

## On-line recognition and retrieval of pd signal by regularity measurement based on computation of lipschitz exponents in wavelet domain

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**Abstract:** The problem of on-line recognition and retrieval of relatively weak industrial signal such as Partial Discharges (PD), buried in excessive noise has been addressed in this paper. The major bottleneck being the recognition and suppression of stochastic pulsive interference (PI), due to, overlapping broad band frequency spectrum of PI and PD pulses. Therefore, on-line, on-site, PD measurement is hardly possible in conventional frequency based DSP techniques.

In Authors methodology, the observed noisy signal is enhanced and non-pulsive noises are removed by implementing a wavelet based soft-thresholding scheme. Then, the pulses are detected using simple peak-detector and further analysis is done on localized pulses by taking appropriate window at the detected location. The features of the PD and PI pulses are obtained by measuring the regularity of the pulses at the detected location, which is accomplished by estimating Lipschitz exponents (LE). In this regard, a wavelet based methodology has been used in estimating LE, which is then, used as an index for classifying the pulses in to PD and PI. The method proposed by the Authors were found to be effective in, automatic retrieval of PD pulses.

### Introduction

In spite of advances in the areas of manufacturing, processing, optimal design and quality control, the high voltage (HV), high power apparatus have continued to fail in service prematurely. Investigations reveal that, in most cases insulation failure is the primary cause. In this context, power utilities are increasingly engaging in on-line, on-site diagnostic measurements to appraise the condition of power system. Amongst others, the PD measurement is emerged as an indispensable, non-destructive, sensitive and most powerful diagnostic tool. Partial discharges are essentially charge pulses of very short duration, the rise time being in the range of a few nano-seconds. The PD pattern appears to be quite sensitive to the amount of aging and hence reveals the status of insulation at any point in time, reasonably accurately. A major constrain encountered with on-line digital PD measurements is the coupling of external interferences that directly affects the sensitivity and reliability of the acquired PD data. The more important of them be-

ing, discrete spectral interferences (DSI), periodic pulse shaped interferences, external random pulsive interferences and random noise generic to measuring system itself.

In most of the cases, external interferences yield false indications, thereby reducing the credibility of the PD as a diagnostic tool [1]. Ever since this problem was recognized, extensive research work has been pursued in this area and several techniques for suppressing these external interferences have been proposed. Narrow band detectors achieved some success; better results were obtained by balanced bridge arrangements and pulse discrimination circuits [2]. Digital PD measurement techniques are now available for noise suppression, signal identification and retrieval, such as, FFT thresholding techniques, digital filtering, (finite and infinite impulse response filtering (FIR,IIR)), adaptive filtering and wavelet de-noising methods. These methods have yielded varying degree of success. But acquisition of pure PD signal on-site and on-line is still elusive which forms the subject matter of this paper.

### Problem Definition:

Discrete spectral interferences can be identified and eliminated in frequency domain as they have a distinct narrow-band frequency spectrum concentrated around the dominant frequency, whereas, PD pulses have relatively a broad band frequency spectrum. Periodic pulse shaped interferences can be gated-off in time domain (any PD occurring in that time interval is lost). But, it is very difficult to identify and suppress the stochastic pulse-shaped interferences (PI) as they have many characteristics in common (both in time and frequency domain) with PD pulses. Also, pulsive noise is a random occurrence like PD pulse which aggravates the process of separation. Thus, pulse shaped interference continues to pose problems for reliable on-line, on-site PD measurement.

Locating the PD/PI pulses are the first step in further analysis of the signal. In this regard, we enhance the observed noisy signal using wavelet based soft thresholding method and the pulses are detected using simple peak-detector. Windowed signal of appropriate length is then taken around the detected location and further anal-

ysis is undertaken on the windowed signal.

The features of the PD and PI pulses need to be obtained for an effective classification, which is done by measuring the regularity of the pulses at the detected location. The regularity of any function can be measured by estimating the Lipschitz exponents (LE). In this regard, an wavelet based methodology has been used in estimating LE, which is then, used as a index to classify the pulses in to PD and PI. The procedure adopted in this work is completely different from the research work reported in the literature, which is generally based on derived signal frequency and noise frequency.

### PD/PI Pulse Detection

It has been observed that, PD and PI pulses randomly occur in time. Therefore detection of the pulses is a primary requirement in further analysis of the signal. The signal-to-noise ratio of the PD signal is generally less (around -25dB) and it is difficult to visualize the location and the form of pulses in the observed noisy signal. In this regard, we denoise the signal using wavelet soft thresholding method and make use of a simple peak detector to detect the location of pulsive activity.

