Low-complexity, Full-diversity Space-Time-Frequency Block Codes for MIMO-OFDM

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***Abstract*— We present a new class of Space-Time-Frequency Block Codes (STFBC) for Multiantenna Orthogonal Frequency Division Multiplexing (MIMO-OFDM) transmissions over frequency selective Rayleigh fading channels. We show that these codes admit symbol-by-symbol decoding (decoupled decoding) when the number of nonzero taps of the channel impulse response is equal to two and they admit reduced complexity (1/2 of that of known schemes) for more than two channel taps. We also present simulation results to show that our codes perform better than the known codes.**

I. Introduction & Preliminaries

It has been proved that when the channel is a frequency flat, Rayleigh and quasi-static fading channel, we can achieve a diversity order equal to the product of number of transmit and the number of receive antennas with appropriate coding across the spatial and temporal domains called Space-Time Coding (STC) [1], [2]. Several authors have constructed Space-Time Codes achieving full diversity [1]–[7]. When the channel is frequency selective, coding intelligently across the spatial, temporal and frequency domains results in a diversity order equal to the product of the channel order, the number of transmit and the number of receive antennas [8]. Coding for such channels is called Space-Time-Frequency Coding (STFC). In this paper, we deal with Space-Time-Frequency Block Codes (STFBCs).

Let the number of frequency carriers be N_c, the number of transmit antennas be N_t, the number of receive antennas be N_r and the number of OFDM symbol intervals be N_x. An STFBC for such a system is a finite set of $N_t \times N_x \times N_c$ 3-D matrices with entries from complex field. If X is a codeword of such a code, then

$$X = \begin{bmatrix} X(0) & X(1) & \cdots & X(N_c-1) \end{bmatrix} \quad (1)$$

where $X(p)$ is a $N_t \times N_x$ matrix. Thus, the codeword X can be seen as a three-dimensional array with transmit antennas, time and subcarriers as the three dimensions.

The impulse response of the channel between the i-th transmit and the j-th receive antenna is assumed to be known at the receiver and is equal to $\mathbf{h}_{ij} = [h_{ij}(0)\ h_{ij}(1)\ \ldots\ h_{ij}(L)]^T \in \mathbb{C}^{(L+1)\times 1}$, where L is the channel order, $\mathbb{C}$ denotes the complex field and $h_{ij}(l),\ l = 0, 1, \ldots L$ are zero mean complex Gaussian random variables with variance equal to $1/(2L+2)$ per dimension. The tap coefficients $h_{ij}(l)$, $j = 0, 1, \ldots, N_r-1$ are assumed to be statistically independent of each other. And the tap coefficients $h_{ij}(l)$, $i = 0, 1, \ldots, N_t-1$ and $l = 0, 1, \ldots, L$, are assumed to be correlated with each other. The covariance matrix of the vector $(h_{ij}(l))_{i\in[1,N_t],l\in[0,L]}$ is denoted as R_h. Let $x_n^i(p)$, the (i,n)-th element of $X(p)$, be the complex symbol (baseband signal) transmitted from the i-th transmit antenna on the p-th subcarrier during the n-th OFDM symbol interval. We assume ideal carrier synchronization, timing, along with perfect symbol-rate sampling. A cyclic prefix (CP) of length L has been inserted at the transmitter and removed at the receiver to eliminate the Inter-symbol Interference (ISI).

The output of the FFT on the j-th receive antenna during the n-th OFDM symbol interval is

$$y_n^j(p) = \sum_{i=1}^{N_t} H_{ij}(p)x_n^i(p) + w_n^j(p), \ \ p = 0, 1, \ldots, N_c-1 \quad (2)$$

which when written in matrix notation is

$$Y(p) = H(p)X(p) + W(p),\ p = 0, \ldots, N_c-1 \quad (3)$$

where

$$H_{ij}(p) = \sum_{l=0}^{L} h_{ij}(l)e^{-j(2\pi/N_c)lp}.$$

The design criteria [8] for constructing good STFBCs based on the Pair-wise Error Probability (PEP) analysis with ML decoding are

Diversity gain: [8] Maximize the rank of the matrix $\bar{\Lambda}_e$ given by[1]

$$\bar{\Lambda}_e := B_h^T \left(\sum_{p=0}^{N_c-1} (I_{N_t} \otimes \omega(p)) \triangle(p)\triangle^{\mathcal{H}}(p) (I_{N_t} \otimes \omega(p))^{\mathcal{H}} \right) B_h^*$$

where

- I_{N_t} is the $N_t \times N_t$ identity matrix

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[1] $\mathcal{H}, T, *$ denote Hermitian, transpose and conjugate respectively

