Polynomial Coefficient Based Multi-Tone Testing of Analog Circuits

Suraj Sindia[1], Virendra Singh[2], and Vishwani Agrawal[3]
[1]ssuraj@cedt.iisc.ernet.in, [2]viren@serc.iisc.ernet.in, [3]vagrawal@eng.auburn.edu

[1]Centre for Electronic Design and Technology, Indian Institute of Science, Bangalore, India
[2]Supercomputer Education and Research Centre, Indian Institute of Science, Bangalore, India
[3]Department of Electrical and Computer Engineering, Auburn University, Alabama, AL, USA

***Abstract*—A method of testing for parametric faults of analog circuits based on a polynomial representaion of fault-free function of the circuit is presented. The response of the circuit under test (CUT) is estimated as a polynomial in the applied input voltage at relevant frequencies apart from DC. Classification of CUT is based on a comparison of the estimated polynomial coefficients with those of the fault free circuit. The method needs very little augmentation of circuit to make it testable as only output parameters are used for classification. This procedure is shown to uncover several parametric faults causing smaller than 5% deviations the nominal values. Fault diagnosis based upon sensitivity of polynomial coefficients at relevant frequencies is also proposed.**

***Index Terms*—Multi-tone test, Parametric faults, Analog circuit test, Curve fitting, Polynomial**

I. Introduction

An analog circuit is called either linear or non-linear based on the type of input-output behavior it displays [1], [2]. Linear circuits preserve linearity and homogeneity of output with the input, and can be described by a linear constant coefficient differential equation [3]. Typically, in the time domain, the output $y(t)$ may be expressed as a function of input $x(t)$, as follows:

$$\sum_{m=1}^{M} a_m \frac{d^m y}{dt^m} + a_0 y = \sum_{n=1}^{N} b_n \frac{d^n x}{dt^n} + b_0 x \qquad (M > N) \quad (1)$$

where $a_m, b_n \in \Re \ \forall \ m, n \in$ Z.

The general solution for (1) is of the form (2), where $H(t) \in \Re$ is a real function of t.

$$y(t) = H(t)x(t) \quad (2)$$

Linear circuits are mainly composed of passive components [1]. Typical examples include RC and LC ladder filters and resistive attenuators among others.

In case of non-linear circuits, coefficients $a_m, b_n \ \forall m, \ n$ in (1) are functions of x and a general solution in time domain for such circuits can be expressed as in (3), where $H_n \ \forall n$ are real functions of t.

$$y(t) = \sum_{n=1}^{n=N} H_n(t) x^n(t) \quad (3)$$

Testing of linear circuits is well studied and several methods can be found in the literature [4], [5], [6], [7]. Savir and Guo [4] describe a method in which the circuit is modeled as a linear time-invariant (LTI) system. They obtain the transfer function of the circuit in the frequency domain, which is of the following form:

$$H(s) = \frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{N} b_i s^i} \qquad (M < N) \quad (4)$$

The coefficients of the transfer function, i.e., a_i and b_i, are all functions of circuit parameters and these are tracked to monitor drift in circuit parameters. The CUT is subjected to frequency rich input signals and the output voltage alone is observed. With these input-output pairs they estimate the transfer function coefficients of CUT. Next they compare these transfer function coefficient estimates with the ideal circuit transfer function coefficients, which are known a priori. The CUT is classified faulty if any of the estimated coefficients is beyond the tolerable range. For example, the circuit shown in Figure 1 is a second order low pass filter and has a transfer function given below:

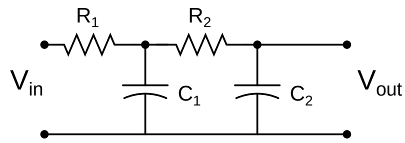

Fig. 1. Second order low pass filter

$$H(s) = \frac{1}{(R_1 R_2 C_1 C_2)\, s^2 + (R_1 C_1 + (R_1 + R_2) C_2)\, s + 1} \quad (5)$$

Clearly the coefficients of the transfer function, $b_0 = 1$, $b_1 = (R_1 C_1 + (R_1 + R_2) C_2)$, $b_2 = R_1 R_2 C_1 C_2$, are functions of circuit parameters R_1, R_2, C_1, C_2. Assuming single parametric faults, they find the minimum drift in any of the circuit component values that will cause the coefficients b_1 or b_2 (b_0 here is a constant) to drift outside a tolerance range. However, this method [4] *necessarily needs the CUT to be linear*, as a frequency domain transfer function is possible only for a LTI system.

Several methods have been proposed for parametric fault testing of non-linear circuits [8], [9], [10], [11], [12], [13], [14], [15]. A prominent method in the industry is the I_{DDQ} testing where quiescent current from the supply rail is monitored and sizable deviations from its expected value are monitored. However, this requires augmentation of the CUT. For example, in the simplest case a regulator supplying power to any sizable circuit has to be augmented with a current sensing resistor and an ADC (for digital output). Subsequently, analysis is performed on the sensed current. I_{DDQ} is found suitable only for catastrophic faults as the current drawn from the supply may be distinguishable when there is some "large enough" fault to change the quiescent current by a distinguishable amount. For example, with resistor R_2 being open in Figure 2, the current drawn from supply can change by 50% of its nominal quiescent value. Such faults can typically be found by monitoring I_{DDQ} using a current sensor. However, parametric deviations, say, less than 10% from their nominal value cannot be observed using this scheme. This is especially so for the very deep submicron circuits where the leakage currents can be comparable to the defect induced current [16]. It is therefore useful to develop a method to detect parametric faults while testing with less circuit augmentation.

To address the issue of parametric deviation, we would typically need more observables to have an idea about the parametric drift in circuit parameters. This would mean an increase in the complexity of the sensing circuit. However, we would also want minimal augmentation to tap any of the internal circuit nodes or currents. To overcome these seemingly contrasting requirements the method intended should have some way of "seeing through" the circuit with only the outputs and inputs at its disposal. References [4], [7] give such strategies for linear circuits as described earlier.

To extend this idea to general non-linear circuits we adopt a strategy where we express the function of the circuit as a polynomial using a Taylor series expansion [17] in terms of input voltage v_{in}, about the point v_{in} = 0 as follows:

$$v_{out} = f(v_{in}) = f(0) + \frac{f'(0)}{1!}v_{in} + \frac{f''(0)}{2!}v_{in}^2 + \frac{f'''(0)}{3!}v_{in}^3 + \cdots + \frac{f^{(n)}(0)}{n!}v_{in}^n + \cdots \quad (6)$$

where $f(x)$ is a real function of x.