Wavelet transform is a very useful tool for the analysis of non-stationary pulsive signals, which is based on breaking up of a signal into shifted and scaled version of mother wavelet. The continuous wavelet transform of a signal  $x(t)$  is given by,

$$CWT_x^\psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t-\tau}{s} \right) dt \quad (1)$$

Where,  $\psi(t)$  is the mother wavelet.  $s$  and  $\tau$  are respectively, dilation and translation parameters. Because of computational and implimentational ease, generally, discrete wavelet transforms (DWT) are used for most of the signal analysis. We rely upon multi resolution analysis (MRA) to obtain the wavelet coefficients [4], with Daubeches wavelet (*db16*) basis function.

Let  $y = x + n$ , be the model for observed noisy signal. Where,  $x, n$  are  $K$ -dimensional clean pulsive signal and non-pulsive noise vectors respectively.

By taking DWT, we have,  $Wy = Wx + Wn$ . Let  $H$  be the operator to modify (threshold) the wavelet coefficients to obtain the enhanced clean signal. Then,  $\hat{x} = W^{-1}HWy$ , where  $W^{-1}$  represents the inverse wavelet transform. A *sureshrink* soft thresholding method proposed by Donoho and Johnston [3] is used for thresholding the wavelet coefficients to estimate the clean signal.

The denoising performance of the wavelet based method was found to be good. The PD/PI pulses, which were completely buried in the noise were visually seen after the enhancement. Therefore a simple peak detector has been used to detect the pulses. A window of appro-

priate length is taken around the detected location and further analysis of the signal is undertaken in the windowed region.

### Measurement of Regularity

Singularities and irregular structures, often carry, essential information in a signal. For example, discontinuities in the intensity of an image indicate the presence of edges in the scene. In electrocardiograms or radar signals, interesting information lies in the sharp transitions. In this section Authors explore the possibility of automatic segregation of PD and PI pulses by regularity measurement.

#### Lipschitz Regularity

To characterize the singular structure, it is necessary to precisely quantify the local regularity of a signal  $f(t)$ . Lipschitz exponents provide uniform regularity measurements over time intervals, but also at any point  $v$ . If  $f$  has singularity at  $v$ , which means that, it is not differentiable at  $v$ , then the Lipschitz exponents at  $v$  characterizes this singular behaviour. The global regularity measurement is useless in analyzing the signal properties at particular location. We measure the local regularity of PD and PI pulses, based on wavelet transforms, as shall be explained later.

A function  $f$  has Lipschitz exponent  $\alpha \geq 0$  at  $v$ , if there exist  $K > 0$  and polynomial  $\rho_v$ , of degree  $m = \lfloor \alpha \rfloor$  such that

$$\forall t \in R, |f(t) - \rho_v(t)| \leq K|t - v|^\alpha \quad (2)$$

At each  $v$ , the polynomial  $\rho_v(t)$  is uniquely defined. if  $m = \lfloor \alpha \rfloor$  times continuously differentiable in a neighbourhood of  $v$ , then  $\rho_v(t)$  is the Taylor expansion of  $f$  at  $v$ . If  $f$  is uniformly Lipschitz  $\alpha > m$  in the neighbourhood of  $v$ , then one can verify that,  $f$  is necessarily  $m$  times continuously differentiable in this neighbourhood. If  $0 \leq \alpha < 1$ , then,  $\rho_v(t) = f(v)$  and the Lipschitz condition 2 becomes,

$$\forall t \in R, |f(t) - f(v)| \leq K|t - v|^\alpha \quad (3)$$

A function discontinuous at  $v$  is Lipschitz 0 at  $v$ . If the Lipschitz regularity is  $\alpha < 1$  at  $v$ , then,  $f$  is not differentiable at  $v$ , but  $\alpha$  characterizes the singularity. We compute the Lipschitz regularity of the pulses (PD/PI), at each detected pulse location, and discriminate the pulses depending on the nature of the singularity. Here, Authors want to make a comment that, being a pulsive signals of different frequent content (although with overlapping spectrum), the nature of the singularities for PD pulses will be different from that of PI pulses, which is a sufficient index to discriminate the pulses. In the forthcoming sections, the relationship between the wavelet transform

and the regularity of the function is explained, in detail.

### Modulus Maxima of Wavelet Transform

A small introduction to the wavelet transform is given in the earlier section. The wavelet transform  $Wf(u, s)$  is a function of the scale and the spatial position  $x$ . It measures the variation in  $f$ , in the neighbourhood of  $u$ , whose size is proportional to  $s$ . The decay of the wavelet coefficients, by the variation in the scale from  $s$  to zero, characterizes the regularity of  $f$  in the neighbourhood of  $u$ .

