High-rate STBC-MTCM Schemes for Quasi-static and Block-fading Channels

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***Abstract*— For the case of quasi-static fading channel, high rate Space-Time Trellis Codes have already been constructed by concatenating Multiple Trellis Coded Modulation (MTCM) and Space-Time Block Codes (STBC) called the STBC-MTCM scheme. The focus in all these constructions, was to increase the rate of transmission by using more than one orthogonal design, while retaining the diversity advantage and little attention was paid to increase the coding gain advantage. In this paper, we present a systematic approach by which STTCs can be constructed by STBC-MTCM scheme, which achieve high rate, full diversity and increased coding gain advantage over the existing codes under certain conditions.**

Also we a present a systematic approach, to construct STTCs by STBC-MTCM codes which can achieve any given diversity for the case of block-fading channel. The codes constructed for block-fading channels trade-off the rate of transmission and the number of states of the trellis.

I. INTRODUCTION

Space-Time Trellis Codes (STTC) have been introduced in [1] to provide improved error performance for wireless systems using multiple transmit antennas. In [2], Alamouti introduced a simple code to provide full diversity for two transmit antennas. In [3], the scheme is generalized to an arbitrary number of antennas and is named space-time block coding. Although a Space-Time Block Code (STBC) provides full diversity and a simple decoding scheme, it does not provide good coding gain, whereas STTC provide full diversity as well as coding gain but at the cost of higher decoding complexity. To achieve additional coding gain, one should concatenate an outer code such as Multiple-Trellis Coded Modulation (MTCM) as defined in [4] with an inner STBC called the STBC-MTCM scheme.

In [5], Alamouti matrix is combined with MTCM to provide more coding gain along with full diversity. The limitation of STBC-MTCM scheme is that the rate of transmission (bits/sec/Hz) gets reduced because the inner block code is at best, a rate-one code and the outer MTCM encoder must have redundancy. In order to enable high data rate via a concatenated STBC-MTCM scheme, the inner block code must be expanded before being concatenated with an outer MTCM encoder.

To increase the transmission rate, the technique adopted in [6]–[9] is to apply some unitary transformations to the original STBC matrices, so that more number of code matrices are available for transmission, but by using this technique, the difference matrix over all possible pairs of matrices is not full rank.

It is well known that the performance of the STBC-MTCM codes is not directly given by the minimum distance between any two codewords but governed by the distance spectrum (i.e. path weights of the error events or the multiplicities of error events). In this paper, we provide an alternative systematic construction for high rate, full diversity achieving STBC-MTCM code, in which the multiplicities of the error events has been reduced leading to better performance under certain conditions, for the case of quasi-static fading channel. We also provide simulation results to show that under certain conditions our codes outperform the best known codes in literature in terms of coding gain.

In a block-fading channel model the codeword is composed of multiple blocks, the fading coefficients are constant over one fading block, but are independent over block to block. It has been shown in [10] that for block-fading channel, if we code across L quasi-static fading intervals (quasi-static fading interval is the time for which the fading coefficients remain constant), the maximum diversity which we can achieve is L times the diversity which we can achieve in one quasi-static fading interval.

For STBC-MTCM codes, if we have to exploit the block-fading channel to get an arbitrary diversity gain, the necessary condition is that, we have to expand the inner block code in such a way, that the the difference matrix of any two distinct STBC matrices is a full rank matrix. Therefore the technique given in [6], cannot be used to, simultaneously increase the rate of transmission as well achieve any given diversity for STBC-MTCM codes in block-fading channel. In this paper we provide a construction of STBC-MTCM codes which can achieve high rate and any given diversity in the case of block-fading channel. Simulation results are also provided.

The remaining content of the paper is organized as follows: We provide a systematic construction for high-rate, full-diversity STBC-MTCM codes for quasi-static fading channel in Section II. In Section III, a novel approach for construction of high-rate and any specified diversity achieving STBC-MTCM code for the block-fading channel is given.

