

New fixed point for Kondo spin coupled to a junction of Luttinger liquids

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We study a system of an impurity spin coupled to a junction of several Tomonaga-Luttinger liquids using a renormalization group scheme. By starting close to the decoupled S -matrix at the junction, we find a *new* stable fixed point at a finite ferromagnetic value of the Kondo coupling, for repulsive inter-electron interactions; this leads to the possibility of spin-flip scatterings even at the lowest temperatures. If the junction is governed by the Griffiths S -matrix, we find that the Kondo coupling flows to the strong coupling fixed point (Furusaki-Nagaosa point) where all the wires are decoupled and the conductance vanishes at zero temperature.

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Although the Kondo effect has been studied for many years and is one of the best understood paradigms of strongly correlated quantum systems [1], it continues to yield new physics, particularly when its recent manifestations in quantum dots [2] are considered. For quantum dots with an odd number of electrons, a Kondo resonance is formed at the Fermi level of the leads, which is seen as a peak in the conductance.

If the leads are one-dimensional, inter-electron interactions convert them into Tomonaga-Luttinger liquids (TLLs) which may completely change the physics. The Kondo effect has been studied by several groups for a system with two TLL leads [3, 4, 5, 6] and for crossed TLL wires [7]. Furusaki and Nagaosa found that at $T = 0$, the strong coupling fixed point is that of decoupled semi-infinite TLLs and a spin singlet is formed if the impurity has spin $1/2$; the conductance vanishes at $T = 0$ via the usual TLL power law [8].

Motivated by recent experiments which probe the Kondo density of states in a three terminal geometry [9], we study in this paper what happens when an impurity spin is coupled to a junction of two or more quantum wires which are modeled as TLLs. The junction is characterized by an S -matrix which controls how the wires are connected with each other, and the couplings of the Kondo spin are described by a J -matrix.

We obtain the renormalization group (RG) equations for the system, and find that the flow of the Kondo couplings involves the elements of the S -matrix. By studying the RG equations near the fixed point where all the wires are decoupled from each other, we find that for a large range of initial values of the Kondo couplings and for any impurity spin S , the system flows to a *new* stable fixed point, where the diagonal couplings are finite and ferromagnetic and the off-diagonal couplings are zero. This could lead to spin-flip scattering of the electrons from the impurity spin. On the other hand, near the Griffiths fixed point of the S -matrix, we find that there is no stable fixed point for finite values of the Kondo couplings, and

the system flows towards strong coupling. We then perform an expansion in the inverse of the Kondo couplings and find that the impurity is strongly and antiferromagnetically coupled to the electron spin at the junction and, hence, the effective spin is now $S - 1/2$. Furthermore, the system is now near the decoupled fixed point of the S -matrix (because the junction site is tightly bound to the impurity spin), and hence it flows to the new stable fixed point. If the impurity spin is $1/2$, it decouples by forming a singlet with the electron spin at the junction.

We begin with N semi-infinite quantum wires which meet at a junction site, where the incoming and outgoing fields are related by an $N \times N$ unitary S -matrix. The wave function corresponding to an electron with spin α ($\alpha = \uparrow, \downarrow$) and wave number k (defined with respect to the Fermi wave number k_F) which is incoming in wire i ($i = 1, 2, \dots, N$) is given by

$$\begin{aligned} \psi_{i\alpha k}(x) &= e^{-ikx} + S_{ii} e^{ikx} \text{ on wire } i, \\ &= S_{ji} e^{ikx} \text{ on wire } j \neq i. \end{aligned} \quad (1)$$

Here k goes from $-\Lambda$ to Λ , and the dispersion is linearized ($E = v_F k$). The second quantized annihilation operator corresponding to the wave function in Eq. (1) is given by $\Psi_{i\alpha k}(x) = c_{i\alpha k} \psi_{i\alpha k}(x)$, where the wire index i runs from 1 to N . If an impurity spin is coupled to the electrons at or near the junction, the Hamiltonian is given by

$$\begin{aligned} H_0 + H_{\text{spin}} &= v_F \sum_i \sum_{\alpha} \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} k c_{i\alpha k}^{\dagger} c_{i\alpha k} \\ &+ \sum_{i,j} \sum_{\alpha,\beta} \int_{-\Lambda}^{\Lambda} \int_{-\Lambda}^{\Lambda} \frac{dk_1}{2\pi} \frac{dk_2}{2\pi} \\ &J_{ij} \vec{S} \cdot c_{i\alpha k_1}^{\dagger} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{j\beta k_2}, \end{aligned} \quad (2)$$

where J_{ij} is a Hermitian matrix, $\vec{\sigma}$ denotes the Pauli matrices, and we are assuming an isotropic spin coupling $J_x = J_y = J_z$ for simplicity.