This method is very general as any analog circuit can be tested using this model. The technique applies equally well to linear circuits, which are a subclass of the general non-linear circuits considered in this paper. The accuracy, resolution and observability of faults uncovered depends on the degree of expansion of the coefficients in (7). Ignoring the higher order terms in (6), we can expand v_{out} up to the n^{th} power of v_{in}, which gives us the approximation in (7). In order to increase the available observables to better track down parametric faults we can expand v_{out} at multiple frequencies. Thus, we will have $m \times (n+1)$ observables where m is the number of tones (frequencies) including DC at which v_{out} is expanded and n is the degree of expansion [18]:

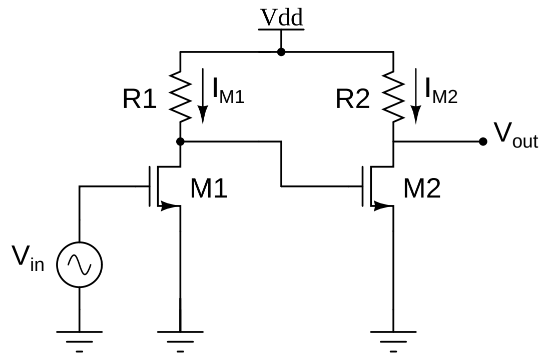

Fig. 2. Cascaded amplifier

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + \cdots + a_n v_{in}^n \quad (7)$$

where $a_0, a_1, a_2, \ldots, a_n$ are all real functions of circuit parameters $p_k \forall\ k$.

The special case of DC test, that detects a subset of faults, was given in a recent paper [19]. Further, we assume that normal parameter variations (normal drift) in a good circuit are within a fraction α of their nominal value, where $\alpha << 1$. That is, every parameter p_i is allowed to vary within the range $p_{k,nom}(1-\alpha) < p_k < p_{k,nom}(1+\alpha)\ \forall k$, where $p_{k,nom}$ is the nominal value of parameter p_k. Whenever one or more of the coefficient values slip outside its individual hypercube we get a different set of coefficients reflecting a detectable fault. Therefore, equation (8) describes the hypercube for all parameters that correspond to either good machine values or undetectable parametric faults [4], [9], [15]:

$$a_{i,\min} < a_i < a_{i,\max} \quad \forall\ i, \quad 0 \le i \le n \quad (8)$$

This paper is organized as follows. Section 2 analyzes the coefficients of the polynomial expansion of the function $f(v_{in})$ and determines the detectable fault sizes of parameters. In Section 3, we describe the problem at hand and discuss the proposed solution with an example. In Section 4, we generalize the solution to an arbitrarily large circuit. Section 5 presents the simulation results for some standard circuits. Section 6 outlines the method of fault diagnosis using the proposed method and we conclude in Section 7.

II. Preliminaries

The coefficients $a_i\ \forall i\ 0 \le i \le n$ are, in general, non-linear functions of circuit parameters $p_k\ \forall k$. The rationale behind using these coefficients as metrics in classifying CUT as faulty or fault free is based on the dependence of the coefficients on circuit parameters.

A. Analysis of Polynomial Coefficients

We derive several significant results.

Theorem 1: If coefficient a_i is a monotonic function of all parameters, then a_i takes its limiting (maximum and minimum) values when at least one or more of the parameters are at the boundaries of their individual hypercube.

Lemma 1: If coefficient a_i is a non-monotonic function of one or more circuit parameters p_i, then a_i can take its limiting values anywhere inside the hypercube enclosing the parameters.

From Theorem 1 and Lemma 1 it is clear that by exhaustively searching the space in the hypercube of each parameter we can get the maximum and minimum values of the polynomial coefficient. Typically this can be formulated as a non-linear optimization problem to find the maximum and minimum values of coefficient with constraints on parameters allowing only a normal drift.

Theorem 2: In polynomial expansion of non-linear analog circuit there exists at least one coefficient that is a monotonic function of all circuit parameters.

From Lemma 1 and Theorem 2 we find that circuit parameter deviations have a bearing on coefficients and monotonically varying coefficients can be used to detect parametric faults of the circuit parameters.

Theorem 3: A continuous non-monotonic function $f : \Re \rightarrow \Re$ can be decomposed into piecewise monotonic functions as follows:

$$\begin{aligned} f(x) = f(x)u(x_0 - x) + f(x)\left(u(x - x_0) - u(x - x_1)\right) + \\ f(x)\left(u(x - x_1) - u(x - x_2)\right) + \cdots \\ + f(x)\left(u(x - x_{n-1}) - u(x - x_n)\right) \end{aligned} \tag{9}$$

where $x_0, x_1, \cdots x_n$ are all stationary points of $f(x)$ and $u(x) = \begin{cases} 1 & \forall\, x \geq 0 \\ 0 & \forall\, x < 0 \end{cases}$

Using Theorem 3, we can express every polynomial coefficient as a monotonic function of circuit parameters and thus we can use every coefficient to track the drifts in circuit parameters.

B. Definitions

Definition 1: Minimum size detectable fault (MSDF), (ρ) of a parameter is defined as the minimum fractional deviation of a circuit parameter from its nominal value for it to be detectable with all other parameters being held at their nominal values. The fractional deviation can be positive or negative and is named upside-MSDF (UMSDF) or downside-MSDF (DMSDF), accordingly.

Definition 2: Nearly-minimum size detectable fault (NMSDF), (ρ^*) of a parameter is defined as some fractional deviation of the circuit parameter from its nominal value with all the other parameters being held at their nominal values that is close to its MSDF with an error, ϵ (infinitesimally small). That is to say,

$$\epsilon = |\rho - \rho*| \qquad \epsilon << 1 \tag{10}$$

NMSDF also has notions of upside and downside as in case of MSDF. In equation (10), ϵ can be perceived as a coefficient of uncertainty about the MSDF of a parameter. Let ψ be the set of all coefficient values spanned by the parameters while varying within their normal drifts, i.e.,

$$\begin{aligned} \psi = \{\upsilon_0, \upsilon_1, \cdots, \upsilon_n \,|\upsilon_0 \in A_0, \upsilon_1 \in A_1, \cdots, \upsilon_n \in A_n\} \\ \forall_k \quad p_{k,nom}(1-\alpha) < p_k < p_{k,nom}(1+\alpha) \end{aligned}$$

Note that by Definitions 1 and 2, ψ includes all possible values of coefficients that are not detectable. Any parametric fault inducing coefficient value outside this set ψ will result in a detectable fault.

III. Problem Description and Sketch of Solution

We shall first give an illustrative example of calculation of limits for polynomial coefficients for a simple circuit using MOS transistors. We shall follow this up with MSDF values for the circuit parameters.