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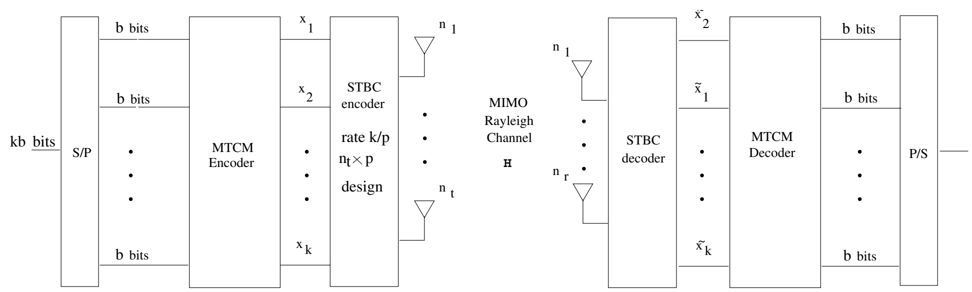

Fig. 1. System Block Diagram

II. QUASI-STATIC FADING CHANNEL

Space-Time Code Design Criteria: The diversity gain of a space-time code is defined by the minimum rank of the matrix $B(s_1, s_2) = (s_1 - s_2)(s_1 - s_2)^H$ over all possible distinct codewords s_1 and s_2 and the coding gain is defined as the minimum of the product of the eigen values of $B(s_1, s_2)$ over all possible pairs of distinct codewords s_1 and s_2 [1]. As in [6], we define the coding gain distance (CGD) between codewords s_1 and s_2 as $d^2(s_1, s_2) = det(B(s_1, s_2))$, where $det(B)$ is the determinant of the matrix B.

Definition 1: A rate-k/p, $n_t \times p$ design is a $n_t \times p$ matrix with entries as linear combination of k complex variables and their conjugates. Restricting the k variables to take values from a finite subset of $\mathbb{C}$, we get a Space-Time Block Code (STBC).

System Model: The system model we consider is a space time wireless communication system with n_t transmit antennas and n_r receive antennas. The channel between a transmit and receive antenna is modeled as a frequency non-selective quasi-static Rayleigh fading process, such that channel coefficients remain same in one frame but are independent from frame to frame and from antenna to antenna.

Let the inner STBC be obtained from a rate-k/p, $n_t \times p$ design $\mathbf{X}$. If the required rate of transmission is b bits/sec/Hz, then the transmitter takes kb bits as input and each of the k streams of b bits undergo Trellis Coded Modulation as shown in Fig. 1. The output of the MTCM scheme is fed to the STBC encoder. STBC encoder which contains the design matrix $\mathbf{X}$, takes these k symbols and forms a $n_t \times p$ codeword matrix.

Each branch of the trellis represents one STBC matrix. This matrix represents the low pass representations of the signal to be transmitted by n_t antennas for the next p symbol durations. The system block diagram is shown in Fig. 1. At the receiver end, the Space Time block decoder followed by a Viterbi decoder can be used to decode the received signals as given in [5].

Code Design Requirements: For designing a rate b bits/sec/Hz STBC-MTCM code, 2^{pb} transitions should come out of each branch of the trellis, but to avoid a catastrophic code, the number of different STBC matrices required for mapping the transitions in the trellis are at least 2^{pb+1} (The catastrophy which we are concerned here, is the catastrophy which can be generated by faulty mapping of the trellis branches and not the catastrophy of the MTCM encoder, as given in [6]).

For simplicity, we take $n_t = 2$, $n_r = 1$ and our STBC to be the Alamouti matrix $\begin{bmatrix} x_0 & x_1 \\ -x_1^* & x_0^* \end{bmatrix}$, $(k = 2, p = 2)$. The number of possible Alamouti matrices over a signal constellation of size 2^b is 2^{2b}, therefore 2^{2b} more Alamouti matrices are required to design a rate b bits/sec/Hz STBC-MCTM non catastrophic code. If we double the size of signal constellation, the number of all possible Alamouti matrices is 2^{2b+2}, out of which only half are required to construct a rate b bits/sec/Hz non catastrophic STBC-MTCM code.