A wavelet  $\psi$ , with  $n$  vanishing moments can be written as the  $n^{\text{th}}$  order derivative of a function  $\theta$ , i.e.  $\psi = (-1)^n \theta^{(n)}$ , thus, the resulting wavelet transform is a multiscale differential operator.

$$Wf(u, s) = s^n \frac{d^n}{du^n} (f * \bar{\theta}_s)(u) \quad (4)$$

The convolution  $f * \bar{\theta}_s(u)$  averages  $f(x)$  over a domain proportional to  $s$ . Let,  $\psi_1 = -\theta'$  and  $\psi_2 = -\theta''$ , be two wavelets. Thus, the wavelet transforms,  $W^1 f(u, s)$  and  $W^2 f(u, s)$ , are respectively are first and second derivative of  $f * \bar{\theta}_s(u)$ . The local maxima of  $W^1 f(u, s)$  and

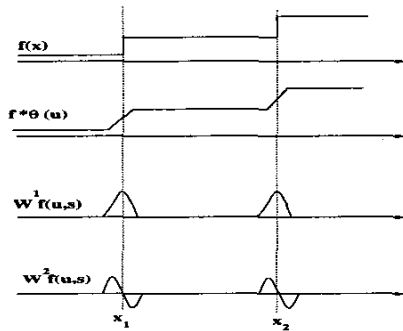


Fig. 1. The position of local maxima and zero crossings of wavelet coefficients

zero crossings of  $W^2 f(u, s)$ , correspond to the inflection point of  $f * \bar{\theta}_s(u)$ , for a fixed scale. For all scales, the local maximum points of  $W^1 f(u, s)$  can be connected as a set of maxima lines in the scale-space plane  $(u, s)$ . Similarly, the zero crossings of  $W^2 f(u, s)$ , define a set of smooth curves, that often look like finger prints. By detecting the position of the local maxima or zero crossings from coarse to fine scale, one can obtain the position of the singularity of a signal. These two methods are very similar, but, the method of obtaining the local maxima has many important advantages [4]. The smoothing function  $\theta$  (scaling function), can be viewed as, impulse response of the low-pass filter. A gaussian scaling function is used as  $\theta$  and the wavelet is defined as the first derivative of the Gaussian function. It should be noted

that, the wavelet function must have, higher vanishing moments compared to that of pulses.

### The Estimation of Lipschitz Regularity

To characterize the singular structure of a signal, it is necessary to quantify the local regularities precisely. Lipschitz exponents provide not only uniform regularity measurements over the time intervals, but also pointwise Lipschitz regularity at any point  $v$  of a signal. The relationship between the decay of wavelet transform amplitude across scales and the pointwise Lipschitz regularity of the signal was described by Jaffard [5], who proved a necessary and sufficient condition in the wavelet domain for estimating the Lipschitz regularity of  $f$  at any given point  $v$ . Assuming that, the wavelet  $\psi$  has  $n$  vanishing moments and  $n$  derivative with fast decay. If  $f \in L^2(\mathbb{R})$  is Lipschitz  $\alpha \leq n$  at  $v$ , then, there exists a constant  $A > 0$ , such that,  $\forall (u, s) \in \mathbb{R} \times \mathbb{R}^+$

$$|Wf(u, s)| \leq As^{(\alpha + \frac{1}{2})} [1 + |\frac{u-v}{s}|^\alpha] \quad (5)$$

We assume that  $\psi$  has a compact support in the interval  $[-C, C]$ . The cone of influence of  $v$  in the scale-space plane is the set of points  $(u, s)$ , such that,  $v$  is included in the support of  $\psi_{u, s(t) = \frac{1}{s}} \psi(\frac{t-u}{s})$ . Since, the support of  $\psi(\frac{t-u}{s})$  is equal to  $[u - Cs, u + Cs]$ , the cone of influence of  $v$  is defined by  $|u - v| \leq Cs$ , which is illustrated in Fig. 2. Since,  $|u - v| \leq Cs$ , the eqn 5 can be written as,

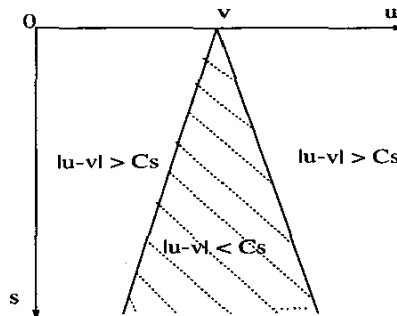


Fig. 2. Cone of influence of  $\psi$  at different  $s$

$$|Wf(u, s)| \leq A' s^{\alpha + 0.5} \quad (6)$$

This is equivalent to the uniform Lipschitz condition given by Mallat [4]. It should be noted that, all modulus maxima converging to  $v$  are assumed to be included in the cone of support. Also, the potential singularity at  $v$  is isolated.