- $\triangle(p) = X(p) - X'(p)$
- B_h is the square root of the covariance matrix R_h
- $\omega(p)$ is the vector containing the first $(L+1)$ entries of the pth column of the N_c-point FFT matrix
- $\otimes$ denotes the Kronecker product

The diversity achieved is equal to $N_r \cdot rank(\bar{\Lambda}_e)$. Notice that the rank of $\bar{\Lambda}_e$ is upper bounded by $N_t(L+1)$. If rank of the matrix $\bar{\Lambda}_e$ is equal to $N_t(L+1)$, then we call the STFBC a full-diversity STFBC.

Coding gain: [8] Maximize the product of the non zero eigen values of $\bar{\Lambda}_e$, i.e., maximize

$$\left[\prod_{i=1}^{rank(\bar{\Lambda}_e)} \frac{1}{L+1}\lambda_{e,i}\right]^{1/rank(\bar{\Lambda}_e)}$$

where $\lambda_{e,i},\ i = 1, \ldots, rank(\bar{\Lambda}_e)$ are the non-zero eigen values of $\bar{\Lambda}_e$. This product of eigen values is called the coding gain. If $\bar{\Lambda}_e$ is a full-rank matrix, then coding gain is equal to the determinant of the matrix $\bar{\Lambda}_e$.

Since $\bar{\Lambda}_e$ could have full rank even if $N_x = 1$, coding across multiple time slots is not necessary. Such a coding is called Space-Frequency Coding (SFC) and have been studied in [9]–[13].

In general N_c is very large and hence, it is very difficult to design STFBCs that maximize the rank of the matrix $\bar{\Lambda}_e$. In order to reduce the design complexity while preserving both diversity and coding advantages, subchannel grouping was proposed in [8]. Though it is actually subcarrier grouping, it was termed as subchannel grouping in [8] and hence we also use the term subchannel grouping. Subchannel grouping is dividing the set of generally correlated OFDM subchannels into several groups of subchannels. Block Codes designed after subchannel grouping are Group STFBCs (GSTFBCs) and it has been proved in [8], that subchannel grouping preserves diversity gain and also reduces decoding complexity at the receiver.

We use sub-channel grouping such that $N_c = N_g(L+1)$ for a certain positive integer N_g denoting the number of groups. Then the three dimensional STF codeword X in (1) is rewritten as $X' := [X_0\ X_1\ \ldots\ X_{N_g-1}]$ which is a permutation of X in (1), where $X_g = [X(g)\ X(N_g + g)\ X(2N_g+g)\ \ldots,\ X(LN_g+g)] \in \mathbb{C}^{N_t \times N_x(L+1)}$. Now, with this subchannel grouping the received matrix is $Y = [Y_0\ Y_1\ \ldots\ Y_{N_g-1}]$ where $Y_g = [Y(g)Y(N_g+g)\ldots Y(N_gL+g)] \in \mathbb{C}^{N_r \times N_x(L+1)}$. Now with this subchannel grouping and the assumption that covariance matrix R_h is the identity matrix, we can reperform the PEP analysis and obtain design criteria as follows:

Sum-of-ranks criterion: Design the STFBC $\mathcal{C}$ such that the set $\mathcal{A} = \{X_g | X \in \mathcal{C}\}$ has the following property: For every pair $X_g \neq X'_g \in \mathcal{A}$, the matrices $\Lambda_e(l)$ given by

$$\Lambda_e(l) = [X(N_gl+g) - X'(N_gl+g)][X(N_gl+g) - X'(N_gl+g)]^{\mathcal{H}} \quad (4)$$

should have full rank, for all $l \in [0, L]$. The minimum of sums of ranks of $\Lambda_e(l)$, $l = 0,\ 1,\ \ldots,\ L$ is equal to the diversity achieved by X.

Product-of-determinants criterion: For the set of matrices satisfying sum-of-ranks criterion, design $\mathcal{A}$ such that $\forall X_g \neq X'_g \in \mathcal{A}$, the minimum of

$$\prod_{l=0}^{L} \det[\Lambda_e(l)], \quad (5)$$

called the coding gain, is maximized.

