

Crow instability in unitary Fermi gas

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We investigate the initiation and subsequent evolution of Crow instability in an inhomogeneous unitary Fermi gas using zero-temperature Galilei-invariant non-linear Schrödinger equation. Considering a cigar-shaped unitary Fermi gas, we generate the vortex-antivortex pair either by phase-imprinting or by moving a Gaussian obstacle potential. We observe that the Crow instability in a unitary Fermi gas leads to the decay of the vortex-antivortex pair into multiple vortex rings and ultimately into sound waves.

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I. INTRODUCTION

A vortex-antivortex pair in a three dimensional inviscid fluid is susceptible to the long wavelength cooperative instability known as Crow instability (CI) [1]. In an undisturbed vortex-antivortex pair, each vortex feels the velocity field produced by the other. As a result of it, the pair moves in the direction perpendicular to the plane containing the pair. Now, some initial disturbance in the fluid can lead to the formation of the sinusoidal distortions in the vortex shape, which are symmetric with respect to the plane separating the two vortices. In this case, the total velocity field experienced by the vortex (antivortex) has three contributions: (a) velocity field induced by the vortex (antivortex) on itself due to its curvature, (b) the straining experienced by the vortex (antivortex) when displaced from its equilibrium position in the velocity field induced by the undisturbed antivortex (vortex), and (c) the velocity field induced on the vortex (antivortex) by the sinusoidally distorted antivortex (vortex). The strain rate is directly proportional to circulation of the vortex and inversely proportional to square of the distance between the vortices. The first and the sum total of the last two contributions are also termed as the velocity field produced due to the self and mutual-induction respectively. In the absence of the second vortex and hence mutual-induction, the sinusoidal displacement disturbance travels along the vortex line and rotates about its unperturbed position, thus distorting the vortex line into a helical shape. These helical waves, which are neutrally stable, propagating along the length of the vortex line are also known as Kelvin waves [2]. In the presence of the second vortex, the self induced rotation can be decreased or increased due to the azimuthal velocity component of strain field. When the two balance each other, the unbalanced radial component of the strain field leads to the exponential growth of the displacement perturbation; this is known as CI. As the amplitude of these sinusoidal oscillations reaches a critical value, the vortex and antivortex reconnect leading to the formation of a chain of vortex rings. It was shown by Crow that a vortex line in the presence of only strain field will be unconditionally unstable, whereas it will be neu-

trally stable of only self induction were present [1]. The interplay between these two opposing effects leads to the CI. The CI is also responsible for the decay of wing-tip generated vortex-antivortex pair in the wake of the aircraft [1]. The CI in this case can be triggered by the turbulence in surrounding air or local variation in the air temperature or density. It was shown by Kuznetsov et al. [3], that only the long-wavelength symmetric modes are unstable due to CI. The long-wavelength antisymmetric modes, on the other hand, are stable [3]. The typical wavelength of the unstable modes is much larger than the size of the vortex core [1].

The CI can occur not only in classical fluids but also in superfluids. The characteristic feature of the superfluids is the quantized circulation in them [4, 5]. Hence, the realization of quantized vortices in Bose-Einstein condensates (BECs) [6–8] and two-component Fermi superfluids [9] has established the superfluid nature of these systems.

In case of homogeneous BECs, the CI has been investigated theoretically in Refs. [3, 10]. In this context, Kuznetsov et al., [3] demonstrated the long wavelength instability of acoustic solitons and vortex pairs in the frame work of three dimensional Schrödinger equation. It was noted in this study that in case of solitons, the instability corresponds to snake instability [11], while in case of vortex-antivortex pairs it takes the form of CI. Recently, CI in cigar shaped BECs was theoretically studied in Ref. [12]. On the experimental front, the snake instability of the solitons leading to the formation of vortex rings has been observed [13, 14], whereas the CI is still eluding the experimental observation. The CI is inhibited in oblate BECs. This is due to the suppression of bending and reconnections of vortices in oblate BECs [15]. Also, the generation of Kelvin waves [16, 17], which aids the development of the CI, is suppressed in oblate condensates [18].