Example . Two stage amplifier

Consider the cascaded amplifier shown in Figure 2. The output voltage V_{out} in terms of input voltage results in a fourth degree polynomial equation as follows:

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4 \tag{11}$$

where the constants a_0, a_1, a_2, a_3 are defined symbolically in (12) for M1 and M2 operating in saturation region. Nominal values of V_{DD}=1.2V, V_T= 400mV, $\left(\frac{W}{L}\right)_1 = \frac{1}{2}\left(\frac{W}{L}\right)_2 = 20$, and $K = 100\mu A/V^2$ are substituted to get coefficients in terms of parameters R_1 and R_2 as given by (13).

$$\begin{aligned}
a_0 &= V_{DD} - R_2 K\left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} (V_{DD} - V_T)^2 + \\ R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^4 - \\ 2(V_{DD} - V_T) R_1 \left(\tfrac{W}{L}\right)_1 V_T^2 \end{array} \right\} \\
a_1 &= R_2 K\left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 4R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^3 \\ +2(V_{DD} - V_T) R_1 K \left(\tfrac{W}{L}\right)_1 V_T \end{array} \right\} \\
a_2 &= R_2 K\left(\tfrac{W}{L}\right)_2 \left\{ \begin{array}{l} 2(V_{DD} - V_T) R_1 K \left(\tfrac{W}{L}\right)_1 \\ -6R_1^2 K^2 \left(\tfrac{W}{L}\right)_1^2 V_T^2 \end{array} \right\} \\
a_3 &= 4V_T K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2 \\
a_4 &= -K^3 \left(\tfrac{W}{L}\right)_1^2 \left(\tfrac{W}{L}\right)_2^2 R_1^2 R_2
\end{aligned} \tag{12}$$

$$\begin{aligned}
a_0 &= 1.2 - R_2 \left(\begin{array}{l} 2.56 \times 10^{-3} + 1.024 \times 10^{-7} R_1^2 \\ -5.12 \times 10^{-4} R_1 \end{array} \right) \\
a_1 &= 4.096 \times 10^{-9} R_1^2 R_2 + 5.12 \times 10^{-6} R_1 R_2 \\
a_2 &= 1.28 \times 10^{-5} R_1 R_2 - 1.536 \times 10^{-8} R_1^2 R_2 \\
a_3 &= 2.56 \times 10^{-8} R_1^2 R_2 \\
a_4 &= 1.6 \times 10^{-8} R_1^2 R_2
\end{aligned} \tag{13}$$

To find the limiting values of the coefficient a_0 we assume the parameters R_1 and R_2 deviate by fractions x and y from their nominal values, respectively. Maximizing a_0 we have the objective function as given by (14), subject to constraints in (15–19). Note that here we have set out to find MSDF of R_1. Similar approach can be used to find the MSDF of R_2.

TABLE I
MSDF FOR CASCADED AMPLIFIER OF FIGURE 2 WITH α = 0.05.

Circuit parameter	%upside MSDF	%downside MSDF
Resistor R_1	10.3	7.4
Resistor R_2	12.3	8.5

$$1.2 - R_{2,nom}(1+y)\left\{\begin{array}{l} 2.56\times10^{-3}+ \\ 1.024\times10^{-7}R_{1,nom}^2(1+x)^2 \\ -5.12\times10^{-4}R_{1,nom}(1+x)\end{array}\right\} \tag{14}$$

$$\begin{aligned} &4.096\times10^{-9}R_{1,nom}^2(1+x)^2R_{2,nom}(1+y) \\ &+5.12\times10^{-6}R_{1,nom}(1+x)R_{2,nom}(1+y) \\ &\quad=4.096\times10^{-9}R_{1,nom}^2(1+\rho)^2R_{2,nom} \\ &\quad+5.12\times10^{-6}R_{1,nom}(1+\rho)R_{2,nom}\end{aligned} \tag{15}$$

$$\begin{aligned} &1.28\times10^{-5}R_{1,nom}(1+x)R_{2,nom}(1+y) \\ &-1.536\times10^{-8}R_{1,nom}^2(1+x)^2R_{2,nom}(1+y) \\ &\quad=1.28\times10^{-5}R_{1,nom}(1+\rho)R_{2,nom} \\ &\quad-1.536\times10^{-8}R_{1,nom}^2(1+\rho)^2R_{2,nom}\end{aligned} \tag{16}$$

$$\begin{aligned} &2.56\times10^{-8}R_{1,nom}^2(1+x)^2R_{2,nom}(1+y) \\ &\quad=2.56\times10^{-8}R_{1,nom}^2(1+\rho)^2R_{2,nom}\end{aligned} \tag{17}$$

$$\begin{aligned} &1.6\times10^{-8}R_{1,nom}^2(1+x)^2R_{2,nom}(1+y) \\ &\quad=1.6\times10^{-8}R_{1,nom}^2(1+\rho)^2R_{2,nom}\end{aligned} \tag{18}$$

$$-\alpha \le x, y \le \alpha \tag{19}$$

The extreme values for x and y on solving the set of equations (15–19) are obtained as, x = $-\alpha$ and y = $-\alpha$, this gives us the MSDF value for R_1, as ρ in (20).

$$\rho = (1-\alpha)^{1.5} - 1 \approx 1.5\alpha - 0.375\alpha^2 \tag{20}$$

Table I gives the MSDF for R_1 and R_2 based on above calculation.

IV. GENERALIZATION

In general, the calculation as described above cannot be done for an arbitrarily large circuit. Such circuits are handled by obtaining a nominal numeric polynomial expansion of the fault free circuit. This is done by sweeping the input voltage across all possible values and noting the corresponding output voltages using any of the standard circuit simulators like SPICE. Now, the output voltage is plotted against the input voltage. A polynomial is fitted to this curve and the coefficients of this polynomial are taken to be the nominal coefficients of the desired polynomial. The circuit is simulated for different drifts in the parameter values at equally spaced points from inside the hypercube enclosing each circuit parameter, spaced at a suitably chosen resolution(=ϵ). Polynomial coefficients are obtained for each of these simulations. The maximum and the minimum values of a coefficient in this search are taken as the limiting values on that coefficient. This process of modeling the circuit as a polynomial expansion and obtaining

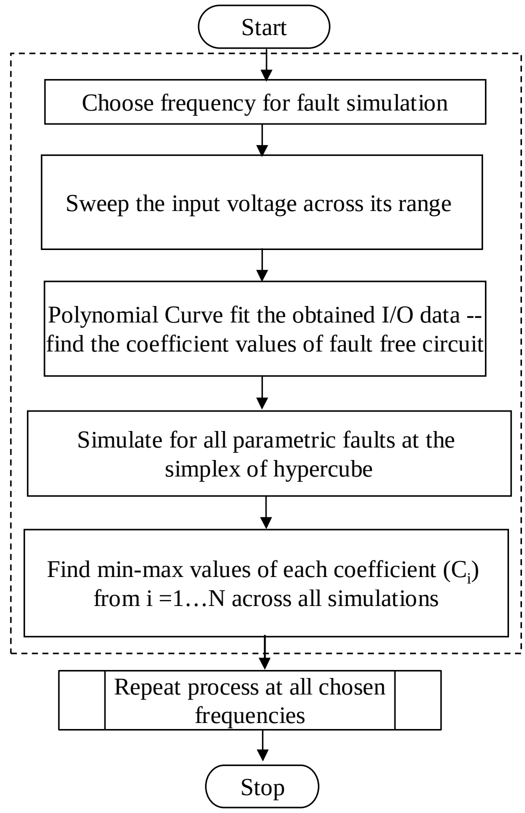

Fig. 3. Flow chart showing fault simulation process and bounding of coefficients.

limit values on coefficients is repeated at "key" frequencies of interest. For example, the cut-off frequency in case of a non-linear filter can be a good candidate for such characterization. Once the limit values on all coefficients have been determined the CUT is subjected to full range of input at DC and each of the "key" frequencies. Its response to input sweep is curve fitted to a polynomial of order same as the fault free circuit. If there are any coefficients that lay outside the limit values of corresponding coefficients of the fault free circuit, we can conclude the CUT is faulty. The converse is also true with a high probability that is inversely proportional to coefficient of uncertainty ϵ. Flow chart in Figure 3 summarizes the process of numerically finding the polynomial and finding the bounds on coefficients. Flow chart in Figure 4 outlines the procedure to test CUT using the described method.