In order to achieve maximum diversity gain, it follows from the sum-of-ranks criterion that we must have $N_x \geq N_t$. Therefore, coding across different time slots is indispensable in GSTFBCs, as compared with SFCs where coding across time is not a must. However, to achieve full diversity GSTFBCs, it is sufficient to code across subcarriers within a group and let the carriers in different groups be independent of each other, whereas to achieve full diversity SFCs coding across all subcarriers is must. Thus, in the case of GSTFBCs the ML decoding of the received matrix can be broken into the ML decoding of each group. Due to this, without loss of generality we consider only the first group $(g = 0)$ for further analysis in this paper and hence, drop the index g of the group. Thus, the system model henceforth is

$$\begin{bmatrix} Y(0) \\ Y(N_g) \\ \vdots \\ Y(N_gL) \end{bmatrix} = \begin{bmatrix} H(0) & 0 & \cdots & 0 \\ 0 & H(N_g) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H(N_gL) \end{bmatrix} \begin{bmatrix} X(0) \\ X(N_g) \\ \vdots \\ X(N_gL) \end{bmatrix} + \begin{bmatrix} W(0) \\ W(N_g) \\ \vdots \\ W(N_gL) \end{bmatrix}$$

Since the analysis is independent of the number of groups N_g, to avoid notational inconvenience we assume that $N_g = 1$ unless specified.

To avoid exhaustive listing of STFBC codewords, let us define a "design" which will enable us to describe an STFBC succinctly, as follows:

Definition 1: A rate-k/n, $n \times l$ design is an $n \times l$ matrix with entries complex linear combinations of k complex variables and their complex conjugates. By restricting these k variables to take values from a finite subset $\mathcal{A}$ of the complex field, we obtain an STFBC, with symbol rate k/n and bit rate

$$R_T = \frac{k}{n|\mathcal{A}|} \quad bps/Hz \quad (6)$$

where k is the number of information symbols, $n = N_cN_x$ and $|\mathcal{A}|$ is size of constellation.

In this paper, we present Group STFBC, using Coordinate Interleaved Designs (CID) [14], [15], (Definition.3 in section.III) for arbitrary number of transmit antennas and for arbitrary number of channel taps.

The rest of the material of this paper is arranged as follows: In Section II, we describe single-symbol decodability of STFBCs and in Section III we present STFBCs from Coordinate Interleaved Orthogonal Designs (CIODs). In Section IV we prove that the proposed STFBCs admit single-symbol decodability for two channel taps and reduced complexity for more than two channel taps. In the same section, we also present the decoding algorithm for two transmit and N_r receive antennas and for two channel taps. Simulation results are presented in Section V for $N_T = 2,\ N_x = 2,\ N_r = 1$

and $N_c = 64$ for channel taps $L = 1$ and $L = 3$ for 4-QAM and 16-QAM constellations. These Simulation results are compared with Alamouti code with precoder and with uncoded system.

II. Single-Symbol Decodable STFBCs

Let X be the transmitted GSTFB codeword. Then, at the receiver, with ML decoding, we decode the received matrix Y to X' if

$$X' = \arg\min \sum_{l=0}^{L} tr\,(Y - HX)\,(Y - HX)^{\mathcal{H}} \tag{7}$$

where $Y = [Y(0)\ \ldots Y(L)]$, is the received matrix and H is the Discrete Fourier Transform (DFT) of the channel impulse matrix.

The term $tr\,(Y - HX)\,(Y - HX)^{\mathcal{H}}$ is called the ML metric and is denoted as $M(X)$. Clearly, this is a function of all the variables in the design X and hence the ML decoding complexity is exponential in the size of the signal set in general, due to joint decoding of the variables.

Definition 2: A design X is to be single-symbol decodable (decoupled decoding) design if the ML metric for X can be written as

$$M(X) = \sum_{i=0}^{k-1} f_i(x_i)$$

where f_i is a function of x_i only and x_i, $i = 0, 1, 2, \ldots, k-1$ are the variables in the design X.

If the design X is single-symbol decodable, then the ML metric is a sum of several functions each of which depends on only one variable and thus minimizing the ML metric is same as minimizing each function separately. Thus, the ML decoding complexity is not exponential, but is linear in the size of the signal set.

