Power Allocation Policies for Convolutional and Turbo Coded Systems over Fading Channels

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Abstract— In this paper we study adaptive power allocation (PA) policies for improving the performance of convolutional and turbo codes on fading channels. The transmitter has an average power constraint. The fading process can be continuous (e.g. Rayleigh distribution). Perfect channel state information at the transmitter (CSIT) and the receiver (CSIR) are assumed. For convolutional codes, we consider block (slow) fading and fast fading environments separately and propose new PA policies that reduce the BER. We do a comparative study of the proposed PA policies with commonly used policies e.g., water filling, (truncated) channel inversion and an optimal policy proposed by Hayes for an uncoded system. For all the cases we study, we show that the proposed policies substantially outperform the commonly used policies. Among the previously known policies only Hayes' policy gives performance improvement over constant PA. We show that inter leaving with PA can improve the performance of coded systems on block fading channels significantly. We also make the important observation that the improvements in BER obtained with PA increase with SNR, which is in sharp contrast to the negligible gain in channel capacity obtained with PA [6]. Since direct optimization for turbo codes is difficult, we use the policies derived for convolutional codes on the constituent convolutional codes of turbo codes and show that significant performance improvements can be obtained.

Index Terms – **Power allocation, block fading, fast fading, inter leaving, convolutional codes, turbo codes.**

1. Introduction

Adaptive communication techniques that utilize the resources efficiently are of great interest in wireless communication. The basic motivation behind adaptive transmission is to obtain improvements in terms of average spectral efficiency or bit-error rate (BER) using the channel knowledge available at the transmitter. Adaptive techniques obtain these gains by varying transmitted power level, transmission rate, constellation size, coding rate/scheme or any combination of these parameters (while keeping the average transmit power fixed) [4].

Adaptive power transmission for binary antipodal signaling was studied in the early work of Hayes [7]. Recently [4] and [8] studied adaptive power transmission policies for MQAM and DPSK modulation schemes respectively. In [6] the optimal power policy that maximizes the channel capacity of fading channels is shown to be of waterfilling type. Information-theoretic power policies are also studied in [3]. Since all the above references study either information-theoretic optimal power policies (which are optimal for long random codes) or power policies for uncoded systems, none of them is guaranteed to work well for practical coded systems.

Efficient power allocation (PA) schemes for minimizing the information bit-error rate (BER) of fixed coded systems have not been considered until recently. In [1] the improved performances of fixed-rate convolutional and turbo coded systems with code-specific adaptive PA policies are reported for a finite-state Markovian fading channel. The policy obtained is shown to be quite different from the commonly used policies like waterfilling, channel inversion, etc., However a functional form of the solution is not obtained in [1] and the problem is (computationally) exponentially complex with the state space of the fading states and the free distance of the convolutional code. Also the solution obtained in [1] is not applicable for continuous fading channels.

In this paper we consider PA policies that improve the performance of fixed-rate convolutional and turbo coded systems over *continuous* fading channels. This problem is relevant to most of the practical systems, since a specific code and hence the rate, are usually fixed. We consider both block and fast fading scenarios. To obtain the optimal power allocation (OPA) policy analytically for a code, we need expressions for the BER of the given code and PA policy. Unfortunately no such expressions are available even for simple convolutional codes. We circumvent the problem by taking the first error event probability as the performance criterion. The policy obtained for fast fading can also be applied for block fading channels with interleaving.

We show that for all the cases, the proposed policies perform better than the previously known policies. We also show that interleaving with PA can significantly improve the performance of coded systems on block fading channels. In general, the OPA policy is code and fading process dependent. However we observe that the code-independent Hayes' policy [7] itself can be quite good on coded systems. See also [9] for new information theoretically optimal policies which perform well for specific coded systems.

It was observed in [6] that doing PA does not give any significant improvement in channel capacity for the Rayleigh fading channel. In this paper we show that the gains obtained in BER with PA are substantial and the gains increase with SNR. This

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shows that information-theoretic observations need not necessarily hold for practical coded systems.