V. EXPERIMENTAL RESULTS

We subjected an elliptic filter shown in Figure 5 to Polynomial Coefficient based test. The circuit parameter values are as in the benchmark circuit maintained by Stroud et al. [20]. We simulated the circuit at four different frequencies. Two of them were chosen close to its 3-dB cut-off frequency(f_c), which is 1000Hz. The estimated polynomial expansion obtained by curve fitting the I/O plots at DC and the frequencies f=100Hz, 900Hz, 1000Hz, 1100Hz are given in (21–25) and the corresponding plots tracing I/O response with polynomial is shown in Figures 6 – 10. The combinations of parameter values

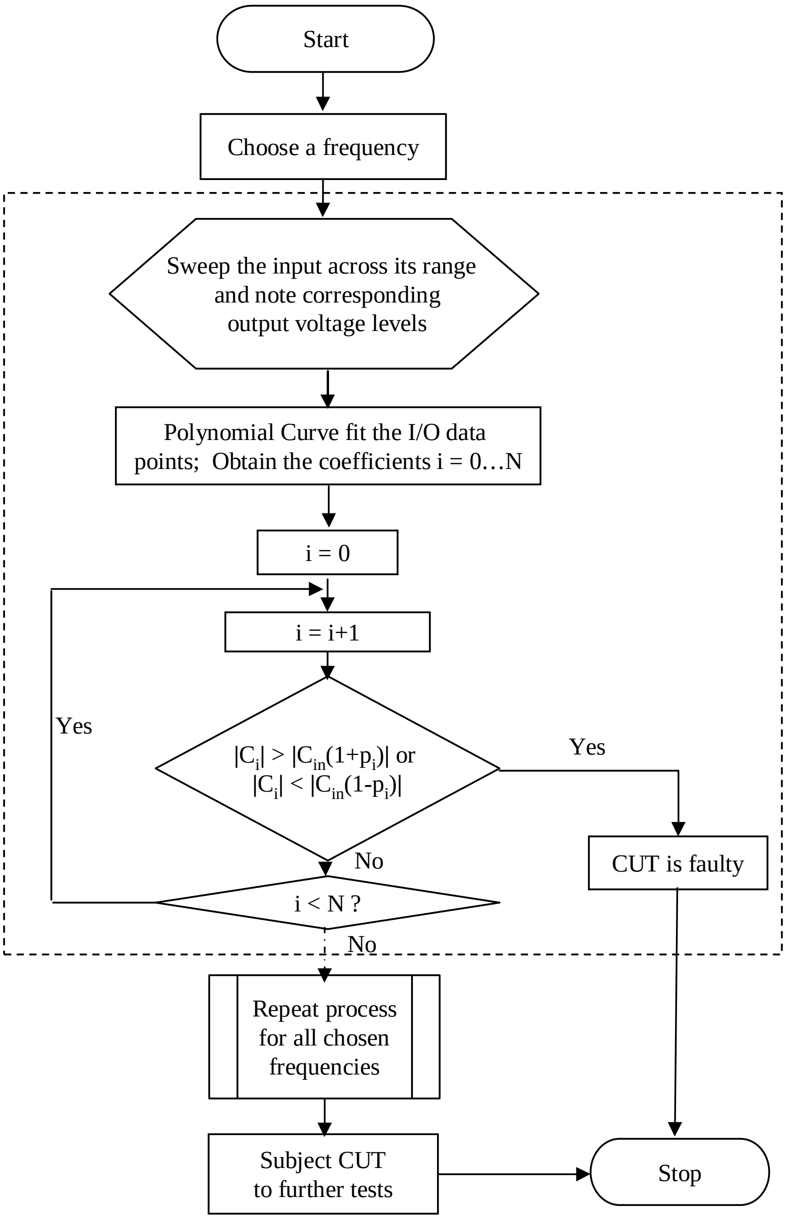

Fig. 4. Flow chart outlining test procedure for CUT.

leading to limits on the coefficients for the tone at 1000Hz are shown in Table II. Further, the pass/fail detectability of several injected faults is tabulated in Table III.

In our ongoing work, we are testing this technique on other common non-linear circuits like logarithmic amplifiers [21].

$$v_{out} = 4.5341 - 3.498v_{in} - 2.5487v_{in}^2 + 2.1309v_{in}^3 - 0.50514v_{in}^4 + 0.039463v_{in}^5 \quad (21)$$

$$v_{out} = 3 + 7.9v_{in} - 11v_{in}^2 + 4.4v_{in}^3 - 0.78v_{in}^4 + 0.049v_{in}^5 \quad (22)$$

$$v_{out} = 2.5 + 5.4v_{in} - 8.6v_{in}^2 + 4v_{in}^3 - 0.77v_{in}^4 + 0.054v_{in}^5 \quad (23)$$

$$v_{out} = 1.1707 + 2.4132v_{in} - 3.8777v_{in}^2 + 1.8035v_{in}^3 - 0.3465v_{in}^4 + 0.023962v_{in}^5 \quad (24)$$

$$v_{out} = 0.23 + 0.48v_{in} - 0.74v_{in}^2 + 0.34v_{in}^3 - 0.063v_{in}^4 + 0.0043v_{in}^5 \quad (25)$$