Example 1: Let the number of channel taps $L + 1 = 1$ (i.e., $L = 0$). Then, all the orthogonal designs are single-symbol decodable. For example, the Alamouti code $\mathcal{S} = \begin{bmatrix} x_0 & x_1 \\ -x_1^* & x_0^* \end{bmatrix}$.

Single-symbol decodable STBCs are well studied in [3], [14], [15], [17]. However, single-symbol decodable STFBCs have not been dealt so far and to the best of our knowledge this is the first paper on this issue.

The main disadvantage with Alamouti code is that, if the number of channel taps is greater than one, then this code will not exploit full diversity because there is no coding across subcarriers and from (4) it is clear that to achieve full diversity, coding across the correlated subcarriers is a must. So, the diversity advantage of the Alamouti code is $2N_r$, instead of $2N_r(L + 1)$ for arbitrary number of channel taps. In the following example, we illustrate a method of obtaining full diversity using Alamouti code and precoding techniques [8]. A constellation precoder for a symbol constellation $\mathcal{A}$ is an $M \times M$ unitary matrix $\mathcal{Q}$ such that for any non-zero M-length vector $\mathrm{s} \in \mathcal{A}^M$, the vector $\mathcal{Q}\mathrm{s}$ has all its components non-zeros [18]. It is known that for any symbol constellation there exists a constellation precoder for that constellation.

Example 2: Let the number of channel taps be 2. And let the symbol constellation of interest be 4-QAM. Let X be the codeword $[X(0)\ X(1)]$, where

$$X(0) = \begin{bmatrix} x_0 & x_1 \\ -x_1^* & x_0^* \end{bmatrix} \quad and \quad X(1) = \begin{bmatrix} x_2 & x_3 \\ -x_3^* & x_2^* \end{bmatrix}$$

From (4), $X(0)$ and $X(1)$ should not be independent of each other. So, we precode x_0 and x_2 using the constellation precoder

$$\mathcal{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & e^{j\pi/4} \\ 1 & e^{j5\pi/4} \end{bmatrix} \tag{8}$$

as $[\hat{x}_0\ \hat{x}_2]^T = \mathcal{Q}[x_0\ x_2]^T$ and similarly, we precode x_1 and x_3 as $[\hat{x}_1\ \hat{x}_3]^T = \mathcal{Q}[x_1\ x_3]^T$. The precoded STFBC is thus full-diversity STFBC because the sum-of-ranks criterion given in (4) is satisfied.

In the above example, in the process of obtaining full diversity from Alamouti code, the most important property of Alamouti code, i.e., the single-symbol decodability, is lost as x_0 and x_2 are entangled and similarly x_1 and x_3 are entangled. Increasing the number of channel taps increases the decoding complexity to be exponential to the size of the signal set. This is true with any other orthogonal design also.

III. STFBCs from CIODs for MIMO-OFDM

In this section, we present a generalized construction of STFBCs using CIODs [14] and show that these STFBCs admit low ML decoding complexity. We recollect the definiton of GLPCOD as they are used in constructing CIODs.

Definition 3: A Generalized Linear Processing Orthogonal Design (GLPCOD) in variables $x_0,\ x_1,\ \ldots,\ x_{k-1}$ is an $n \times m$ matrix $\Theta_{n,m}$ with entries that are complex linear combinations of variables $x_0,\ x_1,\ \ldots,\ x_{k-1}$ and their complex conjugates such that $\Theta_{N,M/2}^*\Theta_{N,M/2} = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix where all diagonal entries are linear combinations of $|x_0|^2,\ |x_1|^2,\ \ldots,\ |x_{k-1}|^2$ with all strictly positive real coefficients.

Definition 4: For any positive integer N and even integers M, K a (N, M, K) Co-ordinate Interleaved Orthogonal Design in variables x_i, $i = 0, ..., K-1$ is an $N \times M$ matrix $\mathcal{S}(x_0, ..x_{K-1})$ given by

$$\mathcal{S} = \left[\ \Theta_{N,M/2}(\tilde{x}_0, .., \tilde{x}_{K/2-1}) \quad \Theta_{N,M/2}(\tilde{x}_0, .., \tilde{x}_{K-1})\ \right] \tag{9}$$

where $\Theta_{N,M/2}(\tilde{x}_0, .., \tilde{x}_{K/2-1})$ is a $N \times M/2$ GLPCOD (Generalized linear processing complex orthogonal Design) [2] and $\tilde{x}_i = Re\{x_i\} + jIm\{x_{(i+K/2)_K}\}$ and where $(a)_K$ denotes $a(modK)$. The term K/M is called the rate of the CIOD.

