

# Reliability and Interval Estimation of Type-II Censored Electrical Insulation Data

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**Abstract**—The paper presents some analytical results pertaining to the estimation of variance of the parameters of a three parameter Weibull distribution (3pW) under type-II censoring. Ageing failure data acquired on an insulating material of considerable application potential has been used to demonstrate the results. The point estimates of the parameters of failure time distribution are obtained using maximum likelihood estimation method. The true value of the variance of the ML estimates for 3pW are hard to obtain and the situation becomes more complex when the data is Censored. The asymptotic variance can be obtained by taking the inverse of the Fisher information matrix, the computation of which is quite involved in the case of censored 3-pW data. Approximations are reported in the literature to simplify the procedure. The Authors have considered the effects of such approximations on the precision of variance estimates when the sample size is greatly limited by practical difficulties in obtaining the authentic data. A detailed study of the effect of censoring on the ML estimates, under this condition is also presented.

## I. INTRODUCTION

The insulation in power equipment are subjected to different kinds of stresses such as electrical, thermal and each of these causes a continuous degradation in their properties. Engineering interest is centered around the information as to the time, reckoned from the instant of application of the stress or stresses, that elapses before failure ensues. In the analysis of times to failure data, the outcome of an analysis is always an estimate of the parameters of the statistical distribution function, deemed to conform to the observed data. The objective of any procedure for processing such stochastic data is to try and assess the degrees of uncertainty associated with the estimated parameters of the proposed statistical model. The accuracy of the parameter estimate is a sensitive function of the quality of acquired data as also the conformity of the data to the assumed model.

Recently, censored data acquisition techniques are often used to reduce the cost and time. The statistical procedure followed in the analysis of censored data is complex and approximations of varying degree are required to reduce the conceptual and computational difficulties, with a consequent loss of accuracy of parameter estimates. If  $\hat{\theta}$  is the unbiased estimator of  $\theta$ , then  $\text{var}(\hat{\theta}) = E\left[-\frac{\partial^2 l}{\partial \theta^2}\right]$ , where,  $l$  is the log likelihood function, shall be explained later. This is known as Fisher information [1], which involves the solution of complicated kernels in the integrand of the expectation integrals and cannot always be obtained in a closed form. Also, the formulation of expressions for computing the asymptotic variance of the point estimate (of 3pW) becomes difficult when the data is censored. Among the more important assumptions made to simplify the problem is the replacement of the global Fisher information with the local information (L), which assumes expectation of function of random variable to be function itself. This is valid only for large sample sizes. In practice, insulation failure data are acquired on a far fewer specimens to economize on time and cost of running

ageing experiments. Authors have considered such a practical situation and tried to study the implications of such approximations in insulation failure data analysis.

One of the methods suggested to validate the parameter estimates is based on the confidence intervals with equal coverage probabilities. There are literature reported dealing with the study of performance of log likelihood ratio procedure of computing the confidence bounds and its relevance to failure data analysis. Escobar [2] presented a statistical prediction technique based on censored life data and provided extensive information for computing the coverage probabilities. Recently, Jeng [3] presented the detailed study of obtaining the confidence intervals and compared those on the basis of coverage probabilities. The terms of reference of the present work are three-fold :

- The computation of global Fisher information using a numerical integration technique and hence evaluation of the parameter variances.
- computation of the confidence interval (CI) using likelihood ratio procedure and method of normal approximations.
- Analysis of confidence intervals and coverage probabilities based on exact Fisher information and local information and to study the effect of censoring.

## II. DATA ACQUISITION

The results of life experiments included here, pertains to epoxy resin bonded mica insulation, an insulation of choice in stator bars as well the stator frame of large turbo generators. A series of electrical ageing experiments were carefully planned, executed and failure data acquired on twenty specimens of a population. The experiments were conducted under statistically identical conditions and under constant accelerated electrical stress. One set of raw data on the time to failure is given in Table. I

TABLE I  
ACQUIRED LIFE DATA

|              |        |       |        |        |        |
|--------------|--------|-------|--------|--------|--------|
| time (hours) | 65     | 161.5 | 234    | 328    | 404.5  |
| time (hours) | 480.5  | 560.5 | 665.75 | 737.5  | 857    |
| time (hours) | 935.5  | 1070  | 1181   | 1325.5 | 1472   |
| time (hours) | 1662.5 | 1891  | 2155.5 | 2530.5 | 3405.2 |

The Weibull probability plots suggested a small degree of nonlinearity. The third (location) parameter was introduced as a correction for the curvature of the Weibull probability plots so as to render them linear. This data is uncensored (complete), which is then deliberately censored at different levels to study the effect of censoring on the maximum likelihood (ML) estimates of the parameters of the three parameter Weibull distribution. This means that although a complete data is available, it was fictitiously truncated at various levels to make it a censored data set.

