

Fatigue Damage Assessment in Cracked RC Beams

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ABSTRACT: In this work, an attempt has been made to assess the fatigue life of reinforced concrete beams, by proposing a crack propagation law which accounts for parameters such as fracture toughness, crack length, loading ratio and structural size. A numerical procedure is developed to compute fatigue life of RC beams. The predicted results are compared with the available experimental data in the literature and seen to agree reasonably well. Further, in order to assess the remaining life of an RC member, the moment carrying capacity is determined as a function of crack extension, based on the crack tip opening displacement and residual strength of the member is computed at an event of unstable fracture.

1 INTRODUCTION

Structures such as concrete bridges, airport runways and highway pavements are subjected to millions of load cycles during their service life. Due to the application of repetitive loading which is generally well below the yield/ultimate limit, stiffness degradation occurs followed by the acceleration in crack opening. The structure undergoes different phases of damage starting with micro-cracking to ultimate failure thereby reducing the load carrying capacity. Hence, for safe performance, it is desirable to know the strength carrying capacity or the remaining life of the existing structure. To achieve this, it is important to understand the response of a reinforced concrete (RC) member when subjected to fatigue loading. Generally, in the design of RC members, a zero tensile strength of concrete is assumed thereby making the design conservative. In real scenario, concrete exhibits tension softening behavior due to the presence of large size fracture process zone ahead of the crack tip and this could be incorporated in the analysis of RC members by using the concepts of fracture mechanics.

In reinforced concrete beams, the reinforcement is modeled as a pair of eccentric axial forces acting at the crack face (Bosco and Carpinteri (1990)). The crack closing force is computed using crack displacement congruence condition. Generally, the effect of reinforcement reaction is incorporated into the fatigue crack propagation models through the loading parameter, ΔK . In the current scenario, the crack propagation in reinforced concrete members due to cyclic loading is predicted using the well known Paris law wherein, the closing force effect is included in ΔK . Paris law is based on linear elastic fracture mechanics (LEFM) criterion, wherein, the softening behaviour in the post-peak region of concrete is neglected thereby, underestimating the overall response of the structure. Very few attempts have been made to incorporate non-linear fracture parameters into the crack propagation model for concrete. Further, the heterogeneity and large size process zone in concrete necessitates the use of global energy approach instead of local stress concept. Various studies have been carried out considering large size process zone ahead of the crack tip which is responsible for nonlinear behavior of concrete. Bazant in his series of papers (Bazant (1976), Bazant and Cadolin (1979), Bazant and Cadolin (1980)) have shown that, the use of fracture energy in the formulation is the appropriate criterion for the crack bands in plain as well as reinforced concrete and can effectively demonstrate objectivity. The use of energy based criterion is more appropriate for analytical and computational modeling of concrete like material.

In this study, a fatigue crack propagation model (Ray and Chandra Kishen (2011)) is employed to predict the crack growth rate in an RC beam specimen due to the application of cyclic loading. The effect of reinforcement is included into the model through set of closing force acting at the crack face. The proposed fatigue crack propagation model considers global energy

parameters in the formulation and also accounts for size. A systematic algorithm has been developed based on the ideas of strength theory of reinforced concrete, to assess the moment carrying capacity of an independent RC member corresponding to the critical crack size.

2 DIMENSIONAL ANALYSES

According to dimensional analysis approach, any physical problem can be written in the form

$$a = f(a_1, \dots, a_k, b_1, \dots, b_n) \quad (1)$$

Here, a is the quantity to be determined in the phenomenon and is found to be dependent on the arguments on the right hand side. In Equation 1, it is assumed that the parameters (b_1, \dots, b_n) have independent physical dimensions. Independent physical dimension means the governing arguments (a_1, \dots, a_k) as well as governed quantity a can be expressed in terms of (b_1, \dots, b_n) . Equation 1 can be expressed as

$$\Pi = \Phi(\Pi_1, \dots, \Pi_k) \quad (2)$$

Where, the dimensionless parameters are defined by the relation

$$\Pi = \frac{a}{a_1^p \dots a_k^r} \quad (3)$$

$$\Pi_1 = \frac{a_1}{a_1^{p_1} \dots a_k^{r_1}} \dots \Pi_k = \frac{a_k}{a_1^{p_k} \dots a_k^{r_k}} \quad (4)$$

$$a = f(a_1, \dots, a_k, b_1, \dots, b_n) = a_1^p \dots a_k^r \Phi\left(\frac{a_1}{a_1^{p_1} \dots a_k^{r_1}} \dots \frac{a_k}{a_1^{p_k} \dots a_k^{r_k}}\right) \quad (5)$$