In contrast to convolutional codes, turbo codes seem to have no well-defined analytical performance measures. Hence instead of attempting a PA policy for turbo codes directly, we use the PA policies that are proposed for convolutional codes on the constituent convolutional codes of turbo codes.

The rest of the paper is organized as follows. Section 2 describes the system model and some commonly used PA policies. Section 3 studies the problem for block fading channels. Section 4 considers fast fading channels. Section 5 concludes the paper.

2. SYSTEM MODEL AND COMMON POWER POLICIES

System Model

We consider a single user communication system. The symbols are transmitted over a fading channel with additive white Gaussian noise (AWGN). The fading gains are assumed to be perfectly known to the transmitter and the receiver. The fading process is assumed to be stationary and ergodic. Coherent BPSK signaling is assumed. With appropriate sampling, the discrete representation of the channel is

$$y_k = a_k\sqrt{s_k}\, x_k + n_k, \quad k = 0, 1, 2, \ldots, \tag{1}$$

where x_k is the output of the encoder, s_k is the transmitted power, a_k is the fading gain and n_k is a white Gaussian process with variance $\sigma_n^2 = N_0/2$. For simplicity we take $E[x_k^2] = 1$. The average power constraint is given by

$$\lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K} s_k \quad \leq \quad \bar{S}, \tag{2}$$

where $\bar{S}$ is the available average power.

Commonly Used Power Allocation Policies

Waterfilling (WF) [6] : It has been shown in [6] that the following algorithm, called waterfilling in time, maximizes channel capacity. The transmitted power $s_k = S(\gamma)$ is given as

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma}, & \gamma \geq \gamma_0, \\ 0, & \gamma < \gamma_0, \end{cases}$$

where

$$\gamma := a_k^2 \bar{S}/RN_0 \tag{4}$$

and R is the rate of the code. The term γ_0 is a constant chosen so that the average power constraint (2) is satisfied with equality.

Channel Inversion (CI) [6], [4]: This policy ensures that the receiver sees a constant SNR, i.e., $a_k^2 s_k = \beta$, where β is chosen to satisfy the average power constraint with equality. This is a commonly used policy in practice.

Truncated Channel Inversion (TCI) [6], [4]: When CI is applied for a channel whose fading level could be very low or zero with a positive probability (like the Rayleigh fading channel), one may require an infinite average power for CI. Then the average power constraint cannot be satisfied for any finite $\bar{S}$. Thus inversion of channel only for $a_k \geq \beta_1$, for some appropriate positive constant β_1 has also been considered. The transmitted power $s_k = S(\gamma)$ is then given as

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{\sigma}{\gamma}, & \gamma \geq \gamma_0, \\ 0, & \gamma < \gamma_0, \end{cases}$$

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where γ is defined in (4) and $\gamma_0 := \beta_1^2 \bar{S}/RN_0$. The term γ_0 (or equivalently β_1) is optimally fixed and then σ is chosen to satisfy the average power constraint.

Hayes' Policy (HP) [7], [8]: The OPA policy that minimizes the BER of an uncoded system on fading channels was studied in [7]. For BPSK signaling, the following OPA policy was derived:

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{\gamma} \ln \frac{\gamma}{\gamma_0}, & \gamma \geq \gamma_0, \\ 0, & \gamma < \gamma_0, \end{cases} \tag{6}$$

where γ is defined in (4) and γ_0 is chosen to satisfy the average power constraint.

We will compare WF, TCI and HP with our proposed PA policies in Sec. 5.

3. CODED SYSTEMS ON BLOCK FADING CHANNELS

In this section the OPA policy for minimizing the bit-error rate of a convolutionally coded system on a block fading channel is presented. By block fading we mean that the fading process is constant over one block of transmission and it is statistically independent between the blocks. Such a channel model is especially suitable for wireless communication systems with slowly moving terminals.

