^3 \mu_i \right], \left[ \mu_4 + M_o \sum_{i=1}^3 \mu_i - L \right] \right)$$

In the above equation,  $U, L, \mu_1, \mu_2, \mu_3, \mu_4$  are all known parameters. Also,  $M_o$ , according to Equation (1), depends only on  $\lambda, R$ , and  $\mu_f$ . Therefore, for a given value of  $R$ ,  $\bar{d}$  is a known parameter. The only unknown quantities in Equation (9) are  $C_p$  and  $C_{pk}$ . Substituting the value of Equation (9) in Equation (6) we get the following relation which is the crux of the problem of converting constraints in terms of decision variables, for a given value of  $R$ .

$$\sigma_4^2 + M_o^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) = \frac{T^2}{9C_p^2} = \frac{\bar{d}^2}{9C_{pk}^2} \quad (11)$$

#### D. Solution of IOPT

The optimization problem here is a mixed integer nonlinear optimization problem. The following provides a step-by-step procedure for solving the IOPT problem.

- 1) Fix a value for  $R$  and solve the resulting subproblem to determine optimal values of  $\sigma_i$ 's to achieve the optimal COST for that value of  $R$ . This requires a careful study and interpretation of the constraints to determine the values of  $C_p$  and  $C_{pk}$  for a given value of  $R$ . This is discussed in the next subsection.
- 2) Repeat Step 1 for all practically feasible values of  $R > 0$ .
- 3) Repeat Step 1 for  $R = 0$ . The case  $R = 0$  is a bit different from that of  $R > 0$  since it leads to a subtly different objective function and subtly different constraints (for details, see [11]).
- 4) Determine the minimum among all such optimal upper bounds on COST computed above. The corresponding  $R$  will give the optimal inventory level to be maintained and the corresponding  $\sigma_i$ 's will give the optimal variabilities to be assigned to individual lead times..

#### E. Determining $C_p$ and $C_{pk}$ for a given Value of $R$

The unknown pair  $(C_p, C_{pk})$  in equation (11) is chosen in a way that it satisfies both the constraints (7) and (8). The idea behind getting such a pair is detailed in [11].

#### F. Solution of IOPT for a Specific Instance

Let us consider the LPG supply chain once again and study the problem in a realistic setting. We have chosen following values for typical known parameters of the IOPT problem in the context of the LPG supply chain.

*Lead Time Parameters:*

$$\mu_1 = 1 \text{ day}, \mu_2 = 3 \text{ days}, \mu_3 = 2 \text{ days}, \mu_4 = 7 \text{ days}$$

*Demand Process Parameters:*

$$\lambda = 1500 \text{ trucks/year}$$

*Cost Parameters:*

These parameters have been chosen so as to capture the negative correlation between cost and mean lead time and between cost and variability of lead time.

$$\mathcal{X}_1 = 10 \left( 1 + \exp\left(\frac{1}{\sigma_1}\right) - \frac{\mu_1}{200} \right) \text{ \$/truck}$$

$$\mathcal{X}_2 = 100 \left( 1 + \exp\left(\frac{1}{\sigma_2}\right) - \frac{\mu_2}{200} \right) \text{ \$/truck}$$

$$\mathcal{X}_3 = 10 \left( 1 + \exp\left(\frac{1}{\sigma_3}\right) - \frac{\mu_3}{200} \right) \text{ \$/truck}$$

$$\mathcal{X}_4 = 100 \left( 1 + \exp\left(\frac{1}{\sigma_4}\right) - \frac{\mu_4}{200} \right) \text{ \$/truck}$$

$$A = 5 \text{ \$/order}; \Pi = 0 \text{ \$/truck}; \hat{\Pi} = 500 \text{ \$/truck-year};$$

$$I = 0.2 \text{ \$/year-\$/invested}; C = 1000 \text{ \$/truck}$$

*Delivery Quality Parameters*

$$\tau = 10 \text{ days}; T = 10 \text{ days}$$

For the sake of numerical experimentations we consider following four different sets of constraints and solve the problem under each case.

- 1) DP=3 $\sigma$  and DS=0.7 for  $L_c$
- 2) DP=4 $\sigma$  and DS=0.8 for  $L_c$
- 3) DP=5 $\sigma$  and DS=0.9 for  $L_c$
- 4) DP=6 $\sigma$  and DS=1.0 for  $L_c$

Assume that it is not possible for the RD to keep more than 40 LPG trucks ready at any given point of time.

We first describe Step 1 of the procedure to solve IOPT, discussed in the last section, for this numerical example. Let us choose Constraint set DP=3 $\sigma$  and DS=0.7 to work with. Step 2 can be carried out in the same manner for all the other values of  $R$ . Step 3 and Step 4 are also trivial. The same procedure can be repeated for other constraints sets also.

To start with, let us fix  $R = 10$ . We first compute the following parameters for the given numerical values.

$$\mu_f = 6 \text{ days}$$

$$M_o = 0.999722639663766$$

$$E = 1499.583959 \text{ trucks/year}$$

$$B = 14.657947016303742 \text{ trucks}$$

$$D = 0.000412769728402651 \text{ trucks}$$

$$\beta = 4.983672090063828 \times 10^6$$

$$\bar{d} = 7.001664162017404 \text{ days}$$

An important problem is to find out values of  $C_p$  and  $C_{pk}$ . This is detailed in [11]. For the present example these indices are  $C_p = 1.25057$  and  $C_{pk} = 0.875609$ . These  $C_p$  and  $C_{pk}$  can further be utilized to determine the value of DP and DS at the global minimum point which come out to be  $4.12678\sigma$  and  $0.83088$  respectively. These quality levels are more than what is desired. Hence, we use  $C_p$  and  $C_{pk}$  as design values. Substituting these in the objective function gives optimal upper bound on COST (2.5921 million \$) of supply chain with  $R = 10$ .

Fortunately, in the present situation the global minimum point becomes a design point so we need not proceed for any further calculation. But if it is not so, then we will be required to solve the underlying optimization problem by the *Lagrange multiplier method* and get stationary points which satisfy the necessary conditions.

Note that we have studied an instance of the IOPT problem assuming  $R = 10$ ,  $DP=3\sigma$ , and  $DS=0.7$  for  $L_c$ . We obtained the optimal allocation of standard deviations to achieve a minimum COST. The standard deviations obtained can be used by a supply chain manager to decide among alternate logistics providers or alternate suppliers, etc.

To now obtain the optimal value of  $R$ , we repeat the solution of the IOPT problem for different values of  $R$ , each time computing the optimal upper bound on COST and the corresponding allocation of variabilities. Detailed results of the above problem are presented in [11].

#### IV. SUMMARY AND FUTURE WORK

In this paper, we have presented a novel approach to achieve variability reduction, synchronization, and therefore delivery performance improvement in supply chain networks. Our approach exploits connections between design tolerancing in mechanical assemblies and lead time compression in supply chain networks. The paper leaves plenty of room for further work in several directions. The design problem that we studied here is only one of a rich variety of design optimization problems that one can address in the framework developed in this paper. Many other problems, as listed in Section II.F can be studied. Also, the supply chain example that we have looked at belongs to the MTO type. Here again, there is no reason why our approach cannot be applied for coordination types other than MTO, such as MTS and BTO (Build to Order). Also, we have assumed presence of inventory at only one of the stages. Generalizing this to multi-echelon networks will be extremely interesting.

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