PRAMANA — journal of physics © Indian Academy of Sciences Vol. 79, No. 4 October 2012 pp. 891–894

Constraints on the K_{l_3} form factors from analyticity and unitarity

GAUHAR ABBAS[1,*], B ANANTHANARAYAN[1], IRINEL CAPRINI[2] and I SENTITEMSU IMSONG[1]

[1]Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India

[2]Horia Hulubei National Institute for Physics and Nuclear Engineering, P.O.B. MG-6, 077125 Magurele, Romania

[*]Corresponding author. E-mail: gauhar@imsc.res.in

Abstract. The $K\pi$ form factors are investigated at low energies by the method of unitarity bounds adapted so as to include information on the phase and modulus along the elastic region of the unitarity cut. Using as input the values of the form factors at $t = 0$, and at the Callan–Treiman point in the scalar case, stringent constraints are obtained on the slope and curvature parameters of the Taylor expansion at the origin.

Keywords. Unitarity; analyticity; zeros.

PACS Nos 11.30.Rd; 13.40.Gp

1. Introduction

K_{ℓ_3} decays, along with the leptonic decay of the kaon, are the gold-plated channels for a precise determination of $|V_{us}|$, where V_{us} is the element of the Cabibbo–Kobayashi–Maskawa matrix. Kaon decay into a pion and a lepton pair is characterized by the vector and scalar form factors. For the scalar form factor, expansion at $t = 0$

$$f_0(t) = f_+(0)\left(1 + \lambda_0' \frac{t}{M_\pi^2} + \frac{1}{2}\lambda_0'' \frac{t^2}{M_\pi^4} + \cdots\right), \tag{1}$$

defines slope and curvature parameters, where $f_+(0) = 0.964(5)$ is known from the lattice and t is the momentum transfer. Analogously we define the expansion for vector form factor. The purpose of this work is to obtain stringent bounds on the slope and curvature parameters which in turn improve the parametrization of the form factors.

2. Formalism: Crux

The formalism exploits the fact that a bound on an integral involving the modulus squared of the form factors along the unitarity cut is known from the dispersion relation satisfied

DOI: 10.1007/s12043-012-0399-x; ***e*Publication: 5 October 2012**

by a certain QCD correlator which for scalar form factor is

$$\chi_0(Q^2) \equiv \frac{\partial}{\partial q^2}[q^2\Pi_0] = \frac{1}{\pi}\int_{t_+}^{\infty} \mathrm{d}t \, \frac{t \, \mathrm{Im}\,\Pi_0(t)}{(t+Q^2)^2} \, , \tag{2}$$

$$\mathrm{Im} \prod\nolimits_0(t) \geq \frac{3}{2}\frac{t_+ t_-}{16\pi}\frac{[(t-t_+)(t-t_-)]^{1/2}}{t^3}|f_0(t)|^2 \, , \tag{3}$$

with $t_\pm = (M_\mathrm{K} \pm M_\pi)^2$. Similar expressions, involving the correlator $\chi_1(Q^2)$, can be written down for the vector form factor. These correlators are calculated in pQCD up to four loops for $Q^2 > 0$. The inequality derived from eqs (2) and (3) can be exploited in order to derive bounds on the form factors and their derivatives in the analyticity domain. For more information and the review of the formalism, see [1,2].

3. Improvement of bounds using theoretical and experimental information

For improving the bounds, we can use real values of form factors known from the theory. In the case of scalar form factor, two low-energy theorems exist. The soft-pion theorem due to Callan and Treiman gives $f_0(M_\mathrm{K}^2 - M_\pi^2) = F_\mathrm{K}/F_\pi + \Delta_\mathrm{CT}$ and $\Delta_\mathrm{CT} \simeq 0$ to two-loops in chiral perturbation theory. The soft-kaon theorem due to Oehme gives $f_0(M_\pi^2 - M_\mathrm{K}^2) = F_\pi/F_\mathrm{K} + \bar{\Delta}_\mathrm{CT}$ and $\bar{\Delta}_\mathrm{CT} = 0.03$ to one-loop in chiral perturbation theory, the two-loops calculation is not yet available. Here F_K and F_π are decay constants of kaon and pion and $F_\mathrm{K}/F_\pi = 1.193 \pm 0.006$ according to recent lattice results. For further improvement of the bounds, we implement the phase of the form factor along the elastic part of the unitarity cut. We can implement phase information through a function known as Omnès