In the case of uncoded antipodal signaling on an AWGN channel, an analytical expression for the BER can be derived. Using this expression Hayes [7] minimized the average BER of the uncoded system subject to an average power constraint and found the OPA given by (6). Unfortunately exact BER expressions for coded systems on an AWGN channel are not available. The available upper bounds on BER are usually infinite summations and it is difficult to find PA policies using them.

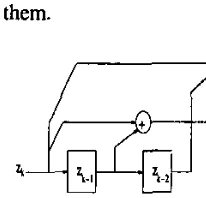

Figure 1. Convolutional encoder

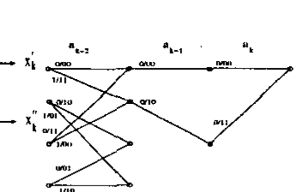

Figure 2. Trellis for the convolutional encoder, d_{free}=5

We circumvent the above difficulty by using the first-error event probability (FEEP) as the performance measure. Now we explain our approach with an example, but the approach is general.

Consider the rate-(1/2) convolutional code specified by the generator matrix $G(D) = \{1 + D^2, 1 + D + D^2\}$(see Figs 1,2). The coded bits are transmitted via BPSK with 0 as -1 and 1 as $+1$. Since we are assuming a block fading channel $a_{k-2} = a_{k-1} = a_k = a$ in Fig. 2. The FEEP for a given fade a is,

$$P(e_1|a) = \int_{\sqrt{\frac{5a^2S(a)}{\sigma^2}}}^{\infty} \frac{\exp(-\frac{x^2}{2})}{\sqrt{2\pi}} dx = Q\left(\sqrt{\frac{5a^2S(a)}{\sigma^2}}\right) \le \frac{1}{2}\exp\left(-\frac{5a^2S(a)}{2\sigma^2}\right). \tag{7}$$

The upper bound in the above inequality is quite tight. Therefore, we consider the following optimization problem:

$$\min \int_a \frac{1}{2}\exp\left(-\frac{5a^2S(a)}{2\sigma^2}\right) f_A(a)da. \tag{8}$$

subject to

$$\int S(a) f_A(a) da \le \bar{S}, \tag{9}$$

where f_A is the probability density of a. Define

$$L(\mu) = \int \left[\frac{1}{2}\exp\left(-\frac{5a^2S(a)}{2\sigma^2}\right) + \mu S(a)\right] f_A(a) - \mu\bar{S}, \tag{10}$$

where μ is a Lagrange multiplier. Minimize (w.r.t. S),

$$g(a, S, \mu)\frac{1}{2} = \exp\left(-\frac{5a^2S(a)}{2\sigma^2}\right) + \mu S(a), \tag{11}$$

for a given a. From

$$\frac{\partial g}{\partial S} = \frac{1}{2}\exp\left(-\frac{5a^2S(a)}{2\sigma^2}\right)\left(-\frac{5a^2}{2\sigma^2}\right) + \mu = 0 \tag{12}$$

we obtain

$$S(a) = \frac{2\sigma^2}{5a^2}\ln\left(\frac{5a^2}{4\sigma^2\mu}\right). \tag{13}$$

If we define $\gamma = \frac{5a^2}{2\sigma^2}$ and $\gamma_0 = \mu$, we get,

$$\frac{S(\gamma)}{\bar{S}} = \begin{cases} \frac{1}{\gamma}\ln\frac{\gamma}{\gamma_0}, & \gamma \ge \gamma_0, \\ 0, & \gamma < \gamma_0. \end{cases} \tag{14}$$

The solution (14) is same as (6), except that (14) depends on the code and the fading process, but (6) depends only on the fading process. Also, the dependency is only to the extent of finding constant γ_0 in (14) and in (6). We compare the performance of the above policy with the optimal policy minimizing BER(OPA) along with other policies provided in section 2. The convolutional code of Fig.1 has been used in Fig.3. In Fig. 3, $\frac{E_b}{N_0}$ is the bit energy to noise power ratio. The policy minimizing BER was obtained in [10] via a curve fitting method. The fading amplitude a_k is modeled with a Rayleigh pdf, $p_A(a_k) = 2a_k e^{-a_k^2}$, for $a_k \ge 0$. We assumed a block length of 200 coded bits for our simulations. For fair comparison of various policies we maintain a fixed transmission rate