VI. Fault Diagnosis

Fault diagnosis using sensitivity of output to circuit parameters has been investigated in the literature [22]. We have extended that approach exploiting the sensitivity of polynomial coefficients to circuit parameters. The advantage of the new approach is an improved fault diagnosis without circuit augmentation. Sensitivity of i^{th} coefficient C_i to k^{th} parameter p_k is represented by $S_{p_k}^{C_i}$ and is given by:

$$S_{Pk}^{Ci} = \frac{p_k}{C_i}\frac{\partial C_i}{\partial p_k} \quad (26)$$

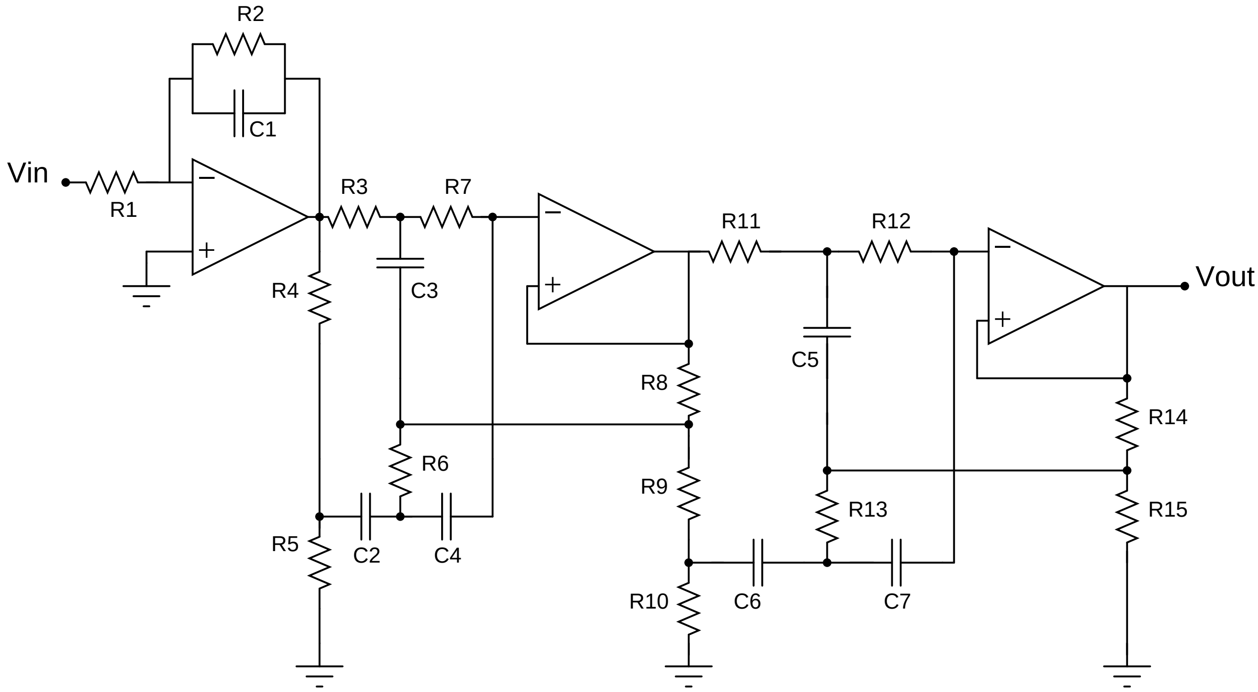

Fig. 5. Elliptic filter

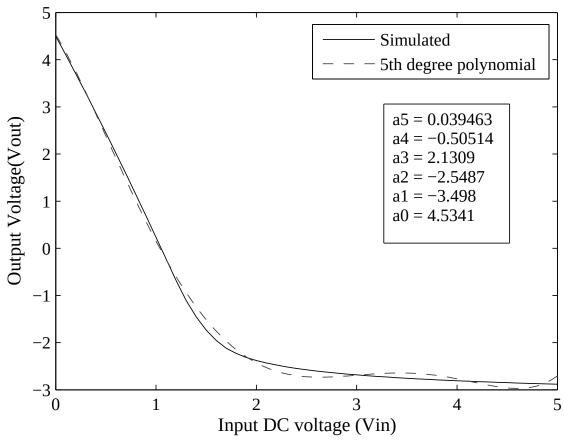

Fig. 6. DC response of Elliptic filter with curve fitting polynomial.

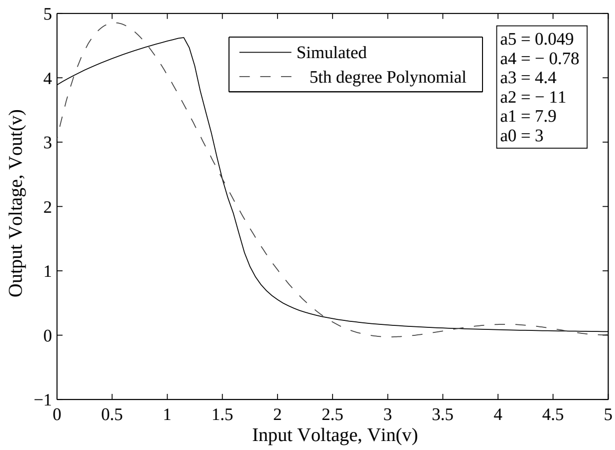

Fig. 7. Curve-fit polynomial with coefficients at frequency = 100Hz.

A. Computation of Sensitivities

Numerical computation of sensitivities given by (26) is accomplished by introducing fractional drifts (=α) in each component ($p_k \ \forall k$); simulating the circuit and measuring the fractional drift in each coefficient of the polynomial resulting from curve fitting operation. This way the numerical sensitivities are computed and a dictionary is maintained for sensitivities. The complexity in computation of sensitivities is linear in the number N of circuit parameters, i.e., $O(N)$.

B. Diagnosing Parametric Faults

Restricting ourselves to single parametric faults, we find the descending order of sensitivities of coefficients (with respect to circuit parameter) that have exceeded their limiting values. The parameter with highest sensitivity is said to be at fault with a probability $\mathrm{P}(\delta \mathrm{p_k}|\delta \mathrm{C_i})$ (which can be interpreted as the confidence in diagnosing fault), given by (27), where δp_k is the suspected drift in parameter p_k and δC_i is the measured drift in coefficient.

$$\mathrm{P}(\delta \mathrm{p_k}|\delta \mathrm{C_i}) = \phi \left(\frac{S_{p_k}^{C_i} \delta p_k}{\delta C_i} \right) \tag{27}$$

Note that ϕ in (27) is a probability measure[23], dependent on $\delta p_k, \delta C_i$ and $S_{p_k}^{C_i}$. For example, if sensitivity of some coefficient, say $\mathrm{C_1}$ to parameter p_1 is 5%, measured drift in

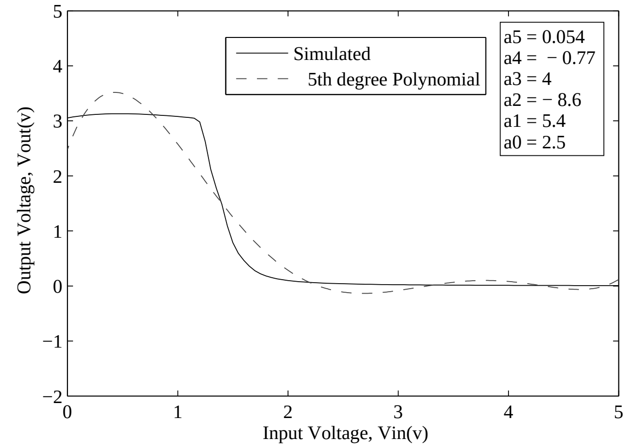

Fig. 8. Curve-fitting polynomial with coefficients at frequency = 900Hz.