Let us consider the parameter a_1 . This parameter is considered as non-essential if the corresponding dimensionless parameter Π_1 is too large or too small (tend to zero or infinity), giving rise to a finite non-zero value of the function Φ with the other similarity parameters remaining constant. The number of arguments can now be reduced by one and we can write Equation 2 as:

$$\Pi = \Phi_1(\Pi_2, \dots, \Pi_k) \quad (6)$$

where, Φ_1 is the limit of the function Φ as Φ_1 tends to 0 or infinity. This is called complete self-similarity or self-similarity of first kind (Barenblatt (1996), Barenblatt (2004)). On the other hand, Φ_1 tending to 0 or infinity, if Φ also tends to zero or infinity, then the quantity Π_1 becomes essential, no matter how large or small it becomes. However, in some exceptional cases, the limit of the function Φ tends to zero or infinity, but the function Φ has power type (scaling) asymptotic representation which can be written as,

$$\Phi \cong \Pi_1^\beta \Phi_1(\Pi_2, \dots, \Pi_k) \quad (7)$$

The constant β and the non-dimensional parameter Φ_1 cannot be obtained from the dimensional analysis alone. This is the case of incomplete self-similarity or self-similarity of second kind. It can be noted here that, the parameter β can only be obtained either from a best fitting procedure on experimental results or according to numerical simulations. Sometimes for Φ_1 tending 0 or infinity, no finite limits exist for the function Φ and also the above exception does not hold. This case can be said to be lack of similarity of the phenomenon in the parameters Π_1 . The main challenge in constructing the self-similar variables is that, we need to know the solution of the

non-idealized problem a priori to make suitable assumptions of different kinds of self-similarity. To handle such problem, each of the similarity parameter is assumed various kinds of similarity depending on the available experimental data and again comparing the derived relation under each assumption with the experimental or numerical data.

3 FORMULATION OF FATIGUE LAW FOR CONCRETE

In the fatigue crack propagation problem, the crack growth rate is the governed parameter which is controlled by various external variables and expressed as follows:

$$\frac{da}{dN} = \Phi(\Delta G, G_f, a, D, \sigma_t, \omega, t, R) \quad (8)$$

In the above Equation, the variables considered are the crack length (a), the loading ratio (R), loading frequency (ω), time (t), the depth of the beam (D), the tensile strength (σ_t) and fracture toughness (G_f). Loading parameter is introduced in the form of change in energy release rate (ΔG). Considering a state of no explicit time dependence and G_f , and σ_t have independent physical dimensions, dimensional analysis gives

$$\frac{da}{dN} = \left(\frac{G_f}{\sigma_t} \right) \Phi \left(\frac{\Delta G}{G_f}, \frac{\sigma_t}{G_f} a, \frac{\sigma_t}{G_f} D, R \right)$$

where, the non-dimensional quantities are

$$\Pi_1 = \frac{\Delta G}{G_f} \quad \Pi_2 = \frac{\sigma_t}{G_f} a \quad \Pi_3 = \frac{\sigma_t}{G_f} D \quad \Pi_4 = R$$

Now, assuming incomplete self-similarity in the parameters Π_1 and Π_2 , it can be written as

$$\begin{aligned} \frac{da}{dN} &= \left(\frac{G_f}{\sigma_t} \right) \left(\frac{\Delta G}{G_f} \right)^{\gamma_1} \left(\frac{\sigma_t}{G_f} a \right)^{\gamma_2} \Phi_2(\Pi_3, \Pi_4) \\ &= G_f^{1-\gamma_1-\gamma_2} \Delta G^{\gamma_1} \sigma_t^{\gamma_2-1} a^{\gamma_2} \Phi_2(\Pi_3, \Pi_4) \end{aligned} \quad (9)$$