In case of two-component Fermi gas, the attractive interaction between the two component spin states is a prerequisite for superfluidity. Depending upon the value of s -wave scattering length and Fermi wave vector k_F , these superfluids can be weakly interacting $k_F|a| \ll 1$ or strongly interacting $k_F r_e \ll 1 \lesssim k_F|a|$, where r_e is the effective range of interatomic interaction poten-

tial. The two limiting cases, $k_F|a| \rightarrow 0$ ($a \rightarrow -0$) and $k_F|a| \rightarrow \infty$ ($a \rightarrow -\infty$) are known as Bardeen-Cooper-Schrieffer (BCS) and unitary limits respectively. The inverse of Fermi wave vector k_F is of the order of average interparticle separation. Thus, the effective range is much less than the interparticle separation which in turn is much less than s -wave scattering length in the unitary limit. It implies that the unitary Fermi gas (UFG) is simultaneously dilute ($r_e \ll k_F^{-1}$) and strongly interacting ($k_F^{-1} \ll |a|$). In the unitary Fermi gas (UFG), the thermodynamic quantities become the universal functions of Fermi energy E_F and the ratio of temperature to Fermi temperature T/T_F [19]. The generation, real time evolution, and interaction among the quantized vortices has been studied in UFG using the time-dependent superfluid local density approximation at zero-temperature [20]. The method involves the computationally intensive task of solving time dependent coupled Bogoliubov-de Gennes equations self-consistently. An alternative method based on the single orbital time dependent density functional theory has been developed in Refs. [21–35] to study the superfluid Fermi gases. Employing this approach, the UFG is well described by a non-linear Schrödinger equation (NLSE) at $T = 0\text{K}$, which leads to same superfluid density as the original many body system. The NLSE is appropriate to study UFG, provided the characteristic wavelength of the phenomenon under study is larger than the coherence length [35]. The NLSE has been used to study various phenomena like collective modes [21–23], expansion dynamics [21, 24, 25], solitons [26, 27], Josephson oscillations [28], shock wave generation [36, 37], etc. Recently, NLSE has also been used to investigate the soliton mediated decay of shock waves into vortex dipoles in the oblate BCS superfluids [38]. In addition to all these, the Bose-Fermi superfluid mixtures can be described by the Gross-Pitaevskii equation (GPE) for the bosons, coupled to the NLSE for the order parameter of the Fermi superfluid [27, 30]. Such coupled equations have been used to study gap solitons [27], mixing-demixing transition [39], the spontaneous symmetry breaking of the Bose-Fermi mixture in double-well potentials [40], localization of a Bose-Fermi mixture in a bichromatic optical lattice [41], etc. The NLSE with quintic repulsive nonlinearity has also been derived to describe one dimensional Tonks-Girardeau (TG) gas [42], and was used to study the formation of shocks and their dynamics [43]. The same equation, with an additional nonlocal cubic term accounting for the dipole-dipole attraction, has been used to study existence, stability and dynamics of bright solitons in dipolar one dimensional TG gas [44].

In the present work, we theoretically study the CI in strongly interacting Fermi superfluids using NLSE. The paper is organized as follows. In Sec. II, we describe the NLSE used to study the UFG. In Sec. II, we study the dynamics of a single vortex in cigar shaped UFG. In Sec. IV, we study the CI in a pair of vortices which are generated in the UFG by imprinting the appropriate phase. In the following Sec. V, we investigate the generation

of vortex-antivortex pair by a Gaussian obstacle potential, which on subsequent evolution decays via CI into vortex rings and sound. We conclude by presenting the summary of the results in the last section.