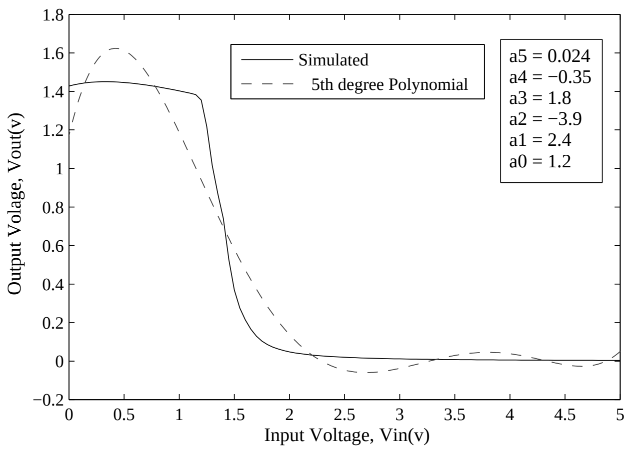

Fig. 9. Curve-fitting polynomial with coefficients at frequency = 1000Hz.

coefficient value is 10% and we suspect that the parameter drift is 10% then the probability of this being true, by assuming ϕ to be an exponential probability measure, is $e^{\frac{-.05\times.1}{.1}} = .95$

C. Fault Deduction

At each frequency, the above process of diagnosis is repeated. This gives the set of fault sites above a certain confidence level at each of these frequencies. The intersection of sets of fault sites at all the frequencies (and at DC) gives a fault site with much higher confidence level. That is, if the confidence of diagnosis of a fault site at one frequency is say $\mathrm{P_i}$, then the resulting confidence level after diagnosis at all the frequencies is as follows[23]:

$$\mathrm{P} = 1 - \prod_{\mathrm{i=1}}^{\mathrm{i=N}} (1 - \mathrm{P_i}) \tag{28}$$

where N is the number of frequencies (including DC) at which the circuit is diagnosed.

The single parametric faults for the elliptic filter in Figure 5 were diagnosable with confidence levels up to 60% at each

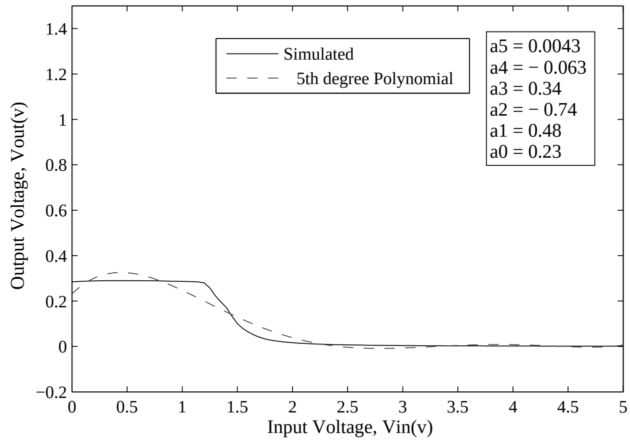

Fig. 10. Curve-fitting polynomial with coefficients at frequency = 1100Hz.

frequency. The resulting confidence level after fault deduction from the four frequencies at which it was diagnosed is about 98.9%. The diagnosis results are tabulated in Table IV for several injected single parametric faults. Another observation worthy of mention here is that the cardinality of set of fault sites detected at frequencies close to cut-off frequency is greater than that at frequencies closer to DC. This can be attributed to higher sensitivity of coefficients to circuit parameters at these frequencies. As a result, fault coverage is better by observing coefficient drifts at frequencies close to f_c. However these frequencies tend to be unfavourable for diagnosis as more than one parameter is likely to have displaced the coefficients out of their respective hypercubes. We can overcome this by looking at the set of fault sites obtained at much lower frequencies than f_c (here DC and 100Hz).

VII. CONCLUSION

A new approach for testing non-linear circuits based on polynomial expansion of the circuit function has been proposed. By expanding polynomial coefficients at critical frequencies the fault coverage is significantly improved, yielding a minimum size of detectable faults in some parameters as low as 5%. The method has been extended to sensitivity based fault diagnosis with probabilistic confidence levels in parameter drifts. Further the expansion at multiple tones leads to a higher confidence level (up to 98.9%) in diagnosing single parametric fault sites.

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TABLE II

PARAMETER COMBINATIONS LEADING TO MAX AND MIN VALUES OF COEFFICIENTS WITH α = 0.05 AT 1000HZ.

Circuit Parameters (Resistance in Ω,Capacitance in Farad)												
Nominal Values	$a_{0,max}$	$a_{1,max}$	$a_{2,max}$	$a_{3,max}$	$a_{4,max}$	$a_{5,max}$	$a_{0,min}$	$a_{1,min}$	$a_{2,min}$	$a_{3,min}$	$a_{4,min}$	$a_{5,min}$
$R_1 = 19.6k$	18.6k	18.6k	20.5k	20.5k	20.5k	18.6k	18.6k	18.6k	18.6k	18.6k	20.5k	20.5k
$R_2 = 196k$	205k	205k	205k	205k	186k	186k	205k	186k	186k	205k	205k	205k
$R_3 = 147k$	139k	139k	154k	139k	139k	139k	139k	139k	154k	139k	139k	139k
$R_4 = 1k$	950	950	1.05k	1.05k	1.05k	1.05k	1.05k	950	1.05k	950	950	1.05k
$R_5 = 71.5$	75	67	75	67	67	75	75	75	67	67	75	67
$R_6 = 37.4k$	35k	39k	39k	35k	35k	39k	39k	39k	35k	35k	35k	35k
$R_7 = 154k$	146k	146k	161k	161k	146k	146k	146k	146k	161k	161k	146k	146k
$R_8 = 260$	247	273	273	247	247	273	273	247	273	247	273	247
$R_9 = 740$	703	777	703	703	777	703	703	703	777	703	703	703
$R_{10} = 500$	475	525	525	475	525	525	475	525	475	475	525	475
$R_{11} = 110k$	115k	115k	115k	104k	104k	104k	115k	115k	104k	115k	104k	104k
$R_{12} = 110k$	104k	104k	115k	115k	115k	115k	115k	115k	104k	104k	115k	104k
$R_{13} = 27.4k$	28.7k	26k	26k	26k	28.7k	28.7k	26k	26k	28.7k	26k	28.7k	26k
$R_{14} = 40$	42	38	42	38	38	42	42	38	42	42	38	42
$R_{15} = 960$	912	912	912	912	912	1k	1k	1k	912	1k	912	912
$C_1 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.5n	2.8n	2.8n	2.8n	2.5n
$C_2 = 2.67n$	2.5n	2.8n	2.8n	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.5n	2.8n
$C_3 = 2.67n$	2.8n	2.8n	2.8n	2.5n	2.8n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.8n
$C_4 = 2.67n$	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.5n	2.5n	2.8n
$C_5 = 2.67n$	2.5n	2.5n	2.5n	2.5n	2.5n	2.8n	2.8n	2.8n	2.8n	2.8n	2.8n	2.8n
$C_6 = 2.67n$	2.5n	2.8n	2.5n	2.8n	2.5n	2.8n	2.5n	2.5n	2.8n	2.8n	2.8n	2.5n
$C_7 = 2.67n$	2.5n	2.8n	2.8n	2.8n	2.8n	2.5n	2.8n	2.5n	2.5n	2.5n	2.5n	2.8n

TABLE III

RESULTS OF SOME INJECTED FAULTS AT DIFFERENT FREQUENCIES.