Example 3: Let $x_i = x_{i,I} + \mathbf{j}x_{i,Q}$, $i = 0, 1, 2, 3$, denote four complex indeterminate, where $x_{i,I}$ and $x_{i,Q}$ denote respectively the real and imaginary parts of x_i. Let $\tilde{x}_i = x_{i,I} + \mathbf{j}x_{(i+2)_4,Q}$, $i = 0, 1, 2, 3$ where $(a)_4$ denotes a (mod 4). The indeterminate $\tilde{x}_i$ will be referred as the coordinate interleaved versions of x_i, i=0,1,2,3. The

CID $\mathcal{S}(x_0, x_1, x_2, x_3)$ in variables x_i, i=0,1,2,3 is defined to be the 2×4 matrix

$$\mathcal{S}(x_0, x_1, x_2, x_3) = \begin{bmatrix} \tilde{x}_0 & \tilde{x}_1 & \tilde{x}_2 & \tilde{x}_3 \\ -\tilde{x}_1^* & \tilde{x}_0^* & -\tilde{x}_3^* & \tilde{x}_2^* \end{bmatrix} \quad (10)$$

$$= \begin{bmatrix} x_{0,I} + \mathbf{j}x_{2,Q} & x_{1,I} + \mathbf{j}x_{3,Q} & x_{2,I} + \mathbf{j}x_{0,Q} & x_{3,I} + \mathbf{j}x_{1,Q} \\ -x_{1,I} + \mathbf{j}x_{3,Q} & x_{0,I} - \mathbf{j}x_{2,Q} & -x_{3,I} + \mathbf{j}x_{1,Q} & x_{2,I} - \mathbf{j}x_{0,Q} \end{bmatrix}$$

A. STFBC Construction for two channel taps

First, we give construction of full-diversity STFBCs for $(L+1) = 2$ taps. In this case, the STFBC for N_t transmit antennas is given by

$$X = \left[\Theta_{N_t,N_x}(\tilde{x}_0, \ldots, \tilde{x}_{K/2-1}) \; \Theta_{N_t,N_x}(\tilde{x}_{K/2}, \ldots, \tilde{x}_{K-1})\right]$$

where $\Theta_{N_t,N_x}(x_0, \ldots, x_{K/2-1})$ is a $N_t \times N_x$ complex orthogonal design and $\tilde{x}_i = x_{i,I} + jx_{i+k,Q}$, $\tilde{x}_{i+k} = x_{i+k,I} + jx_{i,Q}$. Clearly, the above code achieves full-diversity if for any pair of codewords X and X', both the matrices $\Theta_{N_t,N_x}(\tilde{x}_0, \ldots, \tilde{x}_{K/2-1})$ and $\Theta_{N_t,N_x}(\tilde{x}_{K/2}, \ldots, \tilde{x}_{K-1})$ are full-rank matrices. And since $\Theta_{N_t,N_x}(\tilde{x}_0, \ldots, \tilde{x}_{K/2-1})$ is an orthogonal design, a non-zero $\Theta_{N_t,N_x}(\tilde{x}_0, \ldots, \tilde{x}_{K/2-1})$ is a full-rank matrix. So, it is sufficient to ensure that the difference matrices $\Theta_{N_t,N_x}(\tilde{x}_0, \ldots, \tilde{x}_{K/2-1}) - \Theta_{N_t,N_x}(\tilde{x'}_0, \ldots, \tilde{x'}_{K/2-1})$ and $\Theta_{N_t,N_x}(\tilde{x}_{K/2}, \ldots, \tilde{x}_{K-1}) - \Theta_{N_t,N_x}(\tilde{x'}_{K/2}, \ldots, \tilde{x'}_{K-1})$ are non-zero matrices. This condition can be satisfied if the Co-ordinate Product Distance (CPD), defined below, of the signal set $\mathcal{A}$ from which the variables x_i take values from, is non-zero.