Code Construction: In our construction, we will double the signal constellation size to get the required number of Alamouti matrices. To maximize the coding gain, we do set partitioning in terms of CGD, to choose the required number of matrices from the expanded set of STBC matrices, which are optimal in the sense of CGD.

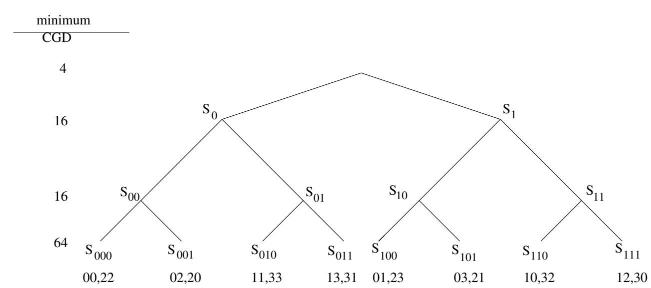

Fig. 2. Set partitioning for QPSK; the numbers at the leaves represent the indexes of the symbols in the space-time block code.

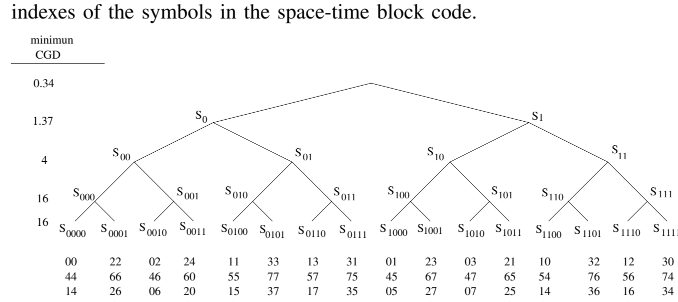

Fig. 3. Set partitioning for 8-PSK; the numbers at the leaves represent the indexes of the symbols in the space-time block code.

In general, this technique of signal set expansion, set partitioning over a M-PSK $(M = 2^b)$ constellation and choosing the restricted set of STBC matrices required for transmission, can be explained as follows:

First Step: Take all possible Alamouti matrices over M-PSK constellation and call this set of matrices A_1 and then do the set partitioning of the matrices in the set A_1, in terms of CGD to form sets $A_{11}, A_{12}, \ldots, A_{1r}$, with each A_{1i} having $2^{2b}/r$ number of elements $\forall\ i, i = 1, 2, \ldots, r$, where r is chosen according to the design requirement of the code.

Second Step: Rotate the M-PSK constellation by angle π/M and then take all possible Alamouti matrices over this rotated constellation and call this set of matrices A_2. Similarly do the set partitioning of the matrices of the set A_2 in terms of CGD, to form sets $A_{21}, A_{22}, \ldots, A_{2r}$ as above.

By using both the sets A_1 and A_2, we choose 2^{2b+1} Alamouti matrices which are optimal in the sense of CGD, required for the construction of rate b bits/sec/Hz STBC-MTCM code, which are full rank.

Code Design Rules: In our scheme, we assign a constituent space-time block code to all transitions from a state. The adjacent states are typically assigned to one of the other constituent space-time block codes from the set A_1 or A_2. The parallel transition branches are assigned STBC matrix from one of the A_{ij} where $i = 1, 2$ and $j = 1, 2, \ldots, r$. Similarly, we can assign the same space-time block code to branches that are merging into a state from either A_1 or A_2. It is thus assured that any path that diverges from (or merges to) the correct path differs by rank 2. In other words, every pair of codewords diverging from (or merging to) a state achieves full diversity because the pair is from the same orthogonal code.