Injected fault	Coefficients out of Bounds at					Detected
	DC	f_1=100Hz	f_2=900Hz	f_3=1000Hz	f_4=1100Hz	
R_1 down 15%	$a_0 - a_4$	$a_1 - a_4$	a_3, a_5	a_2, a_4	a_1, a_2	Yes
R_2 down 5%	a_2, a_5	a_1, a_3	a_1, a_5	a_1, a_2, a_5	a_1, a_2	Yes
R_3 up 10%	a_1, a_2, a_3	a_3, a_5	a_0, a_3, a_4	a_1, a_3, a_4	a_1, a_5	Yes
R_4 down 20%	$a_0 - a_3$	$a_1 - a_2$	a_2, a_3	a_1, a_2, a_3	a_2, a_3	Yes
R_5 up 15%	a_0, a_5	a_1	a_0, a_2	a_0, a_2, a_3	a_3	Yes
R_6 up 5%	$-$	a_1, a_2	a_2, a_3, a_5	a_1, a_3	a_1	Yes
R_7 down 10%	a_2, a_4	a_3, a_5	a_0, a_1, a_2	a_1, a_4, a_5	a_2, a_3	Yes
R_8 up 10%	$-$	a_2	a_0, a_4	a_0, a_2, a_5	a_3, a_4	Yes
R_9 down 5%	$-$	a_3, a_2	a_1, a_2, a_4	a_2, a_3, a_5	a_1, a_3	Yes
R_{10} up 15%	$-$	a_1, a_4	a_1, a_3, a_4	a_0, a_1, a_4	a_1, a_2	Yes
R_{11} down 10%	a_0, a_2	a_3, a_4	a_0, a_1	a_1, a_2, a_4	a_1, a_2	Yes
R_{12} down 15%	a_0, a_4	a_1, a_3	a_1, a_2, a_3	a_1, a_2	a_2, a_5	Yes
R_{13} up 5%	$-$	a_3, a_5	a_1, a_2	a_1, a_2, a_4	a_0, a_2	Yes
R_{14} up 20%	$-$	a_1, a_3	a_0, a_3, a_4	a_0, a_1, a_2	a_3, a_4	Yes
R_{15} up 5%	$-$	a_4	a_3, a_5	a_0, a_1, a_3	a_0, a_5	Yes
C_1 down 10%	$-$	a_4, a_5	a_4, a_5	a_1, a_2, a_3	a_1, a_4	Yes
C_2 up 10%	$-$	a_2, a_3	a_1, a_2	a_2, a_3, a_4	a_0, a_4	Yes
C_3 down 15%	$-$	a_1, a_3	a_0, a_1, a_2	a_4, a_5	a_0, a_1	Yes
C_4 down 10%	$-$	a_0, a_1	a_1, a_2	a_2, a_3	a_2, a_5	Yes
C_5 up 5%	$-$	a_0, a_1	a_1, a_5	a_1, a_2	a_3, a_4	Yes
C_6 up 15%	$-$	a_3, a_4	a_1, a_2, a_4	a_3, a_4, a_5	a_1, a_2	Yes
C_7 up 15%	$-$	a_1, a_4	a_1, a_3, a_4	a_1, a_3, a_5	a_3, a_4	Yes

TABLE IV
PARAMETRIC FAULT DIAGNOSIS WITH CONFIDENCE LEVELS OF $\approx 98.9\%$

Injected fault	Diagnosed fault sites at					Deduced fault site
	DC	100Hz	900Hz	1000Hz	1100Hz	
R_1 down 15%	R_1, R_4	R_1	R_1, R_2	R_1, R_2, C_1	R_1, C_1	R_1
R_2 down 5%	R_2	R_2, C_1	R_2, R_3, C_1	R_2, R_3	R_2, C_1	R_2
R_3 up 10%	R_1, R_3	R_3, C_3	R_3, R_4, C_3	R_3	R_3, C_3	R_3
R_4 down 20%	R_1, R_4	R_1, R_4	R_2, R_4, C_1	R_1, R_2, R_4	R_1, R_2, R_4	R_4
R_5 up 15%	R_5	R_5, C_2	R_4, R_5	R_4, R_5, C_2	R_5, R_6, C_3	R_5
R_6 up 5%	–	R_6, C_2	R_6, R_7	R_6, C_2, C_4	R_6, C_2, C_3	R_6
R_7 down 10%	R_3, R_7	R_7, C_3	R_3, R_7	R_3, R_6, R_7	R_3, R_7, C_3	R_7
R_8 up 10%	–	R_6, R_8	R_8, R_9	R_6, R_8	R_8, R_9	R_8
R_9 down 5%	–	R_8, R_9	R_8, R_9	R_9, R_{10}	R_8, R_9	R_9
R_{10} up 15%	–	R_{10}	R_{10}, C_6	R_{10}	R_{10}, C_6	R_{10}
R_{11} down 10%	R_{11}, R_{12}	R_{11}	R_{11}, C_5	R_{11}, R_{12}	R_{11}, R_{12}, C_5	R_{11}
R_{12} down 15%	R_{11}, R_{12}	R_{11}, R_{12}	R_{12}, C_5	R_{12}, C_5	R_{12}, C_5, C_7	R_{12}
R_{13} up 5%	–	R_{13}, C_5	R_{13}, C_7	R_{13}, C_5, C_6	R_{13}, C_5	R_{13}
R_{14} up 20%	–	R_{14}	R_{14}, R_{15}	R_{14}, R_{15}	R_{14}, R_{15}	R_{14}
R_{15} up 5%	–	R_{13}, R_{15}	R_{14}, R_{15}	R_{14}, R_{15}, C_5	R_{14}, R_{15}	R_{15}
C_1 down 10%	–	R_2, C_1	R_2, C_1	R_2, C_1	R_2, C_1	C_1
C_2 up 10%	–	R_5, C_2	C_2, C_4	C_2	C_2	C_2
C_3 down 15%	–	C_3	R_3, C_3	C_3	C_3	C_3
C_4 down 10%	–	R_6, C_4	C_2, C_4	C_2, C_4	C_2, C_4	C_4
C_5 up 5%	–	C_5	R_{12}, C_5	C_5	C_5	C_5
C_6 up 15%	–	R_{10}, C_6	C_6, C_7	C_6, C_7	C_6, C_7	C_6
C_7 up 15%	–	C_6, C_7	C_7	C_6, C_7	C_6, C_7	C_7

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APPENDIX

Theorem 1. If coefficient a_i is a monotonic function of all parameters, then a_i takes its limit (maximum and minimum) values when at least one or more of the parameters are at the boundaries of their individual hypercube.