Next, let us consider density-density interactions between the electrons of the form

$$H_{\text{int}} = \frac{1}{2} \int \int dx dy \rho(x) U(x-y) \rho(y), \quad (3)$$

where the density ρ is given in terms of the second quantized electron field $\Psi_\alpha(x) = \sum_i \int dk/(2\pi) \Psi_{i\alpha k}(x)$ as $\rho = \Psi_\uparrow^\dagger \Psi_\uparrow + \Psi_\downarrow^\dagger \Psi_\downarrow$. Writing the electron field in terms of outgoing and incoming fields as $\Psi_\alpha(x) = \Psi_{O\alpha}(x) + \Psi_{I\alpha}(x)$, the Hamiltonian in (3) takes the form

$$H_{\text{int}} = \int dx \sum_{\alpha,\beta} [g_1 \Psi_{O\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{O\beta} \Psi_{I\alpha} + g_2 \Psi_{O\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{I\beta} \Psi_{O\alpha} + \frac{1}{2} g_4 (\Psi_{O\alpha}^\dagger \Psi_{O\beta}^\dagger \Psi_{O\beta} \Psi_{O\alpha} + \Psi_{I\alpha}^\dagger \Psi_{I\beta}^\dagger \Psi_{I\beta} \Psi_{I\alpha})], \quad (4)$$

where $g_1 = \tilde{U}(2k_F)$ and $g_2 = g_4 = \tilde{U}(0)$. For repulsive (attractive) interactions, $g_2 > 0$ (< 0) respectively. (We have ignored umklapp scattering here). The interaction parameters g_1 , g_2 and g_4 satisfy some RG equations [10, 11], but we will ignore the flow of these parameters in this work. In general, g_1 , g_2 and g_4 could have different values in different wires.

The junction S -matrix satisfies an RG equation which was derived in Refs. [11, 12] in the absence of a Kondo coupling. We find that the Kondo couplings do not affect the RG flows of the S -matrix. We will therefore assume for simplicity that we are near a fixed point of the RG equations for S_{ij} . We will study what happens near two particular fixed points as discussed below.

We use the technique of ‘poor man’s RG’ [13, 14] to derive the renormalization of the S -matrix and the Kondo coupling matrix J_{ij} . The details of the calculation will be presented elsewhere. We find that the S -matrix contributes to the renormalization of the J_{ij} . Namely,

$$\frac{dJ_{ij}}{d \ln L} = \frac{1}{2\pi v_F} \left[\sum_k J_{ik} J_{kj} + \frac{1}{2} g_{2i} S_{ij} \sum_l J_{il} S_{il}^* + \frac{1}{2} g_{2j} S_{ji}^* \sum_l J_{lj} S_{jl} \right], \quad (5)$$

where L denotes the length scale. Eq. (5) is the key result of this paper. (Note that the parameters g_{1i} and g_{4i} do not appear in Eq. (5)).

We now consider two possibilities for the S -matrix, assuming that g_2 has the same value on all the wires. The first case is that of N disconnected wires for which the S -matrix is given by the $N \times N$ identity matrix (up to phases). We consider a highly symmetric form of the Kondo coupling matrix (consistent with the symmetry of the S -matrix) in which all the diagonal entries are J_1 and all the off-diagonal entries are J_2 , with both J_1 and J_2 being real. Eq. (5) then gives

$$\frac{dJ_1}{d \ln L} = \frac{1}{2\pi v_F} [J_1^2 + (N-1)J_2^2 + g_2 J_1],$$

$$\frac{dJ_2}{d \ln L} = \frac{1}{2\pi v_F} [2J_1 J_2 + (N-2)J_2^2]. \quad (6)$$

These equations have three fixed points at finite values of (J_1, J_2) , namely, at $A = (0, 0)$, $B = (-g_2, 0)$, and $C = (-g_2(N-2)^2/N^2, 2g_2(N-2)/N^2)$. A linear stability analysis shows that if $g_2 > 0$, the fixed point B is stable in both directions, while A and C are unstable in at least one direction. [Note that a stable fixed point like B exists even if the parameters g_{2i} are different in different wires, as long as they are all positive.] A stable fixed point where all the parameters are finite is rather unusual. One might object that the RG equations studied here are only valid at the lowest order in J_{ij} and g_2 . However, one can trust these equations if g_2 and J_{ij} are small, i.e., for weak repulsion and small Kondo couplings.