Design Examples: Now we present several design examples of our new proposed code with rate 1 bit/s/Hz and 2 bits/sec/Hz using QPSK and 8-PSK constellations. The proposed simple design rule is used to construct the STBC-MTCM codes that achieve full diversity. MTCM with multiplicity of 2 is used as an outer encoder, and thus 4 and 16 outgoing transitions are needed to achieve the desired code rate of 1 bit/sec/Hz and 2 bits/sec/Hz respectively.

Figs. 4 and 5, show the new 2-state and 4-state 1 bit/sec/Hz and 2 bits/sec/Hz space-time codes respectively. For the rate 1 bit/sec/Hz and 2 bits/sec/Hz codes we use Alamouti matrices over QPSK and 8-PSK constellation respectively for mapping the branches of the trellis from Figs. 2 and 3.

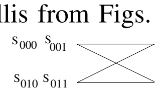

Fig. 4. A two state code; rate 1 bit/sec/Hz using QPSK or 2 bits/sec/Hz using 8-PSK.

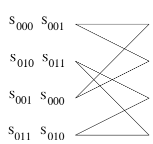

Fig. 5. A four state code; rate 1 bit/sec/Hz using QPSK or 2 bits/sec/Hz using 8-PSK.

Coding Gain Analysis: We calculate distance spectrum for all our codes and compare that with the best known codes in literature i.e. Super Orthogonal Space-Time Trellis Codes (SOSTTC) [6]. We tabulate, the CGD of all our codes in Table I to Table VI and compare them with the appropriate codes as in [6].

Definition 2: In a trellis, two code sequences constitute an error event of length l, if they start from the same state and rejoin at some other state for the first time after l intervals. If the number of input bits is equal to b and the memory of the MTCM encoder is m (number of states of trellis equal to 2^m), then the minimum value of l is $\lfloor m/b \rfloor$. We denote this smallest value of l by P. Effective length of a MTCM code, ν, is the minimum number of distinct symbols between any two codewords and the maximum achievable effective length is given by $\nu = \lfloor m/b \rfloor + 1$ as in [11].

In Table I, we tabulate the CGD on parallel paths and minimum possible CGD on any other path. For parallel transitions (P=1), one can get the CGD from Figs. 2 or 3. For paths other than the parallel transitions, we consider two codewords diverging from state zero and re-merging after P transitions to state zero. For these paths, the CGD is calculated as the determinant of the difference matrix, as already defined.

For 2-state codes, P is equal to 2 and the number of paths coming back after P=2, for rate 1 bit/sec/Hz and 2 bits/sec/Hz codes are equal to 4 and 64 respectively. In Table III and IV, we tabulate the CGD of these paths and compare them with the appropriate code of [6], for 2-state rate 1 bit/sec/Hz and 2 bits/sec/Hz code respectively. For 4-state codes, P=3 and the number of paths coming back after P =3, for rate 1 bit/sec/Hz and 2 bits/sec/Hz codes are 8 and 512 respectively. In Tables V and VI, we tabulate the CGD of these paths and compare them with the equivalent code of [6], for 4-state, rate 1 bit/sec/Hz and 2 bits/sec/Hz code respectively.

Table III to Table VI, give the distance spectrum, of all our codes. It is enough to calculate the CGD's upto minimum possible value of l i.e P, as our codes are non-catastrophic.

From Table I, except for our 2-state rate 2 bits/sec/Hz code, the minimum CGD of our codes is greater than or equal to the minimum CGD of the comparable SOSTTC. Clearly from Tables III-VI, the distance spectrum our codes is better than the distance spectrum for the SOSTTC. The resulting performance improvement is discussed in the following subsection.