Proof: Let a_i be a function of three parameters say x, y and z. Let a_i reach its maximum value for $(\mathrm{x_0, y_0, z_0})$. Further let $\mathrm{x_0, y_0} \neq \alpha$. Now if we can show that the maximum value of the coefficient a_i occurs at $\mathrm{z_0} = \alpha$ we have proved the theorem. From definition of monotonic dependence of a_i on circuit parameters, (29) follows.

$$a_i(\mathrm{x_0, y_0}, \alpha) \geq \mathrm{a_i(x_0, y_0, z_0)}\ \forall \mathrm{z_0} \leq \alpha \tag{29}$$

As the maximum value taken by $z = \alpha$, it follows that $z_0 = \alpha$. With similar arguments we can show that the minimum value for coefficient occurs when $\mathrm{z_0} = -\alpha$. Hence the statement of theorem follows.

Theorem 2. In polynomial expansion of Non-Linear Analog circuit there exists at least one coefficient that is a monotonic function of all the circuit parameters.

Proof: Consider the block diagram in Figure 11 which models an $2n^{th}$ order Non-Linear analog circuit. x is applied input and y is the response, $a_1 \cdots a_n$ are input summed at each stage. The coefficient corresponding to input x raised to the $2n^{th}$ power is given by G in (30).

$$G = \prod_{i=1}^{n} g_i^{2i} \tag{30}$$

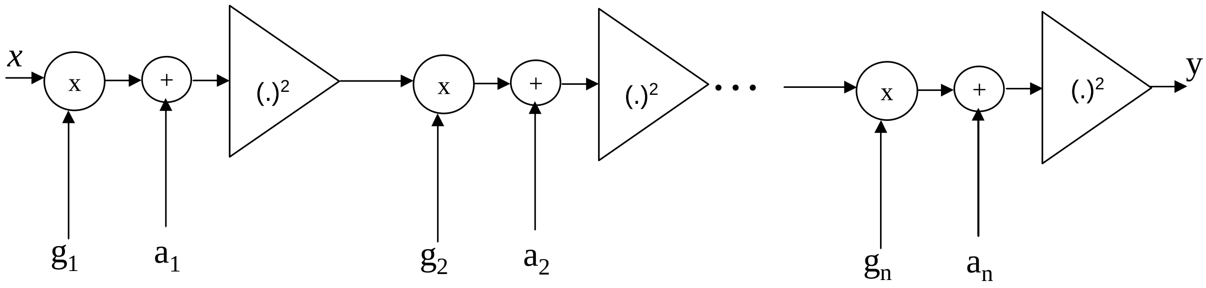

Fig. 11. A possible system model for a non-linear circuit.

where $\mathrm{g_i}\ \forall \mathrm{i} = 1 \ldots \mathrm{n}$ are the monotonic gains of individual stages in the cascaded blocks. As the product of two or more monotonic functions is also monotonic we have G to be a monotonic function. G constitutes the coefficient of the n^{th} power of x in this expansion, as it lies in the main signal flow path from input to output. Thus it is proved that there is at least one monotonically varying coefficient in a polynomial expansion of a non-linear analog circuit. Further, in general the coefficient of $2n^{th}$ power of such a polynomial expansion is monotonic.

Theorem 3. A continuous non-monotonic function $f : \Re \rightarrow \Re$ can be decomposed into piecewise monotonic functions of the form:

$$\begin{aligned} f(x) = f(x)u(x_0 - x) + f(x)\left(u(x - x_0) - u(x - x_1)\right) + \\ f(x)\left(u(x - x_1) - u(x - x_2)\right) + \cdots \\ + f(x)\left(u(x - x_{n-1}) - u(x - x_n)\right) \end{aligned} \tag{31}$$

where $x_0, x_1, \cdots x_n$ are all stationary points of $f(x)$ and $u(x) = \begin{cases} 1 & \forall\ x \geq 0 \\ 0 & \forall\ x < 0 \end{cases}$

Proof: By Rolle's theorem [17], if $f : \Re \rightarrow \Re$ is any continuous and differentiable function in the open interval (a, b) and $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$. To extend this result, suppose $f(x)$ is increasing in the interval (a, c^-), that is $f'(x) > 0\ \forall x \in (a, c^-)$ and decreasing in the interval (c^+, b) that is $f'(x) < 0\ \forall x \in (c^+, b)$ then at point c, $f'(c) = 0$. In general for a continuous function f over arbitrary interval (α, β) there exists countable number of points x_i such that $f'(x_i) = 0$ as $f(x)$ changes its monotonicity. Now that we have shown x_i are stationary points, it follows that $f(x)$ is monotonic between any two stationary points, i.e., in the interval (x_{i-1}, x_i). The windows generated by the step function $u(x)$ ensures that each term in the summation in (31) is monotonic. A typical example is shown in Figure 12, where $\mathrm{f(x)}$ alternates its monotonicity at 6 points namely $\mathrm{x_0}$ through $\mathrm{x_5}$ and at each of these points slope is zero and $\mathrm{f'(x) = 0}$. $\mathrm{f(x)}$ can be expressed as sum of monotonic functions separated by windows in the intervals $(\mathrm{x_0, x_1}), (\mathrm{x_1, x_2}), (\mathrm{x_2, x_3}), (\mathrm{x_3, x_4}), (\mathrm{x_4, x_5})$.

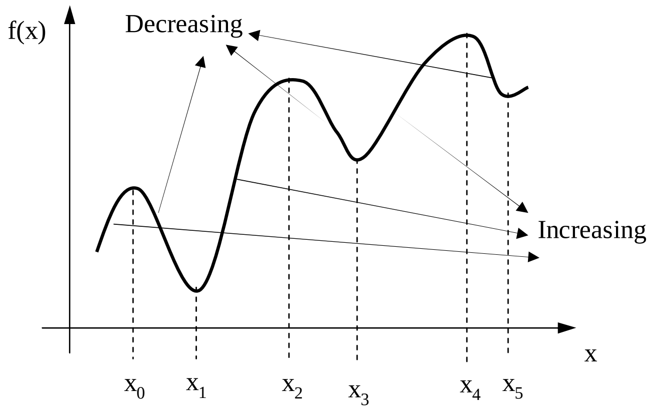

Fig. 12. Non-linear, non-monotonic function decomposed into piecewise monotonic functions.

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