II. NON-LINEAR SCHRÖDINGER EQUATION FOR UNITARY FERMIONIC GAS

The order parameter or the condensate wavefunction for the superfluid two component fermionic system, excluding the normalization factor, is

$$\psi(\mathbf{r}, t) = \langle \hat{\psi}_\downarrow(\mathbf{r}, t) \hat{\psi}_\uparrow(\mathbf{r}, t) \rangle, \quad (1)$$

where $\psi_\uparrow(\mathbf{r}, t)$ and $\psi_\downarrow(\mathbf{r}, t)$ are the fermionic annihilation field operators for \uparrow and \downarrow spin states respectively. The order parameter can be interpreted as the wavefunction of the macroscopically occupied two particle state. The order parameter is normalized to total number of condensed Cooper pairs [19],[45–47]. As mentioned earlier, the s -wave scattering length between two component spin states $a \rightarrow -\infty$ in UFG. The Lagrangian of the UFG with energy E and consisting of equal number of two spin states at $T = 0\text{K}$ is [21],[24], [25], [27], [30], [35]

$$L = \int d\mathbf{r} \frac{i\hbar}{2} \left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t} \right) - E. \quad (2)$$

The energy of the UFG is given by

$$E = \int \left[\frac{\hbar^2}{4m} |\nabla \psi|^2 + \xi \frac{3\hbar^2}{5m} (3\pi^2)^{2/3} |\psi|^{10/3} + V(\mathbf{r}, t) |\psi|^2 \right] d\mathbf{r}, \quad (3)$$

where m is the atomic mass of the fermionic species, $\xi = 0.44$ [30], and $V(\mathbf{r}, t)$ is the trapping potential. Using the action principle

$$\delta \int_{t_1}^{t_2} L dt = 0, \quad (4)$$

the time dependent NLSE describing the UFG at zero temperature is

$$\left[-\frac{\hbar^2}{4m} \nabla^2 + \xi \frac{2\hbar^2}{m_p} (3\pi^2)^{2/3} |\psi(\mathbf{r}, t)|^{4/3} + V(\mathbf{r}, t) \right] \psi(\mathbf{r}, t) = i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t}, \quad (5)$$

where $m_p = 2m$ is the mass of the fermionic pair. The order parameter ψ satisfies the normalization condition

$$\int |\psi(\mathbf{r}, t)|^2 d\mathbf{r} = N. \quad (6)$$

In the present work, we consider cigar-shaped trapping potential

$$V(\mathbf{r}, t) = \frac{m_p \omega^2}{2} (x^2 + y^2 + \alpha^2 z^2), \quad (7)$$

where ω is the radial trapping frequency, $\alpha < 1$ is the ratio of axial to radial trapping frequency. We consider UFG of ^{40}K with $N = 3500$, $\omega = 100\text{Hz}$, and $\alpha = 0.2$. In Refs. [33, 34], effective one dimensional (1D) NLSE for the cigar-shaped Fermi superfluids in the unitary limit has been proposed. The 1D NLSE provides a good approximation to the real three dimensional (3D) systems, provided the dynamics along radial direction is frozen. The CI instability being a three dimensional instability can not be studied using quasi-1D equation. Hence, in the present work, we consider the three dimensional equation and solve it numerically using the split time step Crank-Nicolson method [48]. In this method, the NLSE is discretized in space and time using the finite difference scheme. The time iteration is performed by splitting the Hamiltonian into two parts, one containing the spatial derivative part and the other containing the rest of the terms. The error involved in the splitting of the the Hamiltonian is proportional to the square of the time step.

We can rewrite the Eq. (5) in scaled units using the transformations

$$\begin{aligned} \mathbf{r} &= \mathbf{r}' a_{\text{osc}}, \quad t = t' \omega^{-1}, \\ \psi(\mathbf{r}, t) &= \frac{\sqrt{N} \phi(\mathbf{r}', t')}{a_{\text{osc}}^{3/2}}, \end{aligned} \quad (8)$$

where the primed quantities are in scaled units and $a_{\text{osc}} = \sqrt{\hbar/(m_p \omega)}$ is the oscillator length. After dropping the primes, the scaled NLSE describing the superfluid UFG is

$$\begin{aligned} \left[-\frac{\nabla^2}{2} + 2\xi(3\pi^2 N)^{2/3} |\phi|^{4/3} + V(\mathbf{r}, t) \right] \phi(\mathbf{r}, t) \\ = i \frac{\partial \phi(\mathbf{r}, t)}{\partial t}, \end{aligned} \quad (9)$$

This equation is used to study the dynamics of UFG in the present work. We use $\Delta x = \Delta y = \Delta z = 0.15$ and $\Delta t = 0.001$ as the spatial and time step sizes. The number of time iterations used in the present work is 55000. In terms of unscaled units, the spatial and time step sizes are $0.169 \mu\text{m}$ and $1.59 \times 10^{-3} \text{ms}$ respectively.