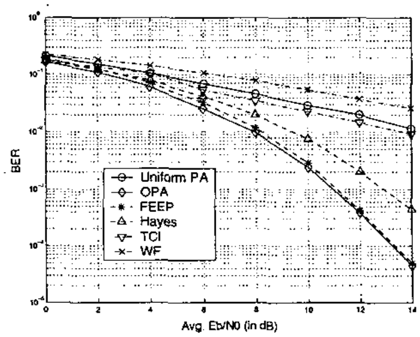

Figure 3. Comparison of the performances of various PA policies for the block fading Rayleigh channel.

and do not wait when the PA policy gives zero power; instead, we transmit the coded bits with zero power. We observe that the performance of OPA and (14) is very close to each other. We also notice that the OPA (and the FEEP) policies significantly outperform all other policies. Even at a BER of 0.02, SNR gain of 7 dB over constant PA is obtained. For lower BERs, the gains are substantial. Among the policies of Sec. 2.2 only HP gives good performance improvement over the constant PA policy. One can expect HP to be a good policy because it minimizes the coded BER and hence should reduce the information BER. WF performs poorer than constant PA, the reason being that WF is optimal for Gaussian signaling (and long random codes) and we are using binary signaling. Information-theoretic (IT) OPA for channels with binary inputs is studied in [9] and it is shown there that performance can be improved using such PAs as well, though gains are not as much as the policy derived here. TCI performs slightly better than constant PA. In Rayleigh fading environment deep fades can occur which causes TCI to allocate no power to deep fading states. On the other hand, TCI performs well in outage capacity sense, where one essentially gets an AWGN channel when there is no outage.

In Fig. 4 we have plotted the various power allocation policies for an average SNR of 3 dB. One can notice that the FEEP is quite different from the other policies. TCI and HP look qualitatively similar to FEEP, though HP initially increases and then decreases, whereas TCI monotonically decreases with the fading gain. WF is qualitatively very different from all the other policies; it is monotonically non-decreasing with the fading gain. Notice that HP and FEEP give power even when the fading gain is quite low. This is one reason for their good performance. Interleaving is usually employed on block fading channels to combat the ill-effects of fading. We will consider interleaved block fading channels in the next section.

4. Coded Systems on Fast Fading Channels

In this section we obtain code-specific PA policies, which minimize the BER of coded systems on fast fading channels. By fast fading we mean that the fading gain changes from symbol to symbol in an independent fashion. In Sec. 4.1 we consider convolutional codes. Sec. 4.2 studies turbo codes.

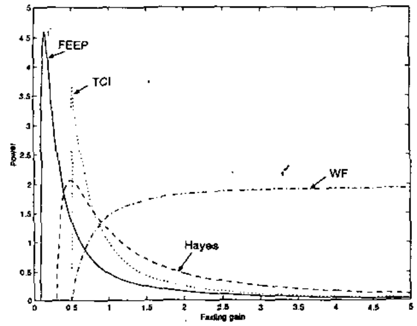

Figure 4. Comparison of various PA policies for the block fading Rayleigh channel.

Convolutional Codes

In this subsection we study the OPA problem for convolutional codes on fast fading channels. Like block fading channels, for fast fading channels too, there are no explicit expressions available for BER of convolutional codes. Hence instead of minimizing the BER of a convolutionally coded system directly, we minimize the first error event probability. We illustrate the scheme for the specific convolutional code (shown in Figs. 1,2) though the approach is general.