The function  $f$  is uniformly Lipschitz  $\alpha$  in the neighbourhood of  $v$  if there exists  $A > 0$ , such that, each

modulus maximum  $W(u, s)$  in the cone satisfies 5. The Eqn. 5 can be rewritten as,

$$\log_2|W_f(u, s)| \leq \log_2 A + (\alpha + 0.5)\log_2 s \quad (7)$$

Therefore, the Lipschitz regularity at any  $v$  is given by the maximum slope of  $\log_2|W_f(u, s)|$  as a function of  $\log_2 s$  along the maxima line converging to  $v$ .

### Regularity Estimation of PD/PI Pulses

The PD pulses, have relatively broader band of frequencies and thus, it is a sharp rising pulse in time domain, whereas, the PI pulses are more regular. The type of singularity of PD and PI can be estimated by computing Lipschitz exponents as explained in the earlier section. Since, being oscillating pulses, Authors, undertake, the regularity measurement of the dominant lobe to characterize the signal.

The pulses (both PD and PI) are localized at the detected location and singularity is estimated. It was found that, the LE of PD pulses were lying in the range of 0.05 to 0.14 and that of PI pulses were found to be in the range of 0.2 to 0.35. Therefore, LE was found to be a useful method in discriminating the pulses, on-line. Theoretically, above observation is true, since PD pulses have relatively a broader frequency spectrum compared to PI, hence, PD pulses are relatively more singular compared to PI. Authors want to inform humbly that, to the best of Authors knowledge the method explained in this paper is novel and has not been reported in the literature.

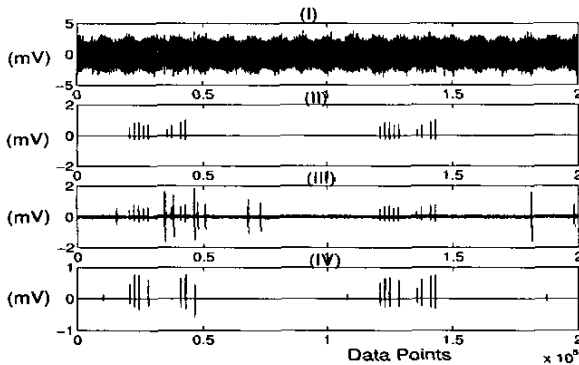


Fig. 3. Output of LE method considering simulated data

The performance of the method on simulated data is shown in figure 3. The added PD pulses are shown in figure 3(II). It can be seen in Fig. 3(IV) that, 14 of the 18 PD pulses have been retrieved without much shift in the pulse position. Also, one PI pulses has been misclassified as PD pulse. A small amount of reduction in pulse height was observed which is attributed to the filtering operation. The performance of the method in deal-

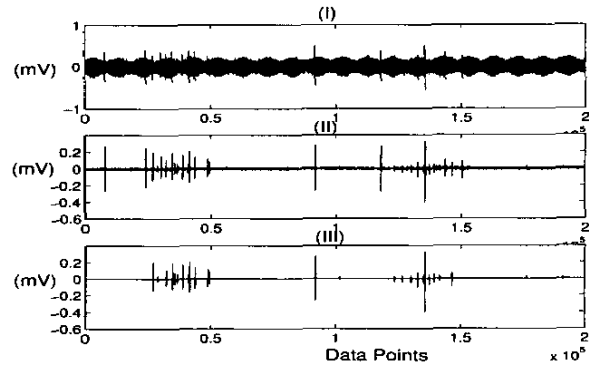


Fig. 4. Output of LE method considering real data

ing with the real signal is shown in figure 4, in which, one PI pulse seems to be mis-classified as PD.

### Conclusion

Based on the work, the following broad conclusions can be drawn.

- The problem of on-line recognition of PD signal is approached in a different perspective than conventional DSP techniques and the theory to model the noisy signal has been developed.
- For the first time, regularity measurement of the signal and noise pulses have been undertaken, which was found to be effective in, on-line retrieval of PD pulses
- The methods proposed is completely automatic and there is no user interference in PD measurement.

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