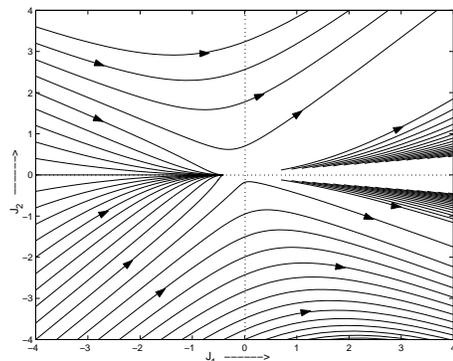


FIG. 1: RG flows of the Kondo couplings for three disconnected wires, with $g_2/(2\pi v_F) = 0.2$. Note the flows to the stable fixed point lying at $(J_1, J_2) = (-0.2, 0)$.

Fig. 1 shows a picture of the RG flows for three wires for $g_2/(2\pi v_F) = 0.2$. [The parameter $g_2/(2\pi v_F)$ is found to be about 0.2 in several experimental systems (see [15] and references therein). In Figs. 1 and 2, the values of g_2 and J_{ij} are shown in units of $2\pi v_F$.] We can see that the RG flows take a large range of initial conditions to the stable fixed point at $(-g_2, 0)$. For all other initial conditions, we see that there are two directions along which the Kondo couplings flow to infinity; these are given by $J_2/J_1 = 1$ and $J_2/J_1 = -1/(N-1)$ (with $N = 3$). This asymptotic behavior can be understood by analyzing Eq. (6) after ignoring the terms of order g_2 .

The second case that we study is called the Griffiths S -matrix. Here all the N wires are connected to each other and there is maximal transmission, subject to the constraint that there is complete symmetry between the N wires. The resultant S -matrix has all the diagonal entries equal to $-1 + 2/N$ and all the off-diagonal entries equal to $2/N$. We again consider a highly symmetric form of the Kondo coupling matrix, with real parameters J_1 and J_2 as the diagonal and off-diagonal entries respectively.

Eq. (5) then gives

$$\begin{aligned} \frac{dJ_1}{d\ln L} &= \frac{1}{2\pi v_F} [J_1^2 + (N-1)J_2^2 - (1 - \frac{2}{N}) g_2 C], \\ \frac{dJ_2}{d\ln L} &= \frac{1}{2\pi v_F} [2J_1J_2 + (N-2)J_2^2 + \frac{2}{N} g_2 C], \\ C &= - (1 - \frac{2}{N}) J_1 + (1 - \frac{1}{N}) 2J_2 . \end{aligned} \quad (7)$$

(For $N = 2$, Eq. (7) agrees with the equations derived in Ref. [4]). One can prove from Eq. (7) that the only fixed points are the fixed point at the origin (which is unstable) and the strong coupling fixed points. Fig. 2 shows a picture of the RG flows for three wires for $g_2/(2\pi v_F) = 0.2$. The couplings are seen to flow to infinity along one of the two directions $J_2/J_1 = 1$ and $-1/(N-1)$.

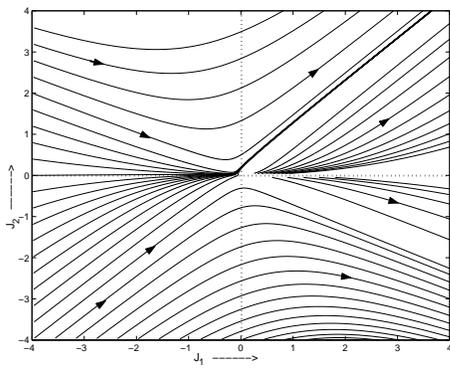


FIG. 2: RG flows of the Kondo couplings for the Griffiths S -matrix for three wires, with $g_2/(2\pi v_F) = 0.2$.

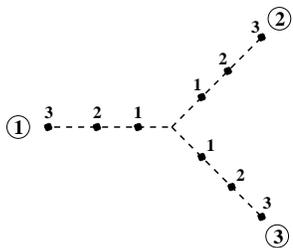


FIG. 3: A lattice model for the S -matrices discussed in the text.

