Analysis of Strong Resonance in Power Systems with STATCOM Supplementary Modulation Controller

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Abstract—The main objective of this paper is to analyze the behaviour of a pair of oscillatory modes of a power system, essentially a swing mode and an exciter mode, that pass near strong resonance. The method of perturbations suggested by Seiranyan [1] to describe the behaviour of a pair of eigenvalues in the neighbourhood of a multiple point has been suitably modified to make it applicable to systems with feedback controllers. In the case of multimachine systems, the swing mode of interest is isolated by making use of the concept of modal transformation.The knowledge of participation factors helps in identifying the exciter mode that interacts with the swing mode. The illustrative examples comprise a Single Machine connected to Infinite Bus (SMIB) system and a 3-machine system with STATCOM supplementary modulation controller.

1. INTRODUCTION

As power systems grow and levels of power exchange increase to meet the ever increasing load demand, the systems tend to be highly stressed. The stability of an equilibrium point can be determined by an investigation of the linearized dynamics of the system. Changes in the system parameters may lead to interaction of two damped modes. The modes which are far away initially move close to each other and collide in such a way that one of the modes may subsequently become unstable. This collision occurs when the system matrix has two complex pairs of eigenvalues that coincide in both real and imaginary parts. If the matrix is not diagonalizable at the point of collision of eigenvalues, the phenomenon is termed strong interaction (strong resonance) and, weak interaction (weak resonance) if it is diagonalizable [2]. Dobson et al [3] have shown that strong resonance is a precursor to oscillatory instability in their study on 3-bus and 9-bus power systems. The generators' power dispatch is varied to study the behaviour of two complex eigenvalues near the point of strong resonance. It is observed that before collision the eigenvalues move together by a change in frequency and it is the strong resonance that transforms this movement into a change in damping. This change in damping ultimately results in one of the eigenvalues moving into the right half plane. Kwatny and Yu [4] have studied the effect of load parameter variations on an undamped stable system whose eigenvalues are on the imaginary axis. Loss of stability is noticed when two modes move towards each other along the imaginary axis as the parameter varies and collide before moving into the right and left half planes. This phenomenon is termed flutter instability and is generic in one parameter Hamiltonian systems.

In the work presented in this paper the behaviour of two modes (essentially a swing mode and an exciter mode) of SMIB system which pass near strong resonance is analyzed by applying the theory developed by Seiranyan [1].This theory is extended to study the strong resonance in the presence of a STATCOM controller located at the load bus .

In the case of 3-machine system with STATCOM damping controller, the two modes which interact near the point of strong resonance are identified. The concept of multi-modal decomposition [5] is applied to isolate the swing mode of interest. The relevant exciter mode is identified from the knowledge of participation factors.

The organisation of the paper is as follows. Section 2 gives the background theory of strong resonance phenomenon. Section 3 presents the case studies of SMIB and 3 machine systems. Sections 4 and 5 present discussion and conclusions respectively.

2. BACKGROUND THEORY

SMIB System without STATCOM controller

The method of perturbations is applied to analyse the interaction of the eigenvalues associated with the swing mode and the exciter mode in the neighbourhood of the point of strong resonance with the help of a family of hyperbolae in the complex s-plane. The coefficients of the equations of these hyperbolae are computed using an eigenvector and an associated vector, an eigenvector of the adjoint problem at the multiple point, and the increments of the parameters.

The linearized model of a SMIB system can be expressed in terms of a second-order vector differential equation as

$$[M]\ddot{q} + [D]\dot{q} + [A]q = 0 \tag{1}$$

where $q = [\Delta\delta, \Delta E_q']^t$.

The matrices M, D and A are defined in Appendix A.

The characteristic equation of the system is given by

$$s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0 \tag{2}$$

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(The above equation is obtained from $\det([M]s^2 + [D]s + [A]) = 0$). The expressions for the coefficients a_1 to a_4 are given in Appendix B.
The eigenvector u_o, the associated eigenvector u_1 and the eigenvector v_o of the adjoint problem are defined by the equations