### III. POINT ESTIMATION

The probability density function,  $f(t)$ , of the 3pW distribution is given by:

$$f(t) = \left(\frac{\beta}{\tau}\right) \left(\frac{t - \tau_0}{\tau}\right)^{(\beta-1)} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta}; \quad t \geq \tau_0. \quad (1)$$

Where, time  $t$  and is the random variable in question.  $\beta, \tau, \tau_0$  are the distribution parameters called the shape, the scale and the location parameters respectively. Let,  $n, r, t_k$  be the total number of samples, number of samples run to fail and failure time of  $k^{th}$  sample respectively. The loglikelihood function of the set of random events for a type-II censored data is given by,

$$L = r \ln \left(\frac{\beta}{\tau}\right) + (\beta - 1) \sum_{k=1}^r \ln \left(\frac{t_k - \tau_0}{\tau}\right) - \sum_{k=1}^r \left(\frac{t_k - \tau_0}{\tau}\right)^\beta - (n - r) \left(\frac{t_f - \tau_0}{\tau}\right)^\beta$$

The ML estimates  $\hat{\beta}, \hat{\tau}$  and  $\hat{\tau}_0$  is the  $\beta, \tau$  and  $\tau_0$  value that maximizes the log likelihood function ( $L$ ) since it is a monotonic function of  $L$ . The ML estimates can be found by the usual calculus method of setting the derivative of the  $L$  to be zero and solving the equations called as likelihood equations shown below.

$$\frac{\partial L}{\partial \beta} = \frac{r}{\beta} + \sum_{k=1}^r \ln \left(\frac{t_k - \tau_0}{\tau}\right) - \sum_{k=1}^r \left(\frac{t_k - \tau_0}{\tau}\right)^\beta \ln \left(\frac{t_k - \tau_0}{\tau}\right) - (n - r) \left(\frac{t_f - \tau_0}{\tau}\right)^\beta \ln \left(\frac{t_f - \tau_0}{\tau}\right) \quad (2)$$

$$\frac{\partial L}{\partial \tau} = -\frac{r}{\tau} - \frac{(\beta - 1)r}{\tau} - \sum_{k=1}^r -\left(\frac{t_k - \tau_0}{\tau}\right)^\beta \beta \tau^{-1} - (n - r) \left(\frac{t_f - \tau_0}{\tau}\right)^\beta \beta \tau^{-1} \quad (3)$$

$$\frac{\partial L}{\partial \tau_0} = (\beta - 1) \sum_{k=1}^r -\left(\frac{t_k - \tau_0}{\tau}\right)^{\beta-1} - \sum_{k=1}^r -\left(\frac{t_k - \tau_0}{\tau}\right)^\beta \beta (t_k - \tau_0)^{-1} + (n - r) \left(\frac{t_f - \tau_0}{\tau}\right)^\beta \beta (t_f - \tau_0)^{-1} \quad (4)$$

The values of the ML estimates under complete data as well the censored data is given in the table II.

TABLE II  
ML ESTIMATES OF THE PARAMETERS OF 3PW UNDER DIFFERENT CENSORING LEVELS (R/N)

| $r/n$          | 1.0   | 0.95  | 0.90  | 0.85  | 0.80  | 0.75 | 0.70   |
|----------------|-------|-------|-------|-------|-------|------|--------|
| $\hat{\beta}$  | 1.178 | 1.157 | 1.166 | 1.152 | 1.147 | 1.14 | 21.129 |
| $\hat{\tau}$   | 1117  | 1113  | 1114  | 1117  | 1117  | 1118 | 1119   |
| $\hat{\tau}_0$ | 46.1  | 45.8  | 46.8  | 47.6  | 47.6  | 47.5 | 48.2   |