We will now see how the different S -matrices and RG flows discussed above can be interpreted in terms of lattice models as was done for the two-wire case in Ref. [4]. The case of N disconnected wires can be realized by the lattice model shown in Fig. 3. The Hamiltonian is taken to be of the tight-binding form, with a hopping amplitude $-t$ on all the bonds, except on the bonds connecting to the $n = 0$ site where they are taken to be zero. (This

is equivalent to removing the junction site from the system). The impurity spin is coupled to the sites $n = 1$ on the different wires by the Hamiltonian

$$\begin{aligned} H_{\text{spin}} &= A \vec{S} \cdot \sum_i \sum_{\alpha,\beta} \Psi_\alpha^\dagger(i,1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_\beta(i,1) \\ &+ B \vec{S} \cdot \sum_{i \neq j} \sum_{\alpha,\beta} \Psi_\alpha^\dagger(i,1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_\beta(j,1) , \end{aligned} \quad (8)$$

where $\Psi_\alpha(i,1)$ denotes the second quantized electron field at site 1 on wire i . Note that this Hamiltonian couples fields on different wires. (Eq. (13) below will provide a justification for this kind of a coupling). We then find that the Kondo coupling matrix J_{ij} in Eq. (2) is as follows: all the diagonal entries are given by J_1 and all the off-diagonal entries are given by J_2 , where

$$J_1 = 4A \sin^2 k_F , \quad \text{and} \quad J_2 = 4B \sin^2 k_F \quad (9)$$

for modes with redefined wave numbers lying close to zero. This is precisely the kind of Kondo matrix whose RG flows were studied in Eq. (6). In particular, the stable fixed point at $(J_1, J_2) = (-g_2, 0)$ describes a fixed point with $A < 0$ and $B = 0$; this corresponds to a ferromagnetic coupling of the impurity spin to the electron spin at the first site of each of the wires.

The case of the Griffiths S -matrix can also be realized by the lattice shown in Fig. 3 and a tight-binding Hamiltonian. The hopping amplitude is now $-t$ on all bonds, except for the bonds connecting to the junction site where it is taken to be $t_1 = -t\sqrt{2/N}$. We then find that the S -matrix is of the Griffiths form for all values of the wave number k . The impurity spin is then coupled to the junction site and the $n = 1$ sites by the Hamiltonian

$$\begin{aligned} H_{\text{spin}} &= A \vec{S} \cdot \sum_{\alpha,\beta} \Psi_\alpha^\dagger(0) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_\beta(0) \\ &+ B \vec{S} \cdot \sum_i \sum_{\alpha,\beta} \Psi_\alpha^\dagger(i,1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_\beta(i,1) , \end{aligned} \quad (10)$$

where $\Psi_\alpha(0)$ denotes the second quantized electron field at the junction site with spin α . Then the Kondo coupling matrix J_{ij} in Eq. (2) has all the diagonal entries equal to J_1 and all the off-diagonal entries equal to J_2 , where

$$\begin{aligned} J_1 &= \frac{4A}{N^2} + 2B [1 - (1 - \frac{2}{N}) \cos 2k_F] , \\ J_2 &= \frac{4A}{N^2} + \frac{4B}{N} \cos 2k_F \end{aligned} \quad (11)$$

for modes with wave numbers lying close to zero. The RG flows of this are given in Eq. (7). Now,

$$\begin{aligned} J_1 - J_2 &= 2B (1 - \cos 2k_F) , \\ J_1 + (N-1)J_2 &= \frac{4A}{N} + 2B (1 + \cos 2k_F) . \end{aligned} \quad (12)$$

Since $0 < k_F < \pi$, $1 \pm \cos 2k_F$ lie between 0 and 2. In the first quadrant of Fig. 2, we see that $J_1 + (N-1)J_2$ goes to ∞ faster than $|J_1 - J_2|$; Eq. (12) then implies that A goes to ∞ and $|B|/A \rightarrow 0$. Similarly in the fourth quadrant, B goes to ∞ and A goes to $-\infty$.

These flows to strong coupling have the following physical interpretations. In the first case, A flows to ∞ which means that the impurity spin is strongly and antiferromagnetically coupled to an electron spin at the junction site $n = 0$; hence those two spins will combine to form an effective spin of $S - 1/2$. In the second case, the impurity spin is coupled strongly and ferromagnetically to an electron spin at the site $n = 0$, and strongly and antiferromagnetically to electron spins at the sites $n = 1$ on each of the N wires to form an effective spin of $S + 1/2 - N/2$.