$$[L]u_o = 0 \quad (3)$$

$$[L]u_1 = -(2\lambda_o[M_o] + [D_o])u_o \quad (4)$$

$$[L^*]v_o = 0 \quad (5)$$

L^* is the conjugate transpose of the matrix L, where

$$[L] = \lambda_o^2[M_o] + \lambda_o[D_o] + [A_o] \quad (6)$$

and $M_o = M(p_o), D_o = D(p_o), A_o = A(p_o)$.
The parameter vector p_o corresponds to a double root λ_o of the characteristic equation . To investigate the behaviour of the eigen values λ in the neighbourhood of the point $p = p_o$ in the parameter space, the vector p_o is given an increment $p = p_o + \epsilon k$, where $k = (k_1, k_2, ..., k_n)$ is an arbitrary normalized variation vector such that $|k| = \sqrt{k_1^2 + k_2^2 + ... + k_n^2} = 1$ and ϵ is a small parameter, $\epsilon > 0$
Note:- For a single parameter case k=±1 and $\epsilon = |\Delta p|$, where $\Delta p = p - p_o$.
The expansion for λ is given by

$$\lambda = \lambda_o + \sqrt{\epsilon}\lambda_1 + \epsilon\lambda_2 + \epsilon^{3/2}\lambda_3... \quad (7)$$

where λ_1 is the first correction given by

$$\lambda_1^2 = \sum_{j=1}^{n} f_j k_j \quad (8)$$

where

$$f_j = -(\left[\frac{\partial L}{\partial p_j}\right] u_o, v_o)[((2\lambda_o[M_o] + [D_o])u_1, v_o) + ([M_o]u_o, v_o)]^{-1} \quad (9)$$

and

$$\left[\frac{\partial L}{\partial p_j}\right] = \lambda_o^2\left[\frac{\partial M}{\partial p_j}\right] + \lambda_o\left[\frac{\partial D}{\partial p_j}\right] + \left[\frac{\partial A}{\partial p_j}\right] \quad (10)$$

Let a_j=real(f_j) and b_j=imag(f_j) and $\Delta p_j = \epsilon k_j$. Then equation (8) after multiplying by ϵ can be written as

$$\sqrt{\epsilon}\lambda_1 = [\sum_{j=1}^{n}(a_j + jb_j) \quad \Delta p_j]^{1/2} \quad (11)$$

Let the increment in the eigenvalue be written in terms of the real and imaginary parts as

$$\sqrt{\epsilon}\lambda_1 = X + jY \quad (12)$$

Using equation (12) and squaring equation (11),

$$X^2 - Y^2 = \sum_{j=1}^{n} a_j\Delta p_j \quad (13)$$

$$2XY = \sum_{j=1}^{n} b_j\Delta p_j \quad (14)$$

Eliminating one of the parameters, say Δp_1, from equations (13) and (14), the equation for the hyperbola with mutually orthogonal asymptotes
$b_1X = Y(a_1 \pm (a_1^2 + b_1^2)^{1/2})$ can be written as

$$b_1(X^2 - Y^2) - 2a_1XY = \Delta\Phi = constant \quad (15)$$

where

$$\Delta\Phi = \sum_{j=2}^{n}(b_1a_j - a_1b_j)\Delta p_j \quad (16)$$

The hyperbola can be constructed using the solution of the eigenvalue problems (3), (4) and (5) (to determine the quantities λ_o, u_o, u_1, v_o), and by computing the constants a_j and b_j.

Systems with STATCOM controller

The block diagram of a STATCOM supplementary modulation controller is shown in Figure 1. The control signal used is known as Thevenin voltage and is synthesized from the locally measurable signal viz., the magnitude of the voltage of the bus at which the controller is connected [6]. The output of the controller is the magnitude of the reactive current injected into the system. The controller gains K_r and X_{th} are both tunable, and T_p is the STATCOM plant time constant (taken as 20msec).The controller is installed in the system to enhance the damping of critical modes.

In the case of multi machine systems the swing mode of interest is isolated by applying the concept of multi-modal decomposition [5]. Thus the modal system representing the swing mode involves the states, the modal angle and the modal speed.The relevant exciter mode is identified from the knowledge of participation factors. The effect of other variables, not relevant to the modes which interact near strong resonance, is neglected.

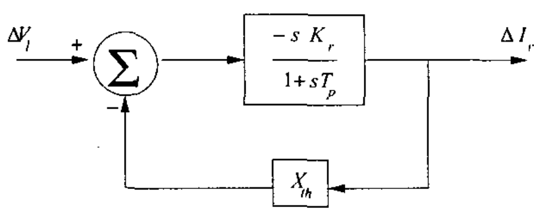

Figure 1. Block diagram of STATCOM damping controller

The system with STATCOM damping controller can be expressed by a vector differential equation

$$[M]\ddot{q} + [D]\dot{q} + [A]q = [B]u + [B_d]\dot{u} \quad (17)$$

where B and B_d are 2-dimensional vectors. (see Appendix.C)