### IV. COMPUTATION OF THE VARIANCE OF ML ESTIMATES

One must calculate the Fisher information (F) to obtain the asymptotic variance of the ML estimator of the distribution parameters to obtain the approximate confidence bounds. The Fisher information is the expectation of the negative of the log likelihood with respect to the parameter[1]. That is, if  $\theta$  is the estimate and  $f(t_1, t_2, \dots, t_n; \theta)$ , the joint probability density function of the random variable  $t$ , then

$$E\left\{-\frac{\partial^2 L}{\partial \theta^2}\right\} = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \left\{-\frac{\partial^2 L}{\partial \theta^2}\right\} f(t_1, \dots, t_n) dy_1 \dots dy_n$$

For a three parameter Weibull distribution function,  $F$  can be represented as:

$$F = E \begin{bmatrix} -\frac{\partial^2 L}{\partial \beta^2} & -\frac{\partial^2 L}{\partial \beta \partial \tau} & -\frac{\partial^2 L}{\partial \beta \partial \tau_0} \\ -\frac{\partial^2 L}{\partial \tau \partial \beta} & -\frac{\partial^2 L}{\partial \tau^2} & -\frac{\partial^2 L}{\partial \tau \partial \tau_0} \\ -\frac{\partial^2 L}{\partial \tau_0 \partial \beta} & -\frac{\partial^2 L}{\partial \tau_0 \partial \tau} & -\frac{\partial^2 L}{\partial \tau_0^2} \end{bmatrix} \quad (5)$$

A elements of  $F$  are as shown below.

$$E\left[-\frac{\partial^2 L}{\partial \beta^2}\right] = \int_{\tau_0}^{\infty} -r \left(-\beta^{-2} - \left(\frac{t - \tau_0}{\tau}\right)^\beta \left(\ln \left(\frac{t - \tau_0}{\tau}\right)^2\right) \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt + \int_{\tau_0}^{\infty} (n - r) \left(\frac{t - \tau_0}{\tau}\right)^\beta \left(\ln \left(\frac{t - \tau_0}{\tau}\right)\right)^2 \times \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt \quad (6)$$

$$E\left[-\frac{\partial^2 L}{\partial \beta \partial \tau}\right] = \int_{\tau_0}^{\infty} -r \left(-\tau^{-1} + \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta \ln \left(\frac{t - \tau_0}{\tau}\right) \tau^{-1} + \left(\frac{t - \tau_0}{\tau}\right)^\beta \tau^{-1}\right) \times \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt + \int_{\tau_0}^{\infty} -(n - r) \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta \ln \left(\frac{t - \tau_0}{\tau}\right) \tau^{-1} - (n - r) \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt \quad (7)$$

$$E\left[-\frac{\partial^2 L}{\partial \beta \partial \tau_0}\right] = \int_{\tau_0}^{\infty} -r \left(-(t - \tau_0)^{-1} + \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta \ln \left(\frac{t - \tau_0}{\tau}\right) (t - \tau_0)^{-1} + \left(\frac{t - \tau_0}{\tau}\right)^\beta (t - \tau_0)^{-1}\right) \times e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt + \int_{\tau_0}^{\infty} -(n - r) \left(\frac{t - \tau_0}{\tau}\right)^\beta \times \beta \ln \left(\frac{t - \tau_0}{\tau}\right) (t - \tau_0)^{-1} - (n - r) \left(\frac{t - \tau_0}{\tau}\right)^\beta \times (t - \tau_0)^{-1} \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt \quad (8)$$

$$E\left[-\frac{\partial^2 L}{\partial \tau^2}\right] = \int_{\tau_0}^{\infty} -r \left(\tau^{-2} + \frac{\beta - 1}{\tau^2} - \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta^2 \tau^{-2} - \left(\frac{t - \tau_0}{\tau}\right)^\beta \beta \tau^{-2}\right) \beta \left(\frac{t - \tau_0}{\tau}\right)^{\beta-1} e^{-\left(\frac{t - \tau_0}{\tau}\right)^\beta} \tau^{-1} dt +$$

$$\int_{\tau_0}^{\infty} ((n-r) \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta^2 \tau^{-2} + (n-r) \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta \tau^{-2}) \times \beta \left( \frac{t-\tau_0}{\tau} \right)^{\beta-1} e^{-\left( \frac{t-\tau_0}{\tau} \right)^{\beta}} \tau^{-1} dt \quad (9)$$

