

STUDY OF DENDRITE GROWTH IN A ROTATIONAL FLOW FIELD

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ABSTRACT

A numerical model to study the growth of dendrites in a pure metal solidification process with an imposed rotational flow field is presented. The micro-scale features of the solidification are modeled by the well-known enthalpy technique. The effect of flow changing the position of the dendrite is captured by the Volume of Fluid (VOF) method. An imposed rigid-body rotational flow is found to gradually transform the dendrite into a globular microstructure. A parametric study is carried out for various angular velocities and the time for merger of dendrite arms is compared with the order estimate obtained from scaling.

Keywords: Dendrite growth; enthalpy method; stirring; transformation; microstructure

INTRODUCTION

Study of microstructures in materials is important for several reasons; some of which are enhancement of mechanical properties and reduction of defects. Grain size is one parameter to evaluate the quality of any cast component. Dendrites are typically highly anisotropic and unstable in nature which can form complex interfacial boundaries and reduce the tensile strength of the component. Equiaxed dendritic microstructures are desired when the components need to possess good tensile and fatigue

strengths. Their morphology is significantly affected by fluid flow. It has been found that globular microstructures are more effective in improving the mechanical properties and hence studies of various kinds have been undertaken to understand such aspects, (Flemings, 1974). It is known that the growth of the equiaxed dendrites is altered by the fluid flow and most of the studies report uniform flow approaching the dendrite, where the arm facing the flow undergoes larger growth rates compared to other arms, (Beckermann et al., 1999). A rotational flow field exists in scenarios such as stirring and it would be useful to see how such a flow would alter the morphology, (Barman et al., 2009).

In this study, effect of rotational flow field on the dendritic microstructures is examined numerically. Firstly, a simple scaling analysis is carried out using the results from studies on dendritic growth in presence of uniform flow by (Bhattacharya, 2007 & Al-Rawahi and Tryggvason, 2002). An estimate of time scale for lateral merging of two dendritic arms is obtained by assuming constant angular velocity of rotation. Governing equations for mass, momentum and energy equations are solved in a two-dimensional framework. For tracking the growth of solid-liquid interface, an enthalpy based explicit technique is utilized, (Voller, 2008

and Pal et al., 2006). The change in position of the interface due to the fluid flow is captured by the Volume of Fluid (VOF) method, (Gerlach et al., 2006). The time taken in the simulation for change in the dendritic morphology is compared with the order estimate obtained from scaling analysis.

DESCRIPTION OF THE SYSTEM

We consider a square domain with an undercooled melt for a pure metal solidification. The dendrite is assumed to grow from the center which coincides with the axis of rotation. A schematic of the system is shown in Fig. 1.

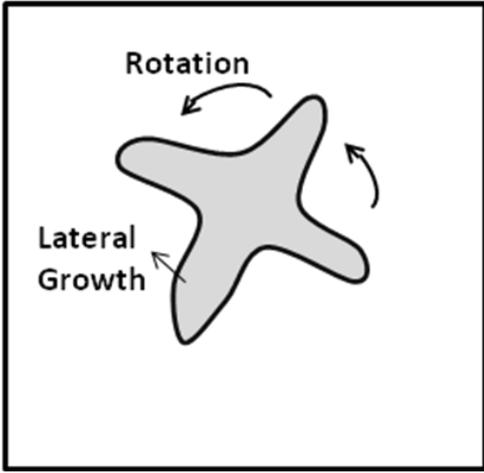


Figure 1. Schematic of the system

MATHEMATICAL FORMULATION

Using the principles of volume-averaging, single fluid conservation equations are written for the system described above. Following (Bhattacharya, 2007 & Voller, 2008), the governing equations are non-dimensionalized and are stated below. The mass conservation can be written as

$$\nabla^* \cdot (\vec{u}^*) = 0 \quad (1)$$

The momentum conservation equation is given by

$$\frac{\partial}{\partial t^*} (\vec{u}^*) + \nabla^* \cdot (\vec{u}^* \vec{u}^*) = \nabla^* \cdot (\text{Pr} \nabla^* \vec{u}^*) - \nabla^* p^* \quad (2)$$