The values of γ_1 and γ_2 cannot be derived from the dimensional analysis. These parameters can only be obtained either from a best fitting procedure using experimental results, or through numerical simulations. Various experimental data available in the literature has been used to calibrate and validate the proposed fatigue law by the present authors (Ray and Chandra Kishen (2011)). In this study, these parameters are determined through a calibration process using the experimental results available in the literature and are found to be 5.4113 and 0.0648. The parameter Φ_2 should accounts for specimen size and geometry and the proposed closed form expression for Φ_2 is (Ray and Chandra Kishen (2011))

$$\text{Log}(\Phi) = 1.3963 \left\{ \text{Log} \left(\frac{\sigma_t}{G_f} D \right) \right\}^2 - 15.399 \left\{ \text{Log} \left(\frac{\sigma_t}{G_f} D \right) \right\} + 34.663 + 2.6R \quad (10)$$

In the next section, the proposed fatigue crack propagation law for plain concrete is applied to study fatigue crack propagation in RC beam and the model prediction results are compared with the available experimental data.

4 IMPLEMENTATION OF CRACK GROWTH LAW

The fatigue crack propagation model proposed above for plain concrete beams is used in conjunction with the closing force concept of rebars (Bosco and Carpinteri (1990)) to analyze reinforced concrete beams. The stress intensity factor is computed by considering the combined effects of bending moment and closing force. Experimental data of Katakalos and Pakonstantinou (2009) is used to compute the fatigue life of an RC beam when subjected to different stress ranges. In their experimental study, the authors have used 6 mm round steel reinforcing bars and all the beams were tested under four point loading configuration. The specimen geometry and material properties are given in Table 1. The tests have been done with a loading frequency of 2 Hz and fatigue life of each beam is computed for different load ranges. The upper limit of applied loads is varied as 15.5, 16.5 and 20.5 kN whereas lower limit of load was kept constant at 0.7 kN. Figure 1 shows the plot of ΔP versus the no. of cycles as predicted by the proposed model together with the experimental ones. In the same plot, the results obtained by using the well known Paris law are also plotted. It is seen that the model predictions are quite encouraging whereas the Paris law does not follow the experimental trend. Ensuring the applicability of the proposed fatigue model for the study of RC beams, a parametric study is carried out next to understand the effect of reinforcement on the crack growth.

Table 1. Geometry and material properties of specimens

Specimen	Depth D (mm)	Span S (mm)	Thickness B (mm)	Notch size a (mm)	G_f (N/mm)	E (N/mm ²)	σ_t (N/mm ²)
Beam	250	1200	150	30	0.3	16500	2.86

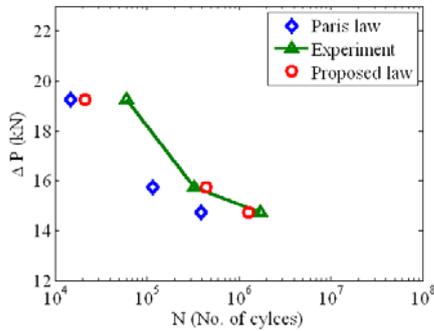


Figure 1. Experimental and computed fatigue life using proposed model and Paris law for different applied Load ranges

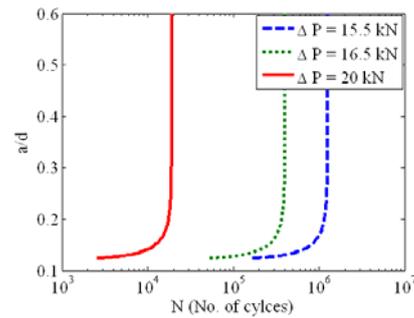


Figure 2. Relative crack depth as a function of loading cycles for applied load ranges

5 PARAMETRIC STUDY ON AN RC BEAM

To understand the influence of reinforcement on fatigue crack growth rate, a parametric study is carried out on an RC beam (Katakalos and Pakonstantinou (2009)). The dimensions and material properties as given in Table 1. Firstly, the variation of load cycles with the crack length are plotted for three different load ranges in Figure 2. Considering the crack propagation curve for $\Delta P = 15.5$ kN, in Figure 2, it is observed that crack starts propagating after 2558 number of load cycles are applied and extends slowly upto a relative crack depth value of 0.22 when no. of cycles reaches 1.8×10^4 . After this, crack grows rapidly, even without significant increase of load cycles. The relative crack depth reaches a value 0.55 at 1.9×10^4 cycles. This behavior can be explained on the basis of yielding of steel. Before the reinforcement yields, the crack propagation is very slow and after which accelerated growth occurs with few load cycles.