$$E\left[-\frac{\partial^2 L}{\partial \tau \partial \tau_0}\right] = \int_{\tau_0}^{\infty} \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta^3 \left( \frac{t-\tau_0}{\tau} \right)^{\beta-1} e^{-\left( \frac{t-\tau_0}{\tau} \right)^{\beta}} (t-\tau_0)^{-1} \tau^{-2} dt + \int_{\tau_0}^{\infty} (n-r) \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta^3 \times \left( \frac{t-\tau_0}{\tau} \right)^{\beta-1} e^{-\left( \frac{t-\tau_0}{\tau} \right)^{\beta}} (t-\tau_0)^{-1} \tau^{-2} dt \quad (10)$$

$$E\left[-\frac{\partial^2 L}{\partial \tau_0^2}\right] = \int_{\tau_0}^{\infty} -r \left( -\frac{\beta-1}{(t-\tau_0)^2} - \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta^2 (t-\tau_0)^{-2} + \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta (t-\tau_0)^{-2} \beta \left( \frac{t-\tau_0}{\tau} \right)^{\beta-1} e^{-\left( \frac{t-\tau_0}{\tau} \right)^{\beta}} \tau^{-1} dt + \int_{\tau_0}^{\infty} ((n-r) \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta^2 (t-\tau_0)^{-2} - (n-r) \times \left( \frac{t-\tau_0}{\tau} \right)^{\beta} \beta (t-\tau_0)^{-2} \beta \left( \frac{t-\tau_0}{\tau} \right)^{\beta-1} e^{-\left( \frac{t-\tau_0}{\tau} \right)^{\beta}} \tau^{-1} dt \quad (11)$$

$F$  is a symmetric matrix. It is quite hard to compute these elements. Therefore, Fisher information is generally replaced with local information. We incorporate numerical integration techniques to obtain the true Fisher information. The asymptotic covariance matrix( $V$ ) of the ML estimators  $\hat{\beta}$ ,  $\hat{\tau}$  and  $\hat{\tau}_0$  is the inverse of the Fisher information matrix. The ML estimate of the covariance matrix is obtained by substituting  $\hat{\beta}$ ,  $\hat{\tau}$  and  $\hat{\tau}_0$  in the equation 5. The diagonal elements of  $V$  yield the variances of the ML estimators of  $\beta$ ,  $\tau$ ,  $\tau_0$  respectively. Variance of the ML estimates computed using  $F$  and local information ( $L$ ) are given in table III. Here,  $r/n$  represents the censoring level.

TABLE III  
VARIANCE OF THE ESTIMATES CONSIDERING  $F$  AND  $L$

| $r/n$ | $\sigma_{\beta F}$ | $\sigma_{\tau F}$ | $\sigma_{\tau_0 F}$ | $\sigma_{\beta L}$ | $\sigma_{\tau L}$ | $\sigma_{\tau_0 L}$ |
|-------|--------------------|-------------------|---------------------|--------------------|-------------------|---------------------|
| 1.00  | 0.220              | 224.7             | 22.8                | 0.28               | 223.7             | 59.2                |
| 0.95  | 0.232              | 222.8             | 23.3                | 0.30               | 225.1             | 64.8                |
| 0.90  | 0.235              | 221.1             | 24.3                | 0.38               | 228.1             | 73.7                |
| 0.85  | 0.245              | 225.6             | 25.7                | 0.49               | 242.5             | 92.4                |
| 0.80  | 0.253              | 225.3             | 26.7                | 0.75               | 271.2             | 137.1               |
| 0.75  | 0.263              | 231.9             | 27.6                | 0.80               | 489.7             | 489.3               |
| 0.70  | 0.280              | 239.9             | 30.2                | 0.72               | 265.1             | 126.2               |
| 0.65  | 0.292              | 242.3             | 31.2                | 0.57               | 284.2             | 98.3                |
| 0.60  | 0.336              | 257.8             | 38.4                | 0.39               | 308.2             | 67.7                |
| 0.55  | 0.318              | 251.3             | 33.3                | 0.45               | 303.1             | 72.4                |
| 0.50  | 0.346              | 260.5             | 38.5                | 0.26               | 362.0             | 47.0                |

$\sigma_{\beta L}$  and  $\sigma_{\beta F}$  are standard deviation of the parameter  $\beta$ , considering  $F$  and  $L$  respectively. The estimated variance of the point estimates considering  $F$  is found to be less than that of  $L$ .