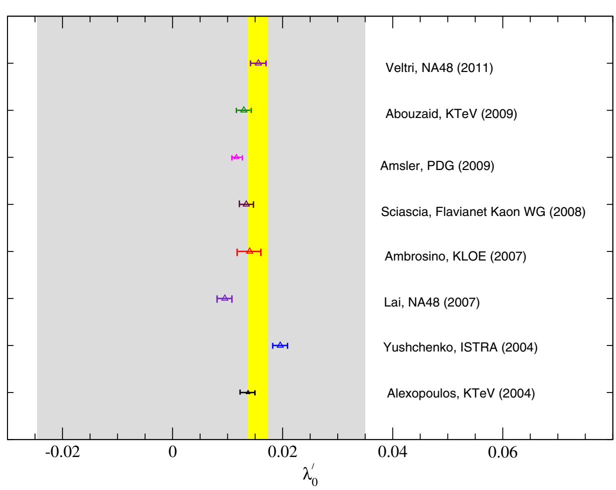

Figure 1. The allowed range for the slope of the scalar form factor, when we include phase, modulus and Callan–Treiman constraint.

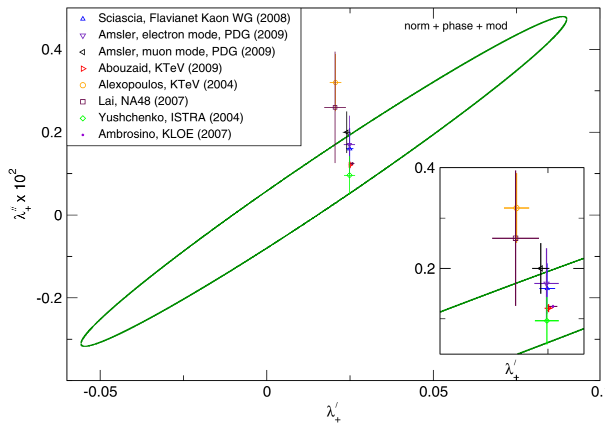

Figure 2. The best constraints for the slope and curvature of the vector form factor.

function. We used the phase of the S-wave of $I = 1/2$ of the elastic $K\pi$ scattering for the scalar form factor, and the phase of the P-wave of $I = 1/2$ for the vector form factor. We can still improve the bounds if we use the modulus as well as the phase of the form factor simultaneously along the unitarity cut. The precise information about the theoretical and experimental inputs is given in [1].

4. Results

In figure 1 we compare the allowed band for the slope λ_0' with the experimental determinations. The slope predicted by NA48 (2007) is not consistent with our predictions. But their recent determination sits inside our predicted range (see Veltri *et al* [3]). The theoretical prediction of ChPT to two loops $\lambda_0' = (13.9^{-0.4}_{+1.3} \pm 0.4) \times 10^{-3}$ is consistent within errors. The same is true for the theoretical prediction $\lambda_0' = (16.00 \pm 1.00) \times 10^{-3}$ obtained from dispersion relations. As shown in figure 2 for vector form factor, except the results from NA48 and KLOE, which have curvatures slightly larger than the allowed values, the experimental data satisfy the constraints (for more results, see [1]). We also found regions where complex as well as real zeros of the form factors are not allowed (for more results, see [1]). We mention that we do not use the soft-kaon theorem as input, but derive bounds on the value at the relevant point. Thus, we are able to predict a narrow range $-0.046 \leq \bar{\Delta}_{\rm CT} \leq 0.014$ for higher order corrections.

5. Conclusions

The results are very stringent in the case of scalar form factor. Note that the most recent results from NA48 [3] respect our prediction for the slope of scalar form factor and restricts the range of the slope to ∼0.01–0.02 which gives a near linear correlation with the curvature. We do not need assumptions on the zeros or the phase above inelasticity threshold. The model independence has a price: we only are able to predict bounds. Nevertheless, the precision of the input is already so large, that the bounds are stringent and improve our knowledge on the form factors.

References

[1] G Abbas, B Ananthanarayan, I Caprini and I Sentitemsu Imsong, *Phys. Rev*. **D82**, 094018 (2010), arXiv:1008.0925 [hep-ph]

[2] G Abbas, B Ananthanarayan, I Caprini, I Sentitemsu Imsong and S Ramanan, *Eur. Phys. J.* **A45**, 389 (2010), arXiv:1004.4257 [hep-ph]

[3] M Veltri, arXiv:1101.5031 [hep-ex]