We considered above two examples of S -matrices and found that the Kondo couplings flow to infinity for many initial conditions. We will now see that the vicinity of the strong coupling fixed points can be studied through an expansion in the inverse of the Kondo coupling [14]. Following the discussion after Eq.(12), let us assume that the RG flows for the case of the Griffiths S -matrix have taken us to a strong coupling fixed point along the line $J_1 = J_2$; thus the impurity spin is coupled to the electron spin at $n = 0$ with a large and positive (antiferromagnetic) value A , while its couplings to the sites $n = 1$ have the value $B = 0$. (The arguments given below do not change significantly if $B \neq 0$, provided that $|B| \ll A$). From the first term in Eq. (10), we see that the impurity spin couples to an electron at $n = 0$ to form an effective spin of $S - 1/2$; the energy of this spin state is $-A(S + 1)/2$. This lies far below the high energy states in which an electron at site $n = 0$ forms a total spin of $S + 1/2$ with the impurity spin (these states have energy $AS/2$), or the states in which the site $n = 0$ is empty or doubly occupied (these states have zero energy).

We now perturb in $1/A$. The unperturbed Hamiltonian corresponds to N disconnected wires along with the spin coupling proportional to A . The perturbation H_{pert} consists of the hopping amplitude t_1 on the bonds connecting the sites $n = 1$ to the junction site. Using this perturbation, we can find an effective Hamiltonian [14]. If $S > 1/2$, we find that the effective Hamiltonian has no terms of order t_1 , and is given by

$$H_{\text{eff}} = A_{\text{eff}} \vec{S}_{\text{eff}} \cdot \sum_i \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(i, 1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(i, 1) \\ + B_{\text{eff}} \vec{S}_{\text{eff}} \cdot \sum_{i \neq j} \sum_{\alpha, \beta} \Psi_{\alpha}^{\dagger}(i, 1) \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta}(j, 1)$$

$$\text{where } A_{\text{eff}} = B_{\text{eff}} = - \frac{8t_1^2}{A(S+1)(2S+1)}, \quad (13)$$

and \vec{S}_{eff} denotes an object with spin $S - 1/2$. We thus find a weak interaction between \vec{S}_{eff} and all the sites labeled as $n = 1$ in Fig. 3. [If the impurity has $S = 1/2$, the

electron at $n = 0$ forms a singlet with the impurity. Then the lowest order terms in the effective Hamiltonian are of order t_1^4/A^3 [16].] For the case $S > 1/2$, we see that Eq. (13) has the same form as in Eqs. (8) and (9), with the effective Kondo couplings

$$J_{1,\text{eff}} = 4A_{\text{eff}} \sin^2 k_F, \quad \text{and} \quad J_{2,\text{eff}} = 4B_{\text{eff}} \sin^2 k_F. \quad (14)$$

The RG flow of this was studied in Eq. (6); there is a range of initial conditions which flow under RG to the stable fixed point at $(J_{1,\text{eff}}, J_{2,\text{eff}}) = (-g_2, 0)$.

We therefore obtain a complete picture of the RG flows at both short and large length scales. We start with the Griffiths S -matrix with certain values of the Kondo coupling matrix, and we eventually end at the stable fixed point of the disconnected S -matrix.

To summarize, we have studied the Kondo effect at a junction of N quantum wires. We find that the scaling of the Kondo couplings depends on the S -matrix at the junction of the quantum wires. For the case of disconnected wires and repulsive interactions, the Kondo couplings flow to $(J_1, J_2) = (-g_2, 0)$. Since J_1 goes to a finite value, this could lead to spin-flip scattering processes at even the lowest temperatures. It may be possible to observe such a scattering by placing a quantum dot with a spin at the junction of several wires with interacting electrons. This is in contrast to the case of Fermi liquid leads (i.e., with $g_2 = 0$) since the stable fixed point in that case lies at $(J_1, J_2) = (0, 0)$, and no spin-flip scattering would be observed at the lowest temperatures.

At the fully connected or Griffiths fixed point, we find that the Kondo couplings flow to the strong coupling fixed point, where their fate is decided by a $1/J$ analysis. There is a range of initial conditions which again lead to the stable fixed point $(J_{1,\text{eff}}, J_{2,\text{eff}}) = (-g_2, 0)$.

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