Definition 5: Let $\mathcal{A}$ be any signal set which is a finite subset of the complex field. Then the Co-ordinate Product Distance of $\mathcal{A}$ is $\Gamma_{\mathcal{A}}$ given by,

$$\Gamma_{\mathcal{A}} = \min_{x_k \neq x'_k \in \mathcal{A}} |x_{kI} - x'_{kI}||x_{kQ} - x'_{kQ}|.$$

The STFBC from CIODs constructed as above achieve full diversity iff CPD of the signal set is non-zero. For constellations with CPD=0, like QAM, we can obtain another signal constellation with non-zero CPD by simply rotating the constellation. For square lattice constellations, the CPD is maximized when the angle of rotation $\theta = \frac{arctan(2)}{2} = 31.7175°$ [16].

Example 4: The design given in Example 2, when x_0, x_1, x_2, x_3 are allowed to take values from a rotated QAM signal set, gives rise to a full-diversity STFBC for two transmit antennas and two channel taps, with codewords as follows:

$$X = \begin{bmatrix} \underbrace{\begin{matrix} \tilde{x}_0 & \tilde{x}_1 \\ -\tilde{x}_1^* & \tilde{x}_0^* \end{matrix}}_{X(0)} & \underbrace{\begin{matrix} \tilde{x}_2 & \tilde{x}_3 \\ -\tilde{x}_3^* & \tilde{x}_2^* \end{matrix}}_{X(1)} \end{bmatrix}$$

where x_i take values from a rotated QAM signal set.

B. STFBC construction for more than two channel taps

When the number of channel taps $L + 1$ is greater than two and is even, we construct full-diversity STFBCs in the following way: Let $L' = (L + 1)/2$ and $X^{(i)}$, $i = 0, 1, \ldots, L' - 1$, be the $N_t \times 2N_t$ CIODs in the variables $x_{Ki}, x_{Ki+1}, \ldots, x_{Ki+K-1}$. Then, consider the design

$$X = \left[X^{(0)} \; X^{(1)} \; \cdots \; X^{(L'-1)}\right]. \quad (11)$$

The above design gives rise to an STFBC for N_t transmit antennas and $L + 1$ taps. However, this is not a full-diversity STFBC. To make this STFBC, a full-diversity one, we use the technique of precoding [18]. In this technique, we replace the symbols $x_i, x_{Ki}, \ldots x_{K(L'-1)}$ with $\hat{x}_i, \hat{x}_{Ki}, \ldots, \hat{x}_{K(L'-1)}$ where

$$[\hat{x}_i \hat{x}_{Ki} \cdots \hat{x}_{K(L'-1)}]^T = \mathcal{Q}[x_i x_{Ki} \ldots x_{K(L'-1)}]^T$$

where $\mathcal{Q}$ is a constellation precoder of size $L' \times L'$ [18]. With this precoding and the assumption that x_i take values from a signal set of non-zero CPD, for any pair of distinct codewords X and X', the difference matrix $X - X' = \left[X^{(0)} - X'^{(0)} X^{(1)} - X'^{(1)} \ldots X^{(L'-1)} - X'^{(L'-1)}\right]$ has the property that $X^{(i)} - X'^{(i)}$ is a non-zero matrix. Thus, the design (11) with appropriate precoding and signal set $(CPD \neq 0)$ gives rise to a full-diversity STFBC.

Example 5: Let the number of channel taps be four and the number of transmit antennas be two then the STFBC is

$$X = \begin{bmatrix} \underbrace{\begin{matrix} \hat{x}_0 & \hat{x}_1 \\ -\hat{x}_1^* & \hat{x}_0^* \end{matrix}}_{X(0)} & \underbrace{\begin{matrix} \hat{x}_2 & \hat{x}_3 \\ -\hat{x}_3^* & \hat{x}_2^* \end{matrix}}_{X(1)} & \underbrace{\begin{matrix} \hat{x}_4 & \hat{x}_5 \\ -\hat{x}_5^* & \hat{x}_4^* \end{matrix}}_{X(2)} & \underbrace{\begin{matrix} \hat{x}_6 & \hat{x}_7 \\ -\hat{x}_7^* & \hat{x}_6^* \end{matrix}}_{X(3)} \end{bmatrix}$$

where $[\hat{x}_i \; \hat{x}_{i+4}] = \mathcal{Q}[\tilde{x}_i \; \tilde{x}_{i+4}]$, $i = 0, 1, 2, 3$, and each variable x_k, $k = 0, 1, \ldots, 7$ takes values a rotated QAM signal set and $\mathcal{Q}$ is the constellation precoder given in (8).

IV. DECODING

We now compare the decoding complexity of our STFBCs with that of some well known STFBCs. First, we have the following theorem.