#### V. CONFIDENCE INTERVAL ESTIMATION AND COVERAGE PROBABILITY

Lawless [4] discussed exact conditional intervals for the parameters and quantiles of the extreme value (EV) and normal

distributions having complete or failure censored data. Recently Jeng [3] has made a survey of the performance of the CI procedures and compared the performance based on coverage probability (CP) whose practical aspects are discussed in [5]. The exact intervals are difficult to obtain in practice and are unavailable in some cases. As a consequence approximate large sample intervals are widely used. Authors consider the two methods namely, the method of normal approximations and log likelihood ratio method (LLR) to demonstrate the effect of censoring on CI's.

With a completely specified continuous probability distribution  $F(t; \theta)$ , an exact  $100(1 - \alpha)\%$  CI is given as:

$$CI(1 - \alpha) = (T, \bar{T}) = (t_{\alpha/2}, t_{1-\alpha/2}) \quad (12)$$

Where,  $t_p$  is the  $p$  quantile of  $F(t; \theta)$ .

$T$  and  $\bar{T}$  are lower and upper bound for the estimate of  $T$ . The probability of the coverage of the interval is,

$$\begin{aligned} P[T \in CI(1 - \alpha); \theta] &= P(T \leq T \leq \bar{T}; \theta) \\ &= P(t_{\alpha/2} \leq T \leq t_{1-\alpha/2}; \theta) \\ &= 1 - \alpha \end{aligned} \quad (13)$$

The coverage probability for the CI procedure is given by,

$$CP[CI(1 - \alpha)|\hat{\theta}] = F(T; \theta) - F(\bar{T}; \theta) \quad (14)$$

for ML estimate of  $\theta$  this turns out to be,

$$CP[CI(1 - \alpha)|\hat{\theta}] = E_{\hat{\theta}} \{ CP[CI(1 - \alpha)|\hat{\theta}; \theta] \} \quad (15)$$

where the expectation is with respect to random  $\hat{\theta}$ .

#### A. Confidence Intervals using Asymptotic Normality of ML Estimators

Let  $\hat{\theta}$  be the ML estimator for a parameter  $\theta$  with the estimate of its variance  $var(\hat{\theta})$  obtained from the inverse of observed  $F$  or  $L$ . The upper ( $\theta_U$ ) and lower ( $\theta_L$ ) confidence limits for  $\theta$  are:

$$\begin{aligned} \theta_U &= \hat{\theta} + k_{1-\alpha/2} \sqrt{var(\hat{\theta})} \\ \theta_L &= \hat{\theta} - k_{1-\alpha/2} \sqrt{var(\hat{\theta})} \end{aligned} \quad (16)$$

where  $k_{\alpha}$  denotes the  $\alpha$  quantile of the standard normal distribution and  $100(1 - \alpha)$  is called as the percentage confidence level. Here, the authors intention is to compare the confidence intervals using the estimated asymptotic variances of the distribution parameters  $\beta$ ,  $\tau$  and  $\tau_0$  considering  $F$  and  $L$ . Although, there are more accurate methods of CI estimation for extreme value distributions, the method explained above gives is good enough for the comparison purpose, since, both CI's (using the variance obtained from  $F$  and  $L$ ) are obtained with the same method which is shown in table IV. The coverage probabilities of the CI's considering the variance of ML estimates computed using  $F$  and  $L$  are given in the table V. CI length (that of  $F$ ) is taken as the reference and that of  $L$  is compared. The coverage probability of CI's considering  $L$  was found to be less than that of  $F$ .