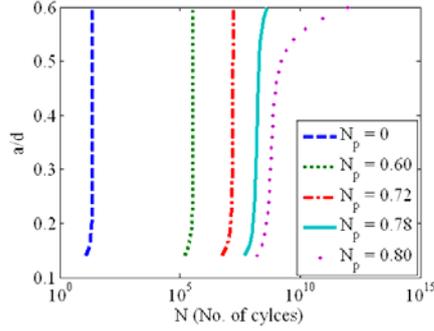


Figure 3. Variation of load cycles as function of relative crack depth with different area of steel.

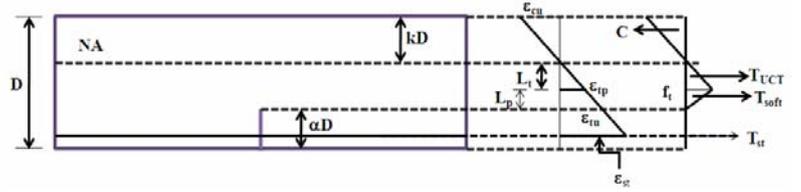


Figure 4. Stress strain distribution of a RC beam with linear soften law

Further, another study is carried out on the same RC beam with varying percentage of steel ($A_s = 0, 90, 108, 116, 120 \text{ mm}^2$) i.e. for different N_p . Here N_p is defined as $\frac{f_y h^{1/2} A_s}{K_{Ic} A}$ (Carpinteri and

Carpinteri (1984)), a brittleness number, dependent on mechanical and geometrical properties of the RC beam cross-section. For $N_p = 0, 0.60, 0.72, 0.78, 0.80$, the crack length versus number of load cycles are plotted in Figure 3. $N_p = 0$, represents the beam without any reinforcement. Analysing the results of numerical study, it has been observed that, for N_p values upto 0.60, the slope of a versus N (no. of load cycles versus relative crack length) curves that is, crack growth rate decreases slowly. Beyond 0.6, the drop in crack growth rate beomes significant and the decrease can be clearly seen for $N_p = 0.78$ and 0.8. An explanation to this can be, as N_p increases, the ductility of the member increases and hence crack growth rate decreases.

6 RESIDUAL STRENGTH ASSESSMENT OF RC BEAMS

For the safe performance of an existing structure during its service life, it is important to assess the strength carrying capacity of the member in terms of crack size. By knowing the critical size of crack length, the corresponding load carrying capacity can be determined. Baluch et al. (1990) have determined the moment capacity of a RC beam with an assumption of steel force. To compute the undetermined steel force, an iterative procedure was followed based on stress-strain curves. The above methodology of determining the steel force is made simpler by using stress-strain constitutive law (Sain and Chandra Kishen (2008)) and the moment carrying capacity is computed based on crack tip opening displacement. A linear softening law has been assumed, to account for the tension softening behaviour in the post-peak region of concrete. In this study, the residual moment is computed by making use of crack tip opening concept which was initially proposed by Sain and Chandra Kishen (2008) and suitably modified it into a simpler form. The length of fracture process zone is determined as the distance ahead of the crack tip for which crack tip opening displacement w reaches its critical value w_c . After knowing the length of fracture process zone, the moment carrying capacity against crack propagation (due to tensile forces) can be computed which includes determination of moment due to steel force, softening force and force in uncracked concrete that is in tension. The concept, average strain ϵ_t in the continuum scale is related to crack opening displacement (COD) in the discrete model by $w = h_s \epsilon_t$ has been used by Sain and Chandra Kishen (2008) and h_s is taken as $0.5D$, where D is the depth of the beam. Steel is assumed to follow elasto- plastic constitutive law and the compressive stress variation in concrete follows a linear elastic law. For computation of moment carrying capacity a systematic algorithm is given below.