Theorem 1: Let $L+1$ be even. Then, the decoding complexity of our STFBCs is equal to that of STFBCs from orthogonal designs with precoding when the number of channel taps is $L' = (L + 1)/2$. In other words, the decoding complexity of our STFBCs is half of that of STFBCs from orthogonal designs with precoding for the same number of channel taps.

Proof: Let X be an STFBC codeword and $X_{even} = [X(0) \;\; X(2) \;\; X(4) \cdots X(2L' - 2)]$ and $X_{odd} = [X(1) \;\; X(3) \;\; X(5) \cdots X(2L' - 1)]$. Then the ML metric $M(X)$ is

$$\begin{aligned} M(X) &= \sum_{l=0}^{L} tr\left(\left(Y(l) - H(l)X(l)\right)\left(Y(l) - H(l)X(l)\right)^{\mathcal{H}}\right) \\ &= \underbrace{\sum_{r=0}^{L'} ||Y(2r) - H(2r)X(2r)||^2}_{M'(X_{even})} + \\ &\quad \underbrace{+\sum_{r=0}^{L'} ||Y(2r+1) - H(2r+1)X(2r+1)||^2}_{M'(X_{odd})} \end{aligned}$$

Since, both the X_{even} and the X_{odd} parts of X are independent of each other, minimizing the metric $M(X)$ over all codewords X is same as minimizing $M'(X_{even})$ over X_{even} part of all codewords and $M'(X_{odd})$ over X_{odd} part of all codewords independently. But this is same as that of STFBCs from orthogonal designs with precoding for $(L+1)/2$ channel taps [8]. ■

Corollary 1: Let $L = 1$. Then, STFBCs from CIODs admit single-symbol decoding (decoupled decoding).

Proof: From Theorem 1, the decoding complexity of X is twice that of X_{even}. But, since X_{even} and X_{odd} are both single-symbol decodable, the ML metric $M(X)$ can be written as

$$\sum_{i=0}^{k-1} f_i(x_i)$$

where x_i, $i = 0, 1, \ldots, k-1$ are the variables of the CIOD. ■

In our proposed model the number of information symbols used to generate an STF codeword is $N_I = N_c N_x$. As per (6), our proposed STFBC is a full rate code. The disadvantage in our proposed model is, it will not allow to choose any arbitrary constellation. However, this is not a serious drawback since given any complex constellation there exists infinitely many angles of rotation for which the $CPD \neq 0$. Choosing any non-zero CPD constellation guarantees all the advantages mentioned in our model.

V. SIMULATIONS

We present simulation results for $L = 1$, $N_t = 2$ and $N_x = 2$ for the STFBCs of Examples 4 and 2 with 4-QAM signal set.

Figure 1 shows simulation results comparing the two codes. It can be observed that our proposed codes perform better than the Alamouti scheme with precoder given in Example 2 by more than $1.5dB$ at 10^{-4} OFDM symbol error rate.

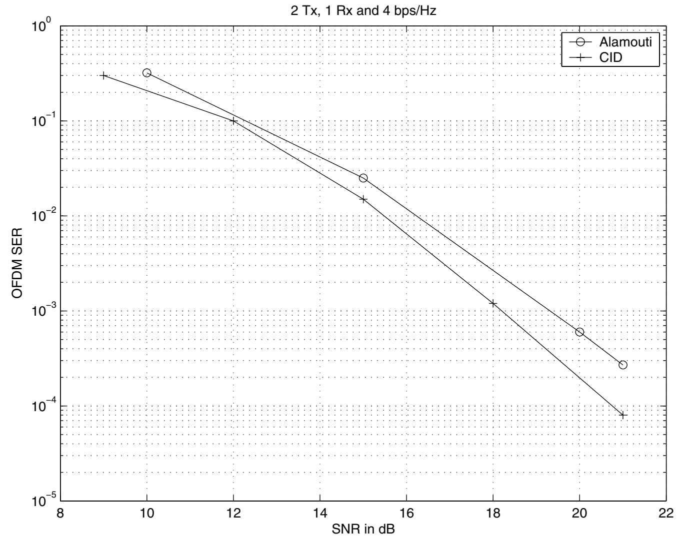

Fig. 1. Performance of CID codes for MIMO-OFDM for $N_t = 2, N_r = 1, L = 1$, $N_c = 48$ and 2 bps/Hz

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