Consider the rate-half convolutional encoder shown in Fig. 1. Here for simplicity we assume that the channel fading gain a_k remains constant during transmission of x'_k and x''_k and the transmitted power s_k is same for both x'_k and x''_k. The coded bits x_k are mapped to BPSK symbols using the mapping $0 \mapsto -1$ and $1 \mapsto +1$. Corresponding to the transmission of (x'_k, x''_k), (y'_k, y''_k) is the received signal vector.

Fig. 2 shows the trellis diagram and the first error event when the all-zero codeword is transmitted. Given the fading gains and PAs till time k, the probability of the first error event e_1 to occur at time index $k \geq 2$ is

$$P(e_1|a_{k-2}, s_{k-2}, a_{k-1}, s_{k-1}, a_k, s_k) = \int_{\frac{\sqrt{\rho_k + 2a_k^2 s_k}}{\sigma_n}}^{\infty} \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}\, dx \tag{15}$$

where $\rho_k := 2a_{k-2}^2 s_{k-2} + a_{k-1}^2 s_{k-1}$, and σ_n^2 denotes the AWGN variance.

For a given $\rho_k = \rho$ we minimize $P(e_1)$ subject to the average power constraint $\bar{S}$. Thus if $\rho_k = \rho$ we can obtain the following equation via Lagrange multipliers:

$$\frac{2a_k^2}{\sqrt{2\pi\sigma_n^2(\rho_k + 2a_k^2 s_k)}} \exp\left(-\frac{\rho_k + 2a_k^2 s_k}{2\sigma_n^2}\right) = \lambda_\rho. \tag{16}$$

Given distribution of a_k, we can obtain the λ_ρ that satisfies the average power constraint $E[s(a_k)|\rho_k = \rho] = \bar{S}$ using (16). Then we obtain $s(a)$ for each a from (16).

Now we study the performance of this PA policy on the fast fading Rayleigh channel. In Fig. 5 we have shown the performances of the policies of Sec. 2.2 along with the proposed policy. We notice from the figure that the proposed PA policy performs better than all the other policies. An SNR gain of 1.5 dB compared to the constant PA case is obtained at a BER of 10^{-3}. HP performs slightly worse than the proposed PA policy and gives an SNR gain of 1.2 dB at a BER of 10^{-3}. On the other hand, WF performs worse than the constant PA case and TCI performs almost the same as the constant PA case. The reasons are the same as those given in Sec. 3.

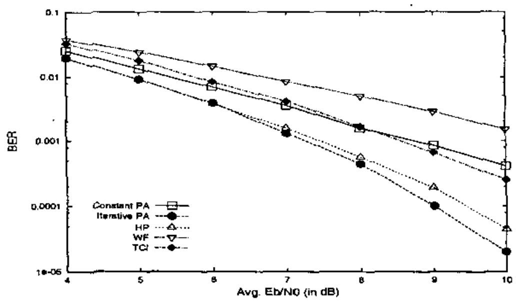

Figure 5. Comparison of various PA policies for the fast fading Rayleigh channel.

Now we apply this policy for interleaved block fading channel. In Fig. 6 we have analyzed the performances of interleaving, proposed PA and proposed PA with interleaving for the block fading Rayleigh channel. We have done interleaving over four coded symbol blocks. One can see from the figure that the proposed policy on an interleaved system performs the best. It gives an improvement of 4.3 dB over the system with interleaving alone at a BER of 10^{-3}. HP with interleaving is 1 dB worse than this scheme. We see from the figure that interleaving itself can provide substantial performance improvement. Thus one can conclude that for block fading channels using OPA with interleaving gives the best performance; HP with interleaving can also give good improvements. (We did simulations with WF and TCI and found they do not perform well for the interleaved case too.)