6.1 Algorithm

The ultimate tensile strain ε_{tu} is computed corresponding to $w = w_c$ and elastic strain ε_{tp} with respect to $w = 0$ using the relationship $w = h_s \varepsilon_t$

1. Suitable values of α (a/D) and k are assumed
2. ε_{st} is computed using the relation $\varepsilon_{st} = \varepsilon_{tu} \frac{1-k}{1-\alpha-k}$
3. Similarly, ε_c , compressive strain is computed, $\varepsilon_c = \varepsilon_{st} \frac{k}{1-k}$
4. Compressive stresses are determined using

$$\begin{aligned} \sigma_c &= \varepsilon_c E & \varepsilon_c \leq \varepsilon_{cp} \\ &= f_{ck} & \varepsilon_{cp} \leq \varepsilon_c \leq \varepsilon_{cu} \end{aligned}$$

ε_{cu} and ε_{cp} are ultimate strain and plastic strain (strain corresponding to peak maximum compressive stress f_{ck} respectively. E is the elasticity modulus for concrete.

5. Compressive force (C), $C = 0.67 \sigma_c \times kD \times b$
6. The steel force is computed as, $T_{st} = \sigma_{st} A_{st}$

The stresses in steel σ_{st} , is calculated by using the following bilinear law.

$$\begin{aligned} \sigma_{st} &= \varepsilon_{st} E_{st} < f_y \\ &= f_y > f_y \end{aligned}$$

where, E_{st} is the modulus of elasticity of steel and ε_{st} is the steel strain.

7. Total tensile force in concrete $T = \frac{1}{2} \times b \times f_t \times D(1-k-\alpha)$
8. Equilibrium condition is checked i.e. $C = T + T_{st}$. If satisfied, next step is followed. Otherwise computation is done from the step (2) with different value of k .
9. Force resisted in the softening part (linear softening law is considered) $T_{soft} = \frac{1}{2} \times b \times L_p \times f_t$,
 L_p is the length of process zone as shown in Figure 4
10. Force on uncracked concrete under tension $T_{UCT} = \frac{1}{2} \times b \times L_t \times f_t$
11. Total moment of resistance M_R is computed as $M_R = T_{st} \times LA1 + T_{soft} \times LA2 + T_{UCT} \times LA3$
 $LA1, LA2, LA3$ are the respective lever arms.
12. Increment α and go to step 2

6.2 Case study

The above methodology is applied to a RC beam (Alaee and Karihaloo (2003)) to predict the moment carrying capacity. The depth and width of the beam are 150 and 100 mm. Various material properties for reinforcement and concrete are given as - $f_y = 544$ MPa, $E = 35.6 \times 10^3$ MPa, $f_{ck} = 45$ MPa and $f_t = 3.75$. For linear softening law w_c is 0.037. Figure 5 shows the plot of normalized moment (normalized with $K_{IC} b D^{3/2}$) as function of relative crack depth. It can be observed that in the elastic regime of steel, as the crack advances the moment carrying capacity also increases and after yielding of steel, load carrying capacity reduces. Upto yield, stress in steel increases continuously and once yielding starts, it reaches a constant value f_y . With the advancement of crack, the size of softening zone and uncracked concrete which are in tension diminishes. Hence, the contribution of load carrying capacity from fracture process zone (FPZ)

and the uncracked concrete reduces with the increase of relative crack depth a/D . Until yielding of steel, stresses in steel increase and thereby raising the moment carrying capacity of the concrete member. Further, after yielding, the steel stresses remain constant and cause a reduction in carrying capacity upto failure. In this study, failure in the concrete member is assumed to occur, when the strain in steel reaches the ultimate strain i.e. $\epsilon_{st-ult} = \frac{0.87f_y}{E_s} + 0.002$

(Park and Paulay (1975)). On this basis, failure occurs at a relative crack depth of 0.362. From this result it is clear that, the assumption of yielding of steel takes place at the incipient of crack growth, in the Carpinteri's model (Carpinteri and Carpinteri (1984)) in real sense is not true. In general, RC members when subjected to external loading, exhibit various type of failure based on the area of steel provided. Depending on the modes of failure, the beams are designed as under-reinforced (UR), over-reinforced (OR) and balanced section and hence, it is very much essential to know how the carrying capacity changes under these situations. The above discussed methodology is employed to predict and compare the moment resistance curves for these three cases and is shown in Figures 5, 6 and 7 respectively. The area of steel used for UR, balanced and OR sections are 113.09 mm^2 , 346.5 mm^2 , 386.5 mm^2 respectively. It is observed that the drop in carrying capacity after yielding of steel is more in case of under-reinforced section and followed by balanced section. With the increase of steel area (346.5 mm^2), the resistance to crack propagation due to the presence of reinforcement becomes more. Moreover, after yielding, the reduction in carrying capacity is negligible with respect to the increase in the moment due to steel. Hence a small drop in carrying capacity is observed in the balanced section (346.5 mm^2) than the under reinforced section (113.09 mm^2). This behavior is observed for both under reinforced and balanced section. In real scenario, over-reinforced concrete members exhibit compression failure due to crushing of concrete before steel reaches its yield point. In this case, the beam fails in a brittle fashion.