The energy conservation equation is written using the enthalpy formulation. Writing the specific enthalpy as $h = cT + f_l L$ and non-

dimensionalizing, the energy equation simplifies to,

$$\frac{\partial}{\partial t^*} (h^*) + \nabla^* \cdot (\vec{u}^* h^*) = \nabla^* \cdot (K^* \nabla^* T^*) \quad (3)$$

Here the variables are, fluid velocity \vec{u} , pressure p , Prandtl no. Pr , thermal conductivity K , temperature T and enthalpy h . The term f_l refers to the liquid volume fraction. The superscript $*$ denotes dimensionless quantities. The details of non-dimensionalizing can be found in (Voller, 2008). For a rigid body vortex, the velocity is given by $\vec{u} = \vec{\omega} \times \vec{r}$. In a two-dimensional framework for a fixed angular velocity ω and a given point at a distance r from the axis of rotation, this reduces to a tangential component given by $U_f = \omega r$, which can be decomposed to obtain its components in x and y directions.

The non-dimensional temperature at the interface ($0 < f_l < 1$) is given by $T_i^* = -\kappa^* d(\theta) / d_0$ where κ^* refers to non-dimensional curvature and anisotropic surface tension is given by $d(\theta) = d_0(1 - 15\varepsilon \cos 4\theta)$ (Voller, 2008). Here ε is the anisotropic strength, d_0 is the length-scale for the dendritic growth and θ is the interface angle. However, for a rotating dendrite, the interface angle is modified to the form $\theta - \phi$, where ϕ is the angle swept by the dendrite in $[0, 2\pi]$.

Motion of dendrites is governed by the flow field, which is obtained by solution of mass and momentum conservation equations, (Eq. (1)-(2)). This motion of a phase can be modeled using the VOF technique, (Gerlach et al., 2006), in which the phase fraction is advected, preserving its shape and conserving its mass. Consider the volume fraction of the liquid phase,

$$f_l = \begin{cases} 0 & \text{if the cell is solid} \\ 1 & \text{if the cell is liquid} \end{cases} \quad (4)$$

Since the fluid type remains the same along the particle path, the volume fraction is advected by

$$\frac{\partial f_l}{\partial t^*} + \vec{u}^* \cdot \nabla^* (f_l) = 0 \quad (5)$$

SCALING FOR DENDRITE GROWTH IN A ROTATIONAL FLOW FIELD

Dendrite growth behavior is known to alter in the presence of fluid flow, (Al-Rawahi and Tryggvason, 2002). The arm facing the flow is found to grow faster as compared to its counterpart, while those perpendicular to the flow are mostly unaffected. This is due to the changes in thermal and solutal boundary layers around the upstream and downstream arm tips. A rotational flow field creates a different scenario when both dendrite and fluid are in motion. As a result, all the dendritic arms face flow directed towards them on one side and the net heat transfer is increased. This causes the dendrite arms to laterally grow and the gradual change in the shape of the dendrite from an expected tree-like four armed structure. A representation is shown in Fig. 2.

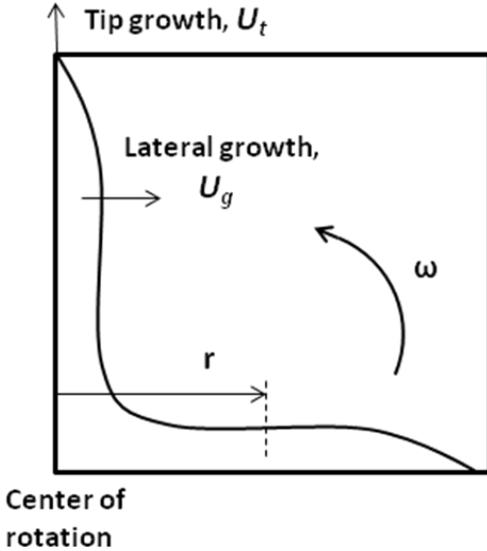


Figure 2. A representative picture showing parameters and section between two perpendicular arms