The output equation of the system with the STATCOM damping controller can be written in terms of the input ΔI_r, the output matrix C and d (the coefficient of the input variable in the output equation of the system) as

$$\Delta V_l = [C]q + d\Delta I_r \tag{18}$$

From the controller input-output relation and equation (18), it is easy to obtain an equation relating ΔI_r and q as

$$\Delta I_r(s) = \frac{-K_r s[C]q(s)}{1 + sT_c} \tag{19}$$

where

$$T_c = T_p + K_r(d - X_{th}) \tag{20}$$

Substitution of equation (19) in equation (17) gives

$$[A_1]\frac{d^3q}{dt^3} + [A_2]\frac{d^2q}{dt^2} + [A_3]\frac{dq}{dt} + [A_4]q = 0 \tag{21}$$

where $[A_1]$=$T_c[M]$,
$[A_2]$=$[M] + T_c[D] + K_r[B_dC]$,
$[A_3]$=$[D] + T_c[A] + K_r[BC]$
and $[A_4]$=$[A]$.

The elements of the matrices A_1, A_2, A_3 and A_4 are functions of the components of the parameter vector $p = (K_r, X_{th})$. Let the parameter vector corresponding to a double root λ_o of the characteristic equation $\det[s^3A_1 + s^2A_2 + sA_3 + A_4] = 0$ be p_o. The corresponding eigenvector u_o, the associated eigenvector u_1, and the adjoint eigenvector v_o are determined respectively from the equations

$$Lu_o = 0 \tag{22}$$

$$Lu_1 = -[3\lambda_o^2 A_{1o} + 2\lambda_o A_{2o} + A_{3o}]u_o \tag{23}$$

$$L^* v_o = 0 \tag{24}$$

where

$$L = [\lambda_o^3 A_{1o} + \lambda_o^2 A_{2o} + \lambda_o A_{3o} + A_{4o}] = 0 \tag{25}$$

(L^* is the conjugate transpose of the matrix L)
and A_{1o}=$A_1(p_o)$, A_{2o}=$A_2(p_o)$, A_{3o}=$A_3(p_o)$ and A_{4o}=$A_4(p_o)$.
To investigate the behaviour of the eigenvalues λ in the neighbourhood of the point $p = p_o$ in the parameter space a family of hyperbolae can be constructed by solving the eigenvalue problems (22), (23) and (24) and following the procedural steps given for the case of systems without STATCOM damping controller.

3. Case Studies

SMIB System without STATCOM supplementary modulation controller

The SMIB system is shown in Figure.2.The generator is represented by (1.0) model and the load is of constant impedance type. A static exciter with a single time constant AVR is considered. The system data are given in Appendix.D

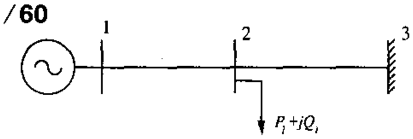

Figure 2. A single machine system

Figure.3 shows that when the generator dispatch is increased maintaining the terminal voltage constant at V_g=V_{go}, where V_{go} is the value of generator terminal voltage at strong resonance (equal to 0.9933 pu), the eigenvalues associated with the exciter mode (EM) and the swing mode (SM) move towards each other and collide at λ_o (-0.5312,5.8878) corresponding to P_g=0.2410 pu. After collision, the direction of movement of the eigenvalues changes by 90 degrees. This collision of the eigenvalues takes place when the deviation in V_g is zero, i.e. ΔV_g=0. When a perturbation is effected in V_g, i.e. $\Delta V_g \neq 0$, the increase in P_g makes the two eigenvalues move away from each other in the opposite directions near the double point. It is interesting to note that the two eigenvalues reverse the quadrants when the deviation in the terminal voltage changes sign. A similar analysis can be carried out by varying V_g and maintaining P_g constant.

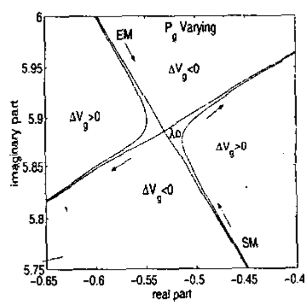

Figure 3. Asymptotic behaviour of eigenvalues of SMIB system when P_g is varying.

