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DMT of Wireless Networks: An overview

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by

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ABSTRACT

The efficient operation of single-source, single-sink wireless networks is considered with the diversity-multiplexing gain tradeoff (DMT) as the measure of performance. Whereas in the case of a point-to-point MIMO channel the DMT is determined by the fading statistics, in the case of a network, the DMT is additionally, a function of the time schedule according to which the network is operated, as well as the protocol that dictates the mode of operation of the intermediate relays. In general, it is only possible at present, to provide upper bounds on the DMT of the network in terms of the DMT of the MIMO channel appearing across cuts in the network. This paper presents a tutorial overview on the DMT of half-duplex multi-hop wireless networks that also attempts to identify where possible, codes that achieve the DMT. For example, it is shown how one can construct codes that achieve the DMT of a network under a given schedule and either an amplify-and-forward or decode-and-forward protocol. Also contained in the paper, are discussions on the DMT of the multiple-access channel as well as the impact of feedback on the DMT of a MIMO channel.

1 Introduction

The DMT [1] was first used to quantify the tradeoff between the rate and the reliability of communication across a point-to-point MIMO channel. It was subsequently used to analyze the performance limits of wireless networks such as the multiple-access channel in [2] and single-source single-sink (ss-ss) half-duplex wireless relay networks in [3, 4, 5]. Since then many other papers have appeared in the literature that study the DMT of wireless networks. In this article, we attempt to provide a tutorial overview of the DMT derivations and code constructions for MIMO channels, ss-ss half-duplex relay networks and the multiple access channel (MAC).

We first review the DMT of a point-to-point MIMO channel and multi-block channel in Section 2. In Section 3, we discuss the criterion for a code to be DMT optimal for a MIMO channel irrespective of the fading statistics and then describe such code constructions for both MIMO and multi-block channels. We then focus on ss-ss half-duplex relay networks in Section 4, where we first describe upperbounds to the DMT of the network (cut-set bounds). We also discuss the DMT of the network under various protocols and schedules for both two-hop and multi-hop networks in this section. In Section 5, we review a DMT optimal code construction for the MAC channel. Finally, we discuss the impact of feedback in MIMO channel in Section 6.

2 Point-to-Point MIMO channel and its DMT

A ss-ss point-to-point wireless channel with n_t transmit antennas and n_r receive antennas is commonly modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \ , \tag{1}$$

where $\mathbf{H}$ is called the channel matrix and is of size $n_r \times n_t$. Here, $\mathbf{x}$ and $\mathbf{y}$ are input and output vectors of sizes $n_t \times 1$ and $n_r \times 1$ respectively. $\mathbf{w}$ is the $n_r \times 1$ noise vector seen at the receiver, whose entries are distributed as i.i.d. $\mathcal{CN}(0,1)$. If the antennas are sufficiently separated and the environment is richly scattered, then the entries of $\mathbf{H}$ can be modeled as i.i.d $\mathbb{C}\mathcal{N}(0,1)$ random variables. Such a channel is said to be a Rayleigh fading MIMO channel.

Communication across this channel is via a sequence of T transmissions $\{\mathbf{x_t}\}_{t=1}^{T}$ and $X = [\mathbf{x_1}\mathbf{x_2}\ldots\mathbf{x_T}]$ is termed as a codeword matrix. The set of all such codeword matrices would be referred to as the space-time code $\mathcal{C}$. The rate of the code is $R = \frac{\log|\mathcal{C}|}{T}$. We impose the average power constraint $\frac{1}{|\mathcal{C}|}\sum_{i=1}^{|\mathcal{C}|}\|X(i)\|_F^2 \leq T.\rho$ on the code, where ρ is the average signal-to-noise ratio (SNR) at the sink. We assume that the sink knows the channel matrix $\mathbf{H}$.

In a quasi-static scenario, the channel is assumed to remain constant over the T-slot time duration of the codeword. If T is the quasi-static interval and one codes over B blocks of T time slots each, the channel is called a multi-block MIMO channel. Here, the channel is assumed to remain constant in each block and vary independently from one block to another. Such a channel can be represented as

$$\mathbf{y}_b = \mathbf{H}_b \mathbf{x}_b + \mathbf{w}_b, \quad 1 \leq b \leq B. \tag{2}$$

The multi-block channel can also be rewritten in the form (1) with $\mathbf{H} = \mathbf{H}_p$, where

$$\mathbf{H}_p = \begin{bmatrix} \mathbf{H}_1 & & & \\ & \mathbf{H}_2 & & \\ & & \ddots & \\ & & & \mathbf{H}_B \end{bmatrix}. \tag{3}$$

However in using this model, one has to take care to normalize the rate by a factor of B. Here, each matrix $\mathbf{H}_i$ is of size $n_r \times n_t$ and we will refer to $\mathbf{H}_p$ as the parallel channel.