An order estimate of time taken for the dendrite to change its morphology to a nearly globular microstructure is obtained based on estimates of flow, rotational speed and growth velocity. Since literature on lateral growth rates of dendrite is not available, we assume that the lateral growth is affected by the flow field in the same manner as that of the tip. Consider a two-dimensional representation of dendrite undergoing

simultaneous rotation and growth, as shown in Fig. 2. For it to become globular, the lateral growth of the arm should extend till its neighbor. This would give us an order estimate of time taken for this transformation. Assume that the center of mass of the dendrite coincides with the axis of rotation. Also the curvature effects on the lateral surfaces of the dendrites can be neglected. Let 'r' be the radius from the center to any given location on the arm. Length of the lateral growth for the equiaxed structure to become circular can be approximated as $r\left(\frac{\pi}{2}\right)$. Let U_g be the lateral growth velocity, which is initiated by the melt flow perpendicular to the tip direction. The order estimate of this is obtained from (Bhattacharya, 2007 and Al-Rawahi and Tryggvason, 2002). Now the time taken for the solidifying front to cover the distance between two arms can be estimated as follows.

$$r\left(\frac{\pi}{2}\right) \approx U_g \times t$$

$$t \approx \frac{r\left(\frac{\pi}{2}\right)}{U_g} \quad (6)$$

From the results in literature, (Bhattacharya, 2007 and Al-Rawahi and Tryggvason, 2002), the lateral growth velocity is proportional to the melt flow velocity which is simply the radial distance times the angular velocity. Hence we can write $U_g \approx kU_f \approx kr\omega$ the equation becomes,

$$t \approx \frac{r\pi}{2kr\omega} \approx \frac{\pi}{2k\omega} \quad (7)$$

Equation (7) gives the order estimate of the time-scale for the change in morphology.

Now, this scaling relation is valid only when the dendrite tip speed is not large enough to overcome the lateral growth, i.e., $U_t < U_g$. The tip speed can be estimated in terms of melt undercooling ΔT and the above condition can be evaluated in terms of ΔT and ω .

From (Bhattacharya, 2007 & Al-Rawahi and Tryggvason, 2002), we have the tip velocity increasing with the flow velocity. Assuming an approximate correlation of type, $U_{gt} = kU_f$,

where U_f is the melt flow velocity (approaching the dendrite tip). The constant k can be approximated to be $\frac{1}{3}$. Therefore, the time-scale estimate simplifies to

$$t \approx \frac{\pi}{2k\omega} \approx \frac{9.5}{2\omega} \approx \frac{O(5)}{\omega} \quad (8)$$

NUMERICAL RESULTS

The dendrite growth code based on the enthalpy method has been validated in (Bhattacharya, 2007). In this study we present the numerical results for the dendrite growth in presence of a vortex field. Firstly, solidification is ignored and the dendrite motion is simulated to verify the correctness of Volume of Fluid implementation. Figure 3 shows initial and final shape after 10000 time steps of clockwise and anticlockwise rotations. It can be seen that the shape of the dendrite has remained intact and the mass lost is negligible, i.e., $10^{-4}\%$.

In this section, the dendrite undergoes simultaneous motion and growth until a change in transformation occurs. To verify whether the dendrite has attained a near circular shape, the deviation from the mean value of the distances between the center and the interface locations is calculated and a value of 10^{-3} is kept as the accuracy limit for the ratio of the deviation to the mean distance.

Figure 4 shows the dendrite undergoing simultaneous rotation and growth and its eventual transformation into a circular shape. An initial undercooling of -0.40 is taken and an angular velocity of 0.001 is imposed.