III. DYNAMICS OF A SINGLE VORTEX LINE

In case of BECs, the dynamics of single vortex line in prolate condensates was studied experimentally in Ref. [49]. It was observed that vortex line in most cases was bent as was predicted in earlier theoretical studies [50–55]. It was shown by Svidzinsky et al. [54] that a straight vortex line is stationary only if it is oriented

along z -axis. The stability of the vortex line in BECs can be inferred from the normal modes, which can be obtained by solving the equation obtained after linearizing the Gross-Pitaevskii equation about the stationary configuration. In oblate condensates with a vortex line near z -axis there is one normal mode with negative frequency and it gives the precession frequency of the vortex in trap. On the other hand, the appearance of more than one normal modes with negative frequency in cigar shaped condensates, makes the vortex line increasingly susceptible to bending[54]. In case of cigar-shaped UFG, a similar instability against bending can aid the reconnection between the vortex and the antivortex line. Hence, before studying the dynamics of vortex-antivortex pair, it can be quite instructive to study the dynamics of a single vortex line in prolate UFG.

We numerically generate the vortex line by phase-imprinting method, in which the order parameter is evolved in imaginary time under the constraint that it has the phase consistent with the presence of the vortex line at all times. To do so, while evolving the NLSE in imaginary time $\tau = it$, the order parameter $\phi(\mathbf{r}, \tau)$ is redefined as

$$\phi(\mathbf{r}, \tau + \delta\tau) = \phi(\mathbf{r}, \tau) \exp \left[i \left(\tan^{-1} \frac{y - y_0}{x - x_0} \right) \right] \quad (10)$$

after each iteration in imaginary time with the temporal step size of $\delta\tau$. Here, (x_0, y_0) is the point of intersection of the vortex line with xy plane, and $\tan^{-1}[(y - y_0)/(x - x_0)]$ is the azimuthal angle measured with the origin shifted to $(x_0, y_0, 0)$. This redefinition of the order parameter ensures that the solution of the NLSE always has vortex line passing through $(x_0, y_0, 0)$ and oriented parallel to z -axis. The solution obtained by this method is then evolved in real time to study the dynamics of the system. We first consider the dynamics of the UFG with an axial vortex line. In this case, the vortex line remains straight, and hence is stable against the bending instability as is shown in Fig. 1. This is not the case for an off-axis vortex line, which tends to bend as it traverses across the condensate. This is evident from Fig. 2, where the upper and lower panel show the dynamics of the vortex line initially located at $x = 1.0 a_{\text{osc}}$ and $x = 3.0 a_{\text{osc}}$ respectively. Hence, like in BECs, an off-axis vortex line is unstable against bending in cigar-shaped UFG.

IV. CI IN VORTEX-ANTIVORTEX PAIR GENERATED BY PHASE-IMPRINTING

We can extend the phase-imprinting method, described in the previous section, to the vortex-antivortex pair. In order to numerically imprint a vortex-antivortex pair, phase contribution from both the vortex and antivortex has to be imprinted on the order parameter after each iteration in imaginary time. This can be ensured by re-

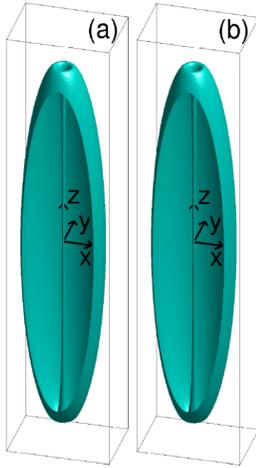


FIG. 1. The time evolution of the UFG with a single axial vortex line. The images are the isosurfaces corresponding to $|\phi(\mathbf{r})| = 0.012 a_{\text{osc}}^{-3/2}$. Images with the labels (a) and (b) are the isosurfaces after 0 ms and 79.6 ms of time evolution respectively. The origin of the coordinate system is located at the center of the box in each image. The dimensions of each box are $12.15 a_{\text{osc}} \times 8.55 a_{\text{osc}} \times 54.15 a_{\text{osc}}$