TABLE I

CGD VALUES FOR DIFFERENT CODES

Rate in bits/sec/Hz	min det(A)	parallel CGD
1 (Fig.5)	64	64
1 (Fig.6)	144	64
2 (Fig.5)	10.05	16
2 (Fig.6)	27.04	16

TABLE II

COMPARISON OF CGD VALUES

Figure	No. of States	Rate (bits/sec/Hz)	minimum CGD	minimum CGD in SOSTTC
5	2	1	64	48
5	2	2	10.05	16
6	4	1	64	64
6	4	2	16	16

TABLE III

CGD VALUES FOR ALL PATHS WITH P=2 FOR RATE 1 BIT/SEC/HZ, 2 STATE CODE

Coding Gain Distance	No. of paths in our code	No. of paths in SOSTTC
64	4	-
48	-	4

Simulation Results: In this subsection, we provide simulation results for our new code design using two transmit and one receive antenna.We compare our results with the SOSTTC for

TABLE IV

CGD VALUES FOR ALL PATHS WITH P=2 FOR RATE 2 BITS/SEC/HZ, 2 STATE CODE

Coding Gain Distance	No. of paths in our code	No. of paths in SOSTTC
10.05	16	-
24	-	64
36	32	-
78	16	-

TABLE V

CGD VALUES FOR ALL PATHS WITH P=3 FOR RATE 1 BIT/SEC/HZ, 4 STATE CODE

Coding Gain Distance	No. of paths in our code	No. of paths in SOSTTC
144	8	-
128	-	8

same number of transmit and receive antennas and for same rate of transmission. In all simulations, a frame consists of 130 transmissions out of each transmit antenna. Figs. 6 and 7, shows the frame error probability results versus signal-to-noise ratio (SNR) for the code given in Figs. 4 and 5 respectively.

Our proposed code for rate 1 bit/sec/Hz for 2 or 4-states, outperforms similar SOSTTC by nearly 0.5 dB, also our proposed 2 bit/sec/Hz, 4-state code, gives a better performance of nearly 0.25 dB than the similar code in [6], but rate 2 bit/sec/Hz, 2-state code in [6] performs better than our code. If we see the simulation results for the 2 bits/sec/Hz, 2 state code, we find out that the performance is not as degraded as indicated by the decrease in minimum CGD, because our performance in terms of multiplicities of error events is better.

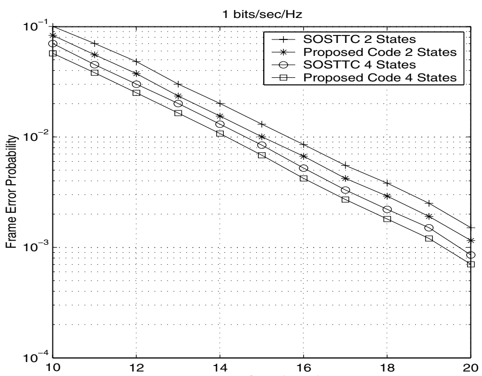

Fig. 6. Performance of rate 1 bit/sec/Hz STBC-MTCM codes in quasi-static fading channel for 2 Transmit and 1 Receive antenna

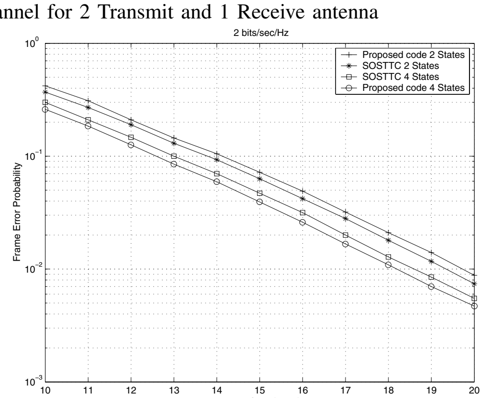

Fig. 7. Performance of rate 2 bit/sec/Hz STBC-MTCM codes in quasi-static fading channel for 2 Transmit and 1 Receive antenna

III. BLOCK-FADING CHANNEL

System Model: The system model we consider remains the same as already shown in Fig.1, except that the channel we consider is a block-fading channel. In block-fading channel the channel coefficients remain same in one block of quasi-static fading interval, but are independent from block to block and from antenna to antenna.