Systems with STATCOM supplementary modulation controller

SMIB System— A STATCOM with its supplementary modulation controller is connected at the load bus of the SMIB system shown in Figure.2. The initial output of the STATCOM is zero.The asymptotic behaviour of the eigenvalues associated with the exciter mode and the swing mode near the point of strong resonance (-0.2353,5.9212) corresponding to P_g=0.5 pu and V_g=1.0 pu, is analyzed by drawing a hyperbola (see Figure. 4) by varying K_r monotonically, keeping X_{th}constant. It is observed from Figure.4 that the damping of the swing mode keeps increasing and that of the exciter mode

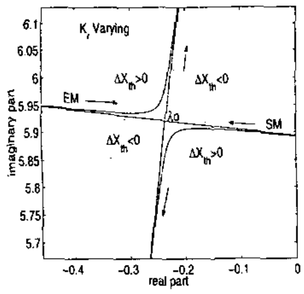

Figure 4. Asymptotic behaviour of eigenvalues of SMIB system when K_r is varying.

keeps reducing as the controller gain K_r is increased, till the eigenvalues associated with the swing mode and the exciter mode pass near strong resonance, after which the damping of the modes remains almost constant. Thus in the system under consideration, the phenomenon of strong resonance limits the damping of the modes of interest.

3-Machine System—A 3-machine system (the data of which are given in [6]) is considered for the analysis of eigenvalue behaviour in the neighbourhood of the point of strong resonance. The generators are represented by a (1.1) model and the loads are of constant power type. High gain AVR with a single time constant exciter is considered for each generator. A STATCOM with its supplementary modulation controller is connected at bus no.9 close to generator no.3. The initial output of the STATCOM is zero.While tuning the controller at the given operating point to enhance the damping of swing modes by varying the gain K_r (with a fixed value of X_{th}), it was noticed [7] that there is an interaction (as shown in the Figure.5) between the exciter mode associated with the generator no.3 and the swing mode of frequency 13.109 rad/sec.

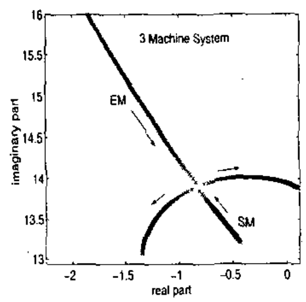

Figure 5. Root loci showing the interaction of two complex modes in 3-machine system when K_r is varying.

For the analysis of the behaviour of these two oscillatory modes by the application of the method of perturbations, the system is reduced by considering the dynamics of the two modes that interact in the neighbourhood of a multiple point. Figure.6 and Figure.7 illustrate the asymptotic behaviour of the eigenvalues of interest in the neighbourhood of the point of strong resonance (-1.1131,13.129) due to the increase in i)K_r for fixed values of X_{th}, and ii) X_{th} for fixed values of K_r respectively.

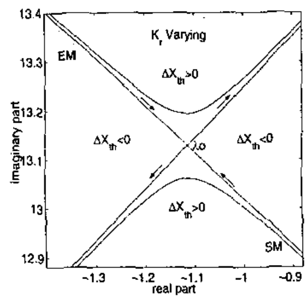

Figure 6. Asymptotic behaviour of eigenvalues of 3-machine system when K_r is varying

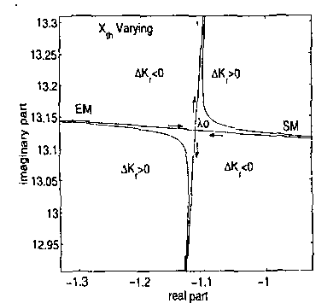

Figure 7. Asymptotic behaviour of eigenvalues of 3-machine system when X_{th} is varying

4. Discussion

This paper extends the work reported in [3] by identifying the modes (a swing mode and an exciter mode) responsible for strong resonance. Previous work [4] focussed on the strong resonance phenomenon resulting from two swing modes.

The analysis of strong resonance (caused by interaction of a swing mode and an exciter mode) due to the variation of controller parameters associated with a STATCOM has been investigated for the first time.

From the results obtained on the eigenvalue interaction near the multiple point, it is observed that the phenomenon of strong resonance results either in instability of one of the modes or in limiting the damping of modes. In the case of three machine system it is interesting to note that the exciter mode, pertaining to the generator close to the bus at which the damping controller is connected, interacts with a swing mode leading to the strong resonance phenomenon. It is also observed that near strong resonance, the eigenvalues move quickly and turn through 90 degrees(approx). A small change in the parameter near strong resonance causes a significantly large change in damping or frequency of the modes concerned.

Although the analysis in a multimachine system is based on the reduced model retaining only the two modes of interest and the controller, the results obtained from the analysis are accurate enough to predict the behaviour of eigenvalues in the neighbourhood of strong resonance for the detailed system.

5. CONCLUSIONS

This paper investigates the phenomenon of strong resonance in power systems with STATCOM supplementary modulation controller.This involves interaction between a swing mode and an exciter mode which can be identified. The analysis is based on a reduced system retaining only the two modes of interest and the controller. The asymptotic behaviour of the two modes in the neighbourhood of strong resonance is investigated and compared with the root loci of the detailed system.