A parametric study is performed to obtain the time taken for the dendrite to become nearly circular in shape. The angular velocity is varied for a given initial melt undercooling and the time taken for the transformation is plotted with the angular velocity. This is compared with the scaling relation obtained in Eq. (8). Figure 5

For eg., for a dimensionless angular speed of 10^{-3} , the approximate time for merging of the arms is 10^4 .

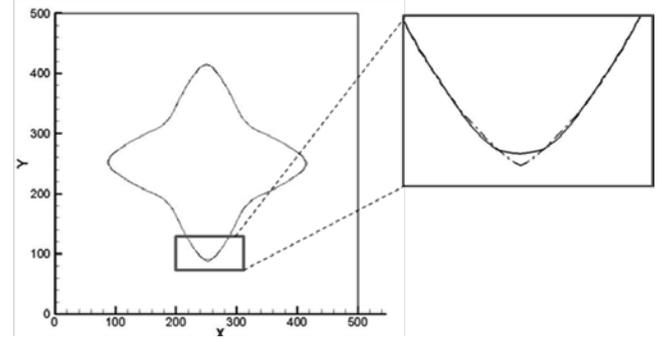


Figure 3: Dendrite rotation in the absence of solidification. The difference between the dendrite shape before and after rotating is shown in the magnified part.

shows the comparison and the reasonable agreement with the scaling analysis.

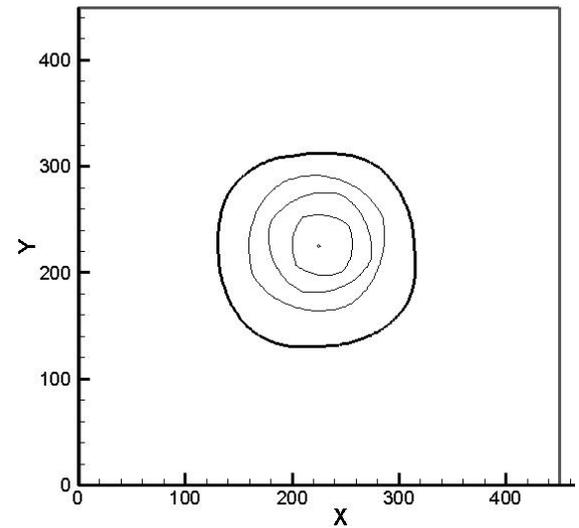


Figure 4: Dendrite growth in presence of a rotational flow field. The shape and position are shown at times - 0, 2000, 4000, 6000 and 10000.

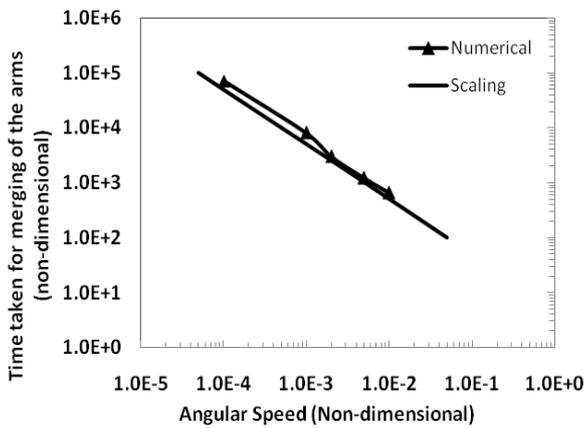


Figure 5: Time taken for merging of the arms. Comparison between values from the scaling analysis and numerical results.

CONCLUSIONS

An existing enthalpy based numerical model for the dendrite growth is utilized to study the effects of rotational flow field. A scaling criterion is obtained for the estimate of order of magnitude of time required for the dendrite to change its morphology due to flow. The transformation is demonstrated and reasonable comparison with the scaling analysis is achieved. Future work would include the assessment of the transformation in presence of stronger melt undercooling. This study gives a practical insight towards estimating the forces needed for stirring based on the dendrite size to obtain change in the morphology.

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