TABLE VI

CGD VALUES FOR ALL PATHS WITH P=3 FOR RATE 2 BITS/SEC/HZ, 4 STATE CODE

Coding Gain Distance	No. of paths in our code	No. of paths in SOSTTC
27.04	128	-
48	-	384
64	256	128
120	128	-

Design Criteria for Block-Fading Channel: Let $\mathcal{C}$ be a code for the channel with n_t transmit and n_r receive antennas. We assume that the code $\mathcal{C}$ has codewords spread over K quasi-static fading blocks, the length of each quasi-static block being p. Thus, the codeword $\mathbf{c} = [\mathbf{c}[1]\ \mathbf{c}[2]\ \ldots\ \mathbf{c}[K]]$, where $\mathbf{c}[i]$ is the part of codeword corresponding to i-th fading block, is a $n_t \times pK$ matrix. The generalized diversity and product distance criteria for space-time codes over MIMO block-fading channels are as follows. Maximize the transmit diversity advantage

$$d = \sum_{i=1}^{K} d_i = \sum_{i=1}^{K} rank(\mathbf{c}[i] - \mathbf{e}[i]) \tag{1}$$

and maximize the coding advantage

$$\mu = \prod_{i=1}^{K} (\lambda_1[i]\lambda_2[i]\cdots\lambda_{d_i}[i])^{1/d_i} \tag{2}$$

over all pairs of distinct codewords **c, e** [10].

Code Construction: If the effective length of a trellis is ν, then any two codewords in the trellis, differ in at least ν positions. Since we are transmitting a STBC matrix on each branch of the trellis, by sum of ranks criteria given in (1), we get diversity νD (where $D = min(n_t, p)$ is the diversity achieved by an STBC), if the difference of any two distinct STBC matrices is a full rank matrix. Therefore, to maximize the diversity gain possible for STB-MCTM codes, we should maximize the effective length of the trellis ν. To design a STBC-MTCM code that achieves diversity gain of νD, the effective length of the trellis should be atleast ν.

Code Design: For simplicity, we take $n_t = 2$, $n_r = 1$ and our STBC to be the Alamouti matrix. To get diversity νD and rate b bits/sec/Hz, for STBC-MTCM scheme in block-fading channel, we will use a MTCM with effective length ν and the same construction for increasing the rate of transmission for the STBC-MTCM code for quasi-static fading channel, of doubling the signal set and set partitioning the set of all possible Alamouti matrices in terms of the CGD. This guarantees that the difference matrix of any two such Alamouti matrices is a full-rank matrix.

Code Design Rules: In our scheme, we assign a constituent STBC to all transitions from a state. The adjacent states are typically assigned to one of the other constituent STBC from the set A_1 or A_2. We avoid parallel transitions in the trellis to get extra diversity gain. By our construction, the difference of any two distinct Alamouti matrices is a full rank matrix, then from the sum of rank criteria, we are guaranteed of at least, a diversity gain equal to 2ν, if the effective length of the trellis is ν as diversity gain given by Alamouti matrix is 2.

Design Examples: We present design examples of our proposed code. Figs. 8 and 9, give the design of diversity 4 STBC-MTCM codes, for rate 1 bit/sec/Hz and 2 bits/sec/Hz, respectively. In both the examples, parallel transitions have been avoided to get extra diversity.

For designing rate 1 bit/s/Hz and 2 bits/sec/Hz diversity 4 STBC-MTCM codes, we use Alamouti matrices over QPSK and 8-PSK constellations respectively for mappings branches of the trellis from Figs. 2 and 3. For the design examples, MTCM with multiplicity of 2 is used as an outer encoder, and thus 4 and 16 outgoing transitions are needed to achieve the desired code rate of 1 bit/sec/Hz and 2 bits/sec/Hz respectively.