TABLE IV  
CONFIDENCE INTERVALS CONSIDERING USING ASYMPTOTIC NORMALITY  
OF ML ESTIMATES CONSIDERING  $F$  AND  $L$

| $r/n$          | 1.00 | 0.95   | 0.90  | 0.85  | 0.80  | 0.75  | 0.70  |
|----------------|------|--------|-------|-------|-------|-------|-------|
| $\hat{\beta}$  | 1.17 | 1.177  | 1.161 | 1.146 | 1.141 | 1.139 | 1.124 |
| $\beta_{LF}$   | 0.73 | 0.72   | 0.69  | 0.66  | 0.64  | 0.62  | 0.57  |
| $\beta_{uF}$   | 1.61 | 1.62   | 1.62  | 1.62  | 1.63  | 1.65  | 1.67  |
| $\beta_{LL}$   | 0.62 | 0.54   | 0.42  | 0.18  | 0.00  | 0.00  | 0.00  |
| $\beta_{uL}$   | 1.73 | 1.81   | 1.90  | 2.11  | 2.61  | 2.80  | 2.54  |
| $\hat{\tau}$   | 1117 | 1113   | 1114  | 1117  | 1117  | 1116  | 1123  |
| $\tau_{LF}$    | 676  | 676    | 681   | 675   | 675   | 662   | 652   |
| $\tau_{uF}$    | 1557 | 1550   | 1548  | 1559  | 1559  | 1571  | 1593  |
| $\tau_{LL}$    | 678  | 672    | 667   | 642   | 586   | 590   | 603   |
| $\tau_{uL}$    | 1556 | 1554.7 | 1562  | 1593  | 1649  | 1650  | 1642  |
| $\hat{\tau}_0$ | 46.1 | 45.8   | 46.8  | 47.6  | 47.6  | 47.5  | 48.2  |
| $\tau_{0LF}$   | 1.47 | 0.24   | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  |
| $\tau_{0uF}$   | 90.8 | 91     | 95    | 98    | 100   | 102   | 107   |
| $\tau_{0LL}$   | 0.00 | 0.00   | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  |
| $\tau_{0uL}$   | 162  | 172    | 194   | 228   | 316   | 316   | 295   |

TABLE V  
COVERAGE PROBABILITY OF CI'S CONSIDERING  $F$  AND  $L$

| $r/n$          | 1.0  | 0.95 | 0.90 | 0.85 | 0.80 | 0.75 | 0.70 |
|----------------|------|------|------|------|------|------|------|
| $CP(\beta)_F$  | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| $CP(\tau)_F$   | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| $CP(\tau_0)_F$ | 0.95 | 0.95 | 0.95 | 0.94 | 0.93 | 0.93 | 0.92 |
| $CP(\beta)_L$  | 0.88 | 0.83 | 0.77 | 0.67 | 0.49 | 0.44 | 0.48 |
| $CP(\tau)_L$   | 0.95 | 0.94 | 0.94 | 0.93 | 0.89 | 0.90 | 0.89 |
| $CP(\tau_0)_L$ | 0.55 | 0.52 | 0.48 | 0.40 | 0.35 | 0.32 | 0.34 |

To graphically demonstrate the effect of censoring as well as the effect of approximations in finding out variances of the estimated parameters, a plot of ML estimates and CI's for different censoring levels is shown in the Fig. 1. The solid line (center) represents the ML estimates of  $\beta$ , under different censoring levels. The dotted lines and dashed lines represent the CI's with  $F$  and  $L$  respectively.

It is evident from the Fig. 1 that, for a given CP, the CI's considering  $F$  has considerable narrower bound compared to that of  $L$ . This vindicates that, in the analysis of extreme value data, approximating  $F$  with  $L$  would give rise to incorrect estimates.

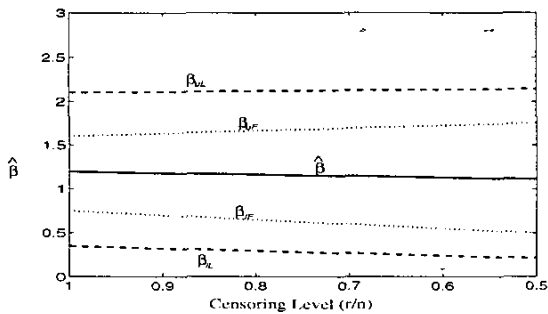


Fig. 1. A regression plot of ML estimate and CI's of  $\beta$  Vs Censoring Level ( $r/n$ ) considering  $F$  and  $L$

### B. Log Likelihood Ratio Procedure

An log likelihood ratio (LLR) procedure is employed for further understanding of effect of censoring on CI's. The profile

likelihood for any given parameter  $\theta_1$  is defined as [3].