defining order parameter $\phi(\mathbf{r}, \tau)$ as

$$\begin{aligned} \phi(\mathbf{r}, \tau + \delta\tau) &= \phi(\mathbf{r}, \tau) \exp \left[i \left(\tan^{-1} \frac{y - y_1}{x - x_1} \right) \right] \\ &\quad \times \exp \left[-i \left(\tan^{-1} \frac{y - y_2}{x - x_2} \right) \right] \\ &= \phi(\mathbf{r}, \tau) \exp \left[i \left(\tan^{-1} \frac{y - y_1}{x - x_1} \right. \right. \\ &\quad \left. \left. - \tan^{-1} \frac{y - y_2}{x - x_2} \right) \right] \end{aligned} \quad (11)$$

after each iteration in imaginary time with the temporal step size of $\delta\tau$. Here $\tan^{-1}[(y - y_1)/(x - x_1)]$ and $\tan^{-1}[(y - y_2)/(x - x_2)]$ are the azimuthal angles measured with the origin shifted to $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ respectively. The converged solution obtained by this method will have a vortex and antivortex at (x_1, y_1) and (x_2, y_2) respectively. This solution is then evolved in real time to study the CI. As a case study, we imprint the vortex and antivortex at $(-a_{\text{osc}}, -a_{\text{osc}})$ and $(a_{\text{osc}}, -a_{\text{osc}})$ respectively. The stationary solution obtained by imaginary time propagation is shown in Fig. 3(a); here the distance between the two parallel vortex lines is equal to $2a_{\text{osc}}$, and the origin of the coordinate system is located at the center of the box. The solution is then evolved in the real time using Eq. (9). During the initial stages of the evolution vortex-antivortex follow the trajectories which is reminiscent of the trajectory followed by a vortex dipole in the pancake-shaped traps [15]. To be precise, the vortex moves in the region to the left of $x = 0$ plane, while the antivortex moves in the region to the right of $x = 0$ plane. In order to understand it, the vortex-antivortex pair at $t = 0$ ms can be considered to