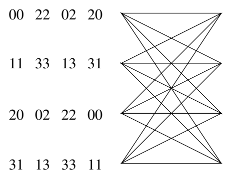

Fig. 8. A four state code; rate 1 bit/sec/Hz using QPSK set partitioning as shown in Fig. 2

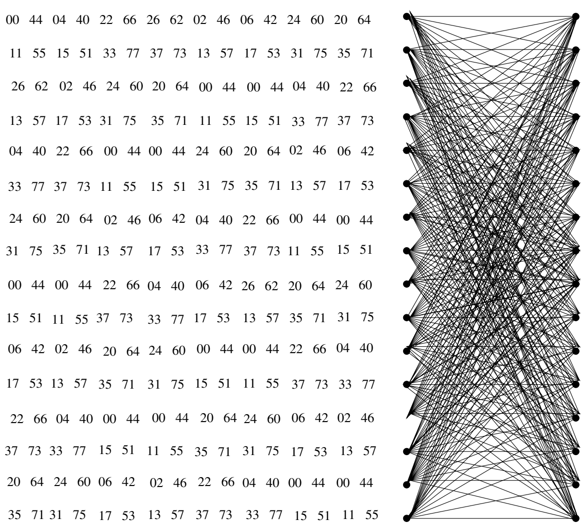

Fig. 9. A sixteen state code; rate 2 bits/sec/Hz using 8-PSK set partitioning as shown in Fig. 3

$Note$: The codes which we present, trades-off the rate of transmission and the number of states of the trellis (decoding complexity). e.g. to achieve diversity 4, for rate 2 bits/sec/Hz, we have to increase the number of states to 16 as shown in Fig. 9. Therefore, fixing the diversity gain, increase in rate is possible only at the cost of increased decoding complexity.

Coding Gain Analysis: For our codes $\nu = 2$, therefore the number of quasi-static fading blocks across which we are able to do coding is 2. The diversity gain of codes given in Figs. 8 and 9 is 4. For any state in our trellis, the diverging branches are mapped from set S_{00} or S_{01} and converging branches are mapped from set S_0, therefore from (2), the coding gain is given by the product of the intraset minimum CGD of matrices S_{00} and S_0 or the product of the intraset minimum CGD S_{01} and S_0, raised to power of $1/4$, where S_{00}, S_{01} and S_0 as given in Figs. 2 and 3 for QPSK and 8-PSK case respectively.

The coding gain for rate 1 bit/sec/Hz, diversity 4, STBC-MTCM code as given in Fig. 8, is $(16 \times 16)^{1/4}$ which is 2 and for rate 2 bit/sec/Hz, diversity 4 STBC-MTCM code as given in Fig. 9, the coding gain is $(4 \times 1.37)^{1/4}$ which is 1.53. Fig. 10, shows the frame error probability results versus signal-to-noise ratio (SNR) for the codes given in Figs. 8 and 9. Clearly, it can be seen from the simulation results that, our codes achieve diversity 4.

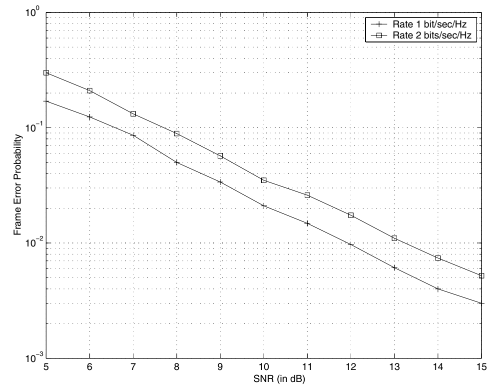

Fig. 10. Performance of rate 1 bit/sec/Hz and 2 bits/sec/Hz. 2 transmit, 1 receive antenna, block fading system

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