Thus far, we have assumed the channel to be frequency flat. If the channel is however, frequency selective, then one can represent the channel by

$$\mathbf{y}(m) = \sum_{l=0}^{L-1} \mathbf{H}_l \mathbf{x}(m-l) + \mathbf{w}(m), \tag{4}$$

where m denotes the time index and L is the number of resolvable paths from the source to the sink. It is well known that with the aid of the notion of a cyclic prefix, OFDM can be used to convert this ISI channel into a bank of parallel (but possibly correlated) channels when viewed in the frequency domain. If N_c denotes the number of sub-carriers used, the resultant MIMO-OFDM parallel channel model takes on the form

$$\widehat{\mathbf{y}}_n = \widehat{\mathbf{H}}_n \widehat{\mathbf{x}}_n + \widehat{\mathbf{w}}_n, \quad 0 \leq n \leq N_c - 1, \tag{5}$$

where

$$\begin{aligned}
[\widehat{\mathbf{x}}_0(i) \ldots \widehat{\mathbf{x}}_{N_c-1}(i)] &= DFT([\mathbf{x}_0(i) \ldots \mathbf{x}_{N_c-1}(i)]), \\
\left[\widehat{\mathbf{H}}_0(j,i) \ldots \widehat{\mathbf{H}}_{N_c-1}(j,i)\right] &= DFT([\mathbf{H}_0(j,i) \ldots \mathbf{H}_{N_c-1}(j,i)]), \\
[\widehat{\mathbf{y}}_0(j) \ldots \widehat{\mathbf{y}}_{N_c-1}(j)] &= DFT([\mathbf{y}_{L-1}(j) \ldots \mathbf{y}_{N_c+L-2}(j)]), \\
[\widehat{\mathbf{w}}_0(j) \ldots \widehat{\mathbf{w}}_{N_c-1}(j)] &= DFT([\mathbf{w}_0(j) \ldots \mathbf{w}_{N_c-1}(j)]), \\
& \quad 1 \leq i \leq n_t, 1 \leq j \leq n_r. \qquad (6)
\end{aligned}$$

In computing the DFT, we set $\mathbf{H}_k(j,i) = 0, L \leq k \leq N_c - 1, \forall i, j$. Note also that the first $L - 1$ samples of the received vector are ignored while computing the DFT since these symbols correspond to the cyclic prefix.

2.1 Diversity-Multiplexing Tradeoff of the channel

A coding scheme $\{C(\rho)\}$ for the MIMO channel is a collection of codes indexed by SNR ρ. Let $R(\rho)$ be the rate of transmission. The spatial multiplexing gain r is given by

$$\lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho} = r. \qquad (7)$$

The average probability of error of this scheme, under a maximum likelihood decoder, is denoted as $P_e(r, \rho)$. The scheme $\{C(\rho)\}$ is said to achieve diversity gain $d(r)$ for a multiplexing gain r, if

$$\lim_{\rho \to \infty} \frac{\log P_e(r, \rho)}{\log \rho} = -d(r). \qquad (8)$$

We use the symbol $\doteq$ to denote exponential equality, i.e., $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = b \Leftrightarrow f(\rho) \doteq \rho^b$. $\dot{\geq}$, $\dot{\leq}$ are similarly defined. The diversity-multiplexing trade-off (DMT) $d^*(r)$ of the channel is the supremum of diversity gains $d(r)$, achieved over all schemes for a given r. We will now characterize the DMT in terms of probability of outage.

The probability that the channel $\mathbf{H}$ is in outage is given by

$$P_{\text{out}}(r \log \rho) \doteq \Pr \left\{ \log \det(I + \rho \mathbf{H}\mathbf{H}^\dagger) < r \log \rho \right\} . \qquad (9)$$

Let $P_{\text{out}}(r \log \rho) \doteq \rho^{-d_{\text{out}}(r)}$. Then, by an application of Fano's inequality, it can be shown that for any coding scheme $\{C(\rho)\}$,

$$d(r) \;\leq\; d_{\text{out}}(r)\ . \tag{10}$$

In [1], it was shown that for a Rayleigh fading MIMO channel, a random Gaussian code with $T \geq n_r + n_t - 1$ achieves the above bound with equality. Explicit coding schemes which attain the above bound with equality, for $T \geq n_t$, has been constructed in [6], [7].

The DMT of the Rayleigh faded MIMO channel [1] is given by

$$d^*_{\text{mimo}}(r) = (n_t - r)(n_r - r)\ , \tag{11}$$

at integer