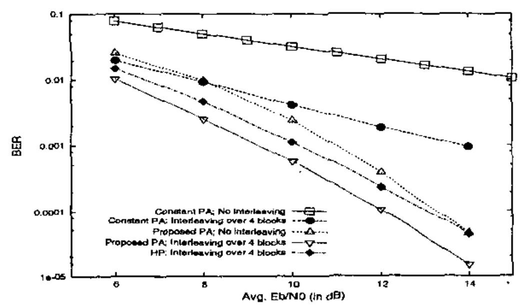

Figure 6. Comparison of interleaving and PA policies for the block fading Rayleigh channel.

An important observation to make from Figs. 3, 5 and 6 is that the BER improvement obtained with PA increases with SNR and the gains can be substantial even at moderate SNRs. This is in direct contrast with the information-theoretic result that PA does not provide any significant increase in channel capacity for Rayleigh fading channel for moderate and high SNRs as was observed in [6]. Hence one can conclude that PA helps to improve the performance of practical coded systems.

Turbo Codes
Turbo codes are parallel concatenated convolutional codes (PCCCs) that offer near Shannon limit performance [2]. Turbo codes usually have large block lengths, so it is inappropriate to assume that fading remains constant over one block. Hence we study turbo codes only for fast fading channels (or interleaved block fading channels). Since direct optimization of transmit power for turbo codes is difficult and turbo codes are concatenation of two convolutional codes, we apply the PA policies proposed for convolutional codes on the constituent convolutional codes of the turbo code.

From Fig. 5, one can notice that the performance of HP on a convolutional code on a fast fading channel is essentially the same as our proposed PA policy for SNRs less than 6 dB. Since turbo codes are usually used at low SNRs, where the two policies perform the same, only HP is studied. The other commonly used PA policies are not studied, since they do not perform well for convolutional codes themselves. (We indeed verified that WF and TCI do not perform well for turbo codes, though we do not present the simulation results here.)

In Fig. 7 we have compared the performances of constant PA and HP for the 7/5 unpunctured turbo code on a fast fading Rayleigh channel for the first eight turbo decoding iterations (or sixteen half iterations) at an SNR of 1.2 dB. The interleaver used was a random interleaver of size 1000. The turbo decoder employed constituent *a posteriori* probability (APP) decoders. It is clear from the figure that the performance of the turbo decoder improves significantly when PA is used. From the figure one can see that PA can help in achieving a target BER in less number of iterations and can also help in reducing the BER that can be achieved at a particular E_b/N_0 with number of iterations (the improvement is substantial as in Fig. 7 when the operation is in the *waterfall* region). This also means that one can reduce the SNR threshold of turbo codes by PA.

We also did simulations with other codes for various SNR values and found that PA can lead to substantial performance improvements.

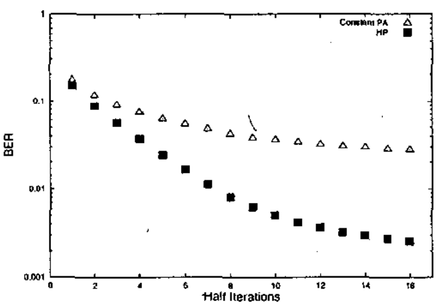

Figure 7. Comparison of constant PA and HP for the 7/5 turbo code on the fast fading Rayleigh channel at an SNR of 1.2 dB.

5. Conclusions and Future Work

In this paper we studied PA policies that are designed to reduce the BER of practical fixed-rate coded systems. It is shown that such code-specific PA policies significantly outperform all previously known PA policies. The OPA policy derived here depends on the code and the fading distribution. Instead one can use HP, which is code-independent and gives good performance improvement, though not as much as the policies derived here. Our results further indicate that doing interleaving with PA can improve the performance by a few dB of SNR for the block fading channel.

We also made the important observation that the improvement obtained in BER with PA increases with SNR and can be substantial even at moderate SNRs. This shows that the observation made in [6] with respect to channel capacity is not a guideline whether to use or not use PA for practical coded systems.

This work can be extended in several directions. It will be interesting to study the effects of imperfect and/or delayed feedback on system performance. Also one can extend the ideas of this paper to space-time codes. Some of these results will be presented in our paper under preparation.

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