$$R(\theta_1) = \max_{\theta_o} \left[ \frac{L(\theta_1, \theta_o)}{L(\hat{\theta})} \right] \quad (17)$$

where,  $\theta_o$  represents remaining parameters of the distribution and  $\hat{\theta}$  is the ML estimates. Let  $W(\theta_1) = -2\log R(\theta_1)$ . The limiting distribution of  $W$  is  $\chi^2_1$ . Let  $\chi^2_{(1-\alpha,1)}$  denote the  $(1-\alpha)$  quantile of the  $\chi^2$  distribution with one degree of freedom. The equation  $W(\theta_1) - \chi^2_{(1-\alpha,1)} = 0$ , generally has two roots one less than and one greater than  $\hat{\theta}_1$ . The LLR CI procedure uses the roots as the lower and upper confidence bounds. The estimated confidence bounds for different censoring levels is as shown in table VI.

TABLE VI  
CONFIDENCE INTERVAL ESTIMATION USING LLR METHOD

| $r/n$          | 1.0  | 0.95 | 0.90 | 0.85 | 0.80 | 0.75 | 0.70 |
|----------------|------|------|------|------|------|------|------|
| $\hat{\beta}$  | 1.17 | 1.17 | 1.16 | 1.14 | 1.14 | 1.14 | 1.13 |
| $\beta_L$      | 1.09 | 1.07 | 1.01 | 0.80 | 0.40 | 0.74 | 0.82 |
| $\beta_U$      | 1.27 | 1.28 | 1.30 | 1.48 | 1.87 | 1.53 | 1.42 |
| $\hat{\tau}$   | 1117 | 1113 | 1114 | 1117 | 1117 | 1116 | 1122 |
| $\tau_L$       | 299  | 293  | 281  | 270  | 265  | 262  | 256  |
| $\tau_U$       | 1934 | 1933 | 1948 | 1964 | 1969 | 1970 | 1990 |
| $\hat{\tau}_0$ | 46.1 | 45.8 | 46.8 | 47.6 | 47.6 | 47.5 | 48.2 |
| $\tau_{0L}$    | 16.8 | 16.0 | 16.8 | 17.2 | 16.7 | 16.1 | 16.0 |
| $\tau_{0U}$    | 75.5 | 75.5 | 76.7 | 78.0 | 78.5 | 78.9 | 80.3 |

As seen in the table, the level of uncertainty grows by increase in the censoring level or the CI's widen by the increment in censoring level. This is basically due to, unavailability of useful data by censoring operation.

### VI. CONCLUSION

Based on the work, following broad conclusions can be drawn.

- The asymptotic variance of estimated parameters, calculated on the basis of local information would lead to inaccurate results. Thus, the approximations in reducing the computation is not valid in case of extreme value distribution.
- The variance of the estimated parameters enhances, by, increasing the censoring level, leading to unreliable life estimates.

### REFERENCES

- [1] Nelson, W.B. Applied Life Data Analysis. John Wiley and Sons.
- [2] Escobar, L. A. and Meeker, W. Q. Statistical Prediction Based on Censored Life Data, Technometrics, Vol. 41, No. 2, May 99, pp. 113-124.
- [3] Geng, S. L. and Meeker, W. Q. Comparison of Approximate Confidence Interval Procedures for Type-I Censored Data, Technometrics, Vol. 42, No. 2, May 2000, pp. 135-148.
- [4] Lawless, H.F. Statistical Models and Methods for Life Time Data, John Wiley and Sons.
- [5] Doganaksoy, N. and Schme, J. Practical Aspects of Corrected Likelihood Ratio Confidence Intervals. A Discussion of Jeng-Meeker and Wong-Lu, Vol. 42, No. 2, May 2000, pp. 156-159.
- [6] Wong, A. C. M and Wu, J. Practical Small Sample Asymptotics for Distribution Used in Life Data Analysis, Vol. 42, No. 2, May 2000, pp. 149-155.
- [7] Meeker, W. Q. and Escobar, L. A. Statistical Analysis for Reliability Data, John Wiley and Sons.
- [8] Ramu, T.S. Hazards of Insulation life estimation using censored data. CEIDP, Wilmington, 1984. pp. 189-194.