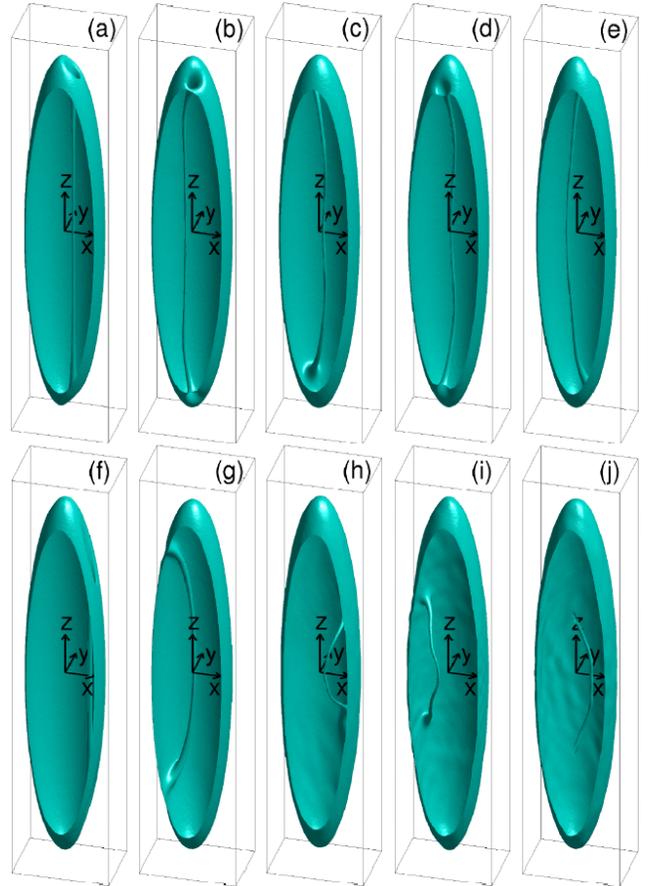


FIG. 2. Upper panel shows the dynamics of the UFG with an off-axis vortex line initially located at $x = 1.0 a_{\text{osc}}$, whereas the lower panel shows the dynamics with an off-axis vortex line initially located at $x = 3.0 a_{\text{osc}}$. The images are the isosurfaces corresponding to $|\phi(\mathbf{r})| = 0.012 a_{\text{osc}}^{-3/2}$. In upper panel, images with the labels (a) and (e) are the isosurfaces after 0 ms, 23.8 ms, 47.7 ms, 63.7 ms, and 79.6 ms of time evolution respectively. In the lower panel, images with the labels (f) and (j) are the isosurfaces after 0 ms, 12.7 ms, 47.7 ms, 63.7 ms, and 82.8 ms respectively. The origin of the coordinate system is located at the center of the box in each image. The dimensions of each box are $12.15 a_{\text{osc}} \times 8.55 a_{\text{osc}} \times 54.15 a_{\text{osc}}$

be equivalent to a highly stretched vortex ring with center at $(0, -a_{\text{osc}})$. The motion of this vortex ring will be due to the contribution from two velocity fields: a) the self induced velocity which will propel the ring in positive y direction and b) the precession induced by the inhomogeneity of the condensate. Initially the self induced velocity dominates and the ring travels forward, later on the precession effects dominate causing the ring to move backward. The precession will result in the expansion of the ring along radial direction for $y > 0$ and contraction for $y < 0$. After the ring returns close to its initial position, each element of the ring (vortex and antivortex) has developed a curvature due to aforementioned expansion as is shown in Fig. 3(b). As the pair

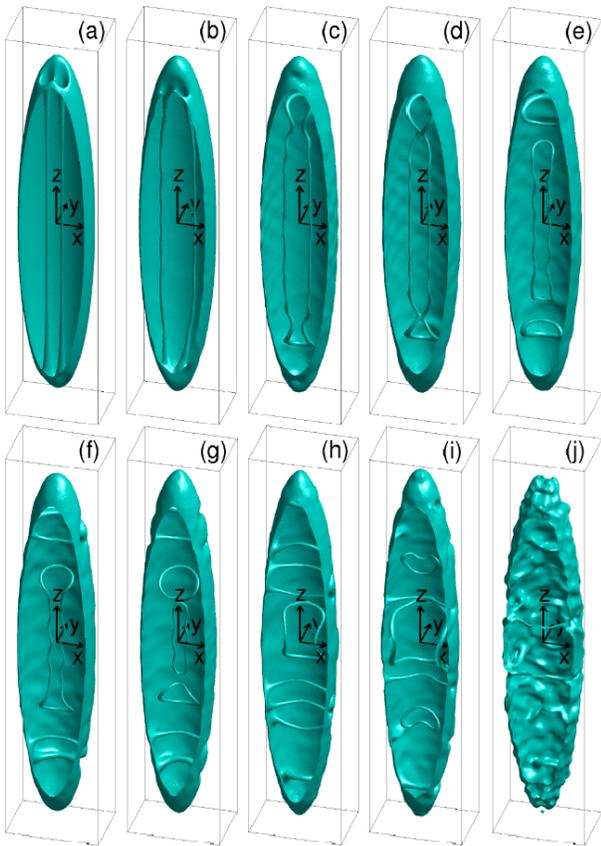


FIG. 3. (Color online) The time evolution of the cigar-shaped UFG with vortex-antivortex imprinted in it at $t = 0$ ms. The images are isosurfaces corresponding to $|\phi(\mathbf{r})| = 0.012 a_{\text{osc}}^{-3/2}$. In the upper panel, images with labels (a) to (e) are the isosurfaces after 0 ms, 20.7 ms, 28.6 ms, 31.8 ms, and 35.0 ms of time evolution respectively. In the lower panel, images with labels (f) to (j) are the isosurfaces after 38.2 ms, 41.4 ms, 47.7 ms, 54.1 ms, and 79.6 ms of time evolution respectively. The origin of the coordinate system is located at the center of the box in each image. The dimensions of each box are $12.15 a_{\text{osc}} \times 8.55 a_{\text{osc}} \times 54.15 a_{\text{osc}}$.