6. APPENDIX

A. Matrices of vector differential equation SMIB System

$$M = \begin{bmatrix} M_g & 0 \\ 0 & T_A T'_{do} K_3 \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{D_g}{w_B} & 0 \\ K_3 K_4 T_A & (T_A + T'_{do} K_3) \end{bmatrix}$$

$$A = \begin{bmatrix} K_1 & K_2 \\ K_3(K_4 + K_A K_5) & (1 + K_A K_3 K_6) \end{bmatrix}$$

where K_1 to K_6 are Heffron Phillip constants [8].

B. Expressions for coefficients of characteristic equation (2) of SMIB System

$$a_1 = \frac{M_g T_A + M_g T'_{do} K_3 + D_g T_A T'_{do} K_3}{M_g T_A T'_{do} K_3}$$

$$a_2 = \frac{M_g + M_g K_A K_3 K_6 + D_g T_A}{M_g T_A T'_{do} K_3} + \frac{D_g T'_{do} K_3 + K_1 T_A T'_{do} K_3}{M_g T_A T'_{do} K_3}$$

$$a_3 = \frac{D_g + D_g K_A K_3 K_6 + K_1 T_A}{M_g T_A T'_{do} K_3} + \frac{K_1 T'_{do} K_3 - K_2 K_3 K_4 T_A}{M_g T_A T'_{do} K_3}$$

$$a_4 = \frac{K_1 + K_1 K_A K_3 K_6 - K_2 K_3 K_4 - K_2 K_3 K_A K_5}{M_g T_A T'_{do} K_3}$$

C. B and B_d matrices of equation (17)

$$B = \begin{bmatrix} -K_{Tr} \\ -K_3(K_{Vr} K_A + K_{Fr}) \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 \\ -K_3 K_{Fr} T_A \end{bmatrix}$$

where

$K_{Tr} = \frac{\partial T_e}{\partial I_R}$,

$K_{Vr} = \frac{\partial V_t}{\partial I_R}$,

and K_{Fr} is defined from the equation

$$\Delta E'_q = \frac{K_3 \Delta E_{fd} - K_3 K_4 \Delta\delta - K_3 K_{Fr} \Delta I_R}{1 + s T'_{do} K_3}$$

D. SMIB System Data

Generator: R_a=0, x_d=2.442, x_q=2.421, x'_d=0.83, T'_{do}=5.33, H=2.832, D_g=0, $M_g = 2H/w_b$

Exciter: K_A=450, T_A=0.6;

Line 1-2: R=0,X=0.168, $B = 2 \times 0.01$;

Line 2-3: R=0,X=0.126, $B = 2 \times 0.008$;

Load: P_L=1.0, Q_L=0.3;

E_b=1.0, f_b=60 Hz.

REFERENCES

[1] A.P.Seiranyan, *Collision of eigenvalues in linear oscillatory systems*, Journal of Applied mathematics and Mechanics, vol. 58, no. 5, 1994, pp 805-813.

[2] Alexander P.Seyranian, *Sensitivity analysis of multiple eigenvalues*, Mechanics of structures and machines, vol. 21,no. 2, 1993, pp 261-284.

[3] Ian Dobson, Jianfeng Zhang, Scott Grene, Henrik Engdahl, and Peter W.Sauer, *Is strong resonance a precursor to power system oscillations?*, IEEE Transactions on circuits and systems-I:Fundamental theory and applications, vol. 48, no. 3, March 2001, pp 340-349.

[4] Harry G.Kwatny, and Xiao-Ming Yu, *Energy anlysis of load-induced flutter instability in classical models of electric power networks*, IEEE Transactions on circuits and systems, vol. 36, no. 12, December 1989, pp 1544-1557.

[5] E.V.Larsen, J.J.Sanchez-Gasca and J.H.Chow, *Concepts for design of FACTS controllers to damp power swings*, IEEE Transactions on power systems, vol. 10, no. 2, May 1995, pp 948-956.

[6] A.M.Kulkarni and K.R.Padiyar, *Damping of power swings using shunt FACTS controllers*, Fourth workshop on EHV Technology, Bangalore, July 1998.

[7] V.Swayam Prakash, *Reactive power modulation in shunt FACTS controllers for damping power swings*, M.Sc(Engg). dissertation, Indian Institute of Science, Bangalore, India, August 2000.

[8] K.R.Padiyar, *Power system Dynamics - Stability and control*, Second edition, B.S.Publication,Hyderabad, 2002.