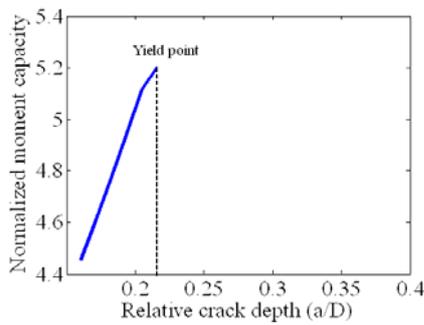


Figure 5. Normalized moment carrying capacity as a function of relative crack length for under-reinforced section

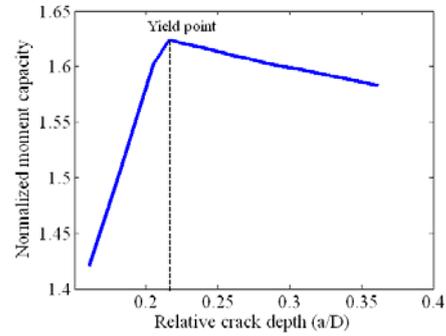


Figure 6. Normalized moment carrying capacity as a function of relative crack length for over-reinforced section

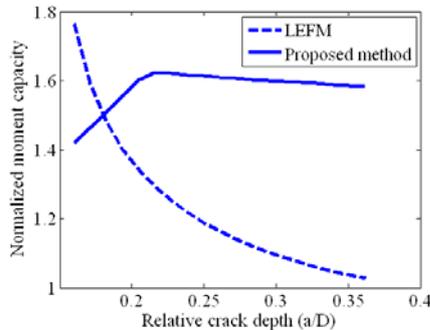


Figure 7. Normalized moment carrying capacity as a function of relative crack length for balanced section

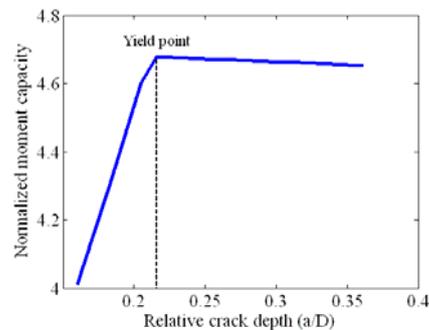


Figure 8. Normalized moment carrying capacity as a function of relative crack length using the proposed method and LEFM approach (Under reinforced section)

In Figure 6, it can be seen that for over-reinforced (OR) section load carrying capacity is higher compared to an UR and balanced section in the initial stage and failure occurs suddenly without showing any indication before steel stress reaches its yield value. Further, to indicate the non-applicability of LEFM approach for concrete, the normalized moment as function of crack length is plotted in Figure 8, for both, using the above method and LEFM approach. In LEFM, moment carrying capacity is computed by making $K_I = K_{IC}$. It is observed that up to certain crack length ($a/D = 0.183$) LEFM predicts high carrying capacity than the present method and beyond that shows lower capacity.

7 CONCLUSIONS

In this study, a fatigue crack propagation law developed for plain concrete using the concepts of dimensional analysis is utilized for crack growth analysis of an RC beam. The developed model uses fracture energy, which is an essential parameter during crack propagation. To ensure the applicability of the present model to RC beams, a comparison with available experimental results is done and is seen to match reasonably well. A parametric study on various crack growth influencing factors are undertaken. With different percentage of steel, the crack length versus number of load cycles curves are plotted. The slope of the curves reduces slightly as the percentage of steel increase indicating the adding-on of ductility. The remaining life/strength of an independent beam is determined in terms of moment carrying capacity for a critical value of crack length. It is observed that, with an increase of relative crack depth the moment carrying capacity increases in the elastic region of steel but the rate of increase reduces as steel approaches yielding and thereafter the moment carrying capacity decreases.

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