moves along positive y direction, the distance between the vortices at their edges decreases further causing the first reconnection event. This results in an actual vortex ring as is shown in Fig. 3(c). The first reconnection event is aided by bending instability of the off-center vortices. Due to this bending instability, vortex and antivortex bend in opposite direction resulting in the decreased separation between them at the edges [see Fig. 3(b)]. The reconnection event is also accompanied by the emission of the sound as is evident from ripples on the isosurface in Fig. 3(c). Also, in the presence of trapping potential, parts of vortex ring in higher density region move slower than those in lower density regions [54]. This leads to the differential velocity field experienced by the ring. The differential velocity field causes the ring to bend at the ends [Fig. 3(c)]; this in turn generates the Kelvin waves which aid the growth of CI as is shown in Figs.

3(c) and (d). For a sufficiently large amplitude of these Kelvin waves, the second reconnection event takes place at 31.8 ms [Fig. 3(d)]. This results in the generation of two daughter vortices and one parent vortex [Fig. 3(e)]. Reconnection events are typically accompanied by the generation sound waves and further generation of Kelvin waves. With further evolution, the parent ring undergoes the same bending at the edges [Fig. 3(f)], followed by the generation of second set of daughter rings [Fig. 3(g)]. As more time elapses, more reconnection events and the penetration of the vortex rings in the UFG's surface take place [Figs. 3(h-j)]. At the end of the time evolution, the initial vortex-antivortex pair has mostly decayed into sound waves due to CI as is shown in Fig. 3(j).

V. CI IN VORTEX-ANTIVORTEX PAIR GENERATED BY A MOVING OBSTACLE POTENTIAL

Experimentally vortex-antivortex pairs have been created in the BEC by moving the condensate across a repulsive Gaussian obstacle [15]. The generation of vortex rings by moving a spherical obstacle across a UFG has also been studied theoretically [20]. The method of generating vortex antivortex pairs by a moving repulsive potential is also a good prospect to experimentally observe CI in BECs [12]. Taking all these into account, we also study the evolution of UFG under the influence of a moving obstacle potential. We employ an obstacle potential

$$V_{\text{obs}}(t) = V_0(t) \frac{w_0^2}{w(z)^2} \exp \left\{ -2 \frac{x^2 + [y - y_0(t)]^2}{w(z)^2} \right\}, \quad (12)$$

created by Gaussian laser beam. Here w_0 is the waist size, $w_z = w_0 \sqrt{1 - z^2/z_R^2}$, where z_R is the Rayleigh range, is beam width or spot size, and $V_0(t)$ is the maximum potential created by the beam at the center of beam waist which lies at $(0, y_0)$. We use the same obstacle potential parameters as used in Ref. [12]: $V_0 = 25 \hbar\omega$, $z_R = 22 \mu\text{m}$, and $w_0 = 2.2 \mu\text{m}$. The stationary state isosurface with the obstacle potential located at $(0, -a_{\text{osc}})$ is shown in Fig. 4(a); the figure clearly shows the depletion of UFG in the region of the obstacle potential. We move the obstacle with the speed of $500 \mu\text{m/s}$ along y -axis. This speed is a fraction of the speed of sound in UFG at the trap center. We estimate the speed of sound at the trap center by using the expression for speed of sound in a homogeneous UFG, i.e., $c = \sqrt{\xi} v_F / \sqrt{3}$, where $v_F = 2(3\pi^2 N |\phi|^2)^{1/3}$ is the Fermi velocity for ideal Fermi gas [19]. Using this expression, the speed of the sound at the trap center is $\sim 1800 \mu\text{m/s}$. While moving the obstacle its amplitude is linearly ramped down. The rate of ramping down is such that the obstacle potential becomes zero at $(0, 4 a_{\text{osc}})$.

As the obstacle moves along positive y direction, it cre-

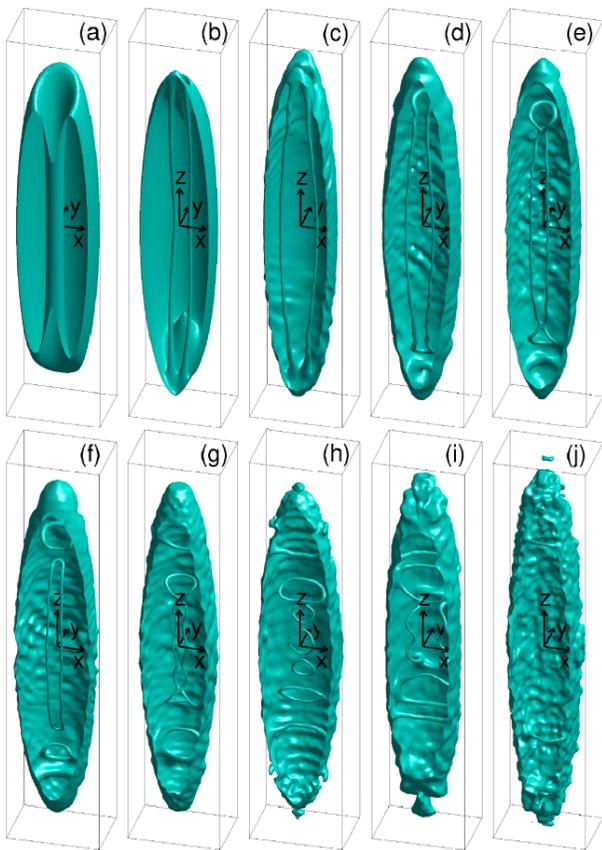


FIG. 4. (Color online) Evolution of the UFG when the obstacle initially located at $(0, -a_{osc})$ is moved with the speed of $550 \mu\text{m/s}$. The images are the isosurfaces corresponding to $|\phi(\mathbf{r})| = 0.012 a_{osc}^{-3/2}$. In the upper panel, images with the labels (a) to (e) are the isosurfaces after 0 ms, 6.4 ms, 20.7 ms, 25.5 ms, and 28.6 ms of time evolution respectively. In the lower panel, images with the labels (f) to (j) are the isosurfaces after 30.2 ms, 35.0 ms, 38.2 ms, 43.0 ms, and 82.8 ms of time evolution respectively. The origin of the coordinate system is located at the center of the box in each image. The dimensions of each box are $12.15 a_{osc} \times 8.55 a_{osc} \times 54.15 a_{osc}$.

ates a vortex antivortex pair in its wake [Fig. 4(b)]. This pair after being reflected from the opposite surface of the UFG returns closer to its initial location [Fig. 4(c)]. This is followed by the first [Fig. 4(d)] and second recon-

nection events [Fig. 4(e)] With further time evolution, CI leads to more reconnection events, penetration of the vortices in the UFG's surface, and generation of sound waves [Figs. 4(f-j)] as happened in the case of phase-imprinted vortex-antivortex pair. At the end of the evolution period, only sound waves are discernible in the system [Fig. 4(j)].

VI. SUMMARY AND CONCLUSIONS

We have numerically investigated the CI in the cigar-shaped superfluid Fermi gas in the unitary limit. We use NLSE based on the time dependent density functional theory to study CI. We numerically solve this equation using split time step Crank-Nicolson method to study the dynamics UFG with a single vortex line as well as a vortex-antivortex pair. In order to generate the vortex-antivortex pair, we employ two methods: (a) phase-imprinting and (b) moving a Gaussian obstacle potential across the system. We have shown that a single vortex line in cigar shaped UFG is stable only if it is oriented along axial direction. On the other hand, an off-axis vortex line tends to bend as it traverses across the condensate. We have shown that a vortex-antivortex pair oriented along the axial direction in a cigar-shaped UFG is susceptible to the CI mechanism. We observe that frequent vortex reconnections, inhomogeneity generated Kelvin waves, and sound generation characterize the CI in an inhomogeneous UFG like in a BEC. We have shown, using the experimentally realizable parameters for trapping potential and number of atoms, that it should be possible to observe the signatures of CI in a vortex-antivortex pair generated by moving a blue detuned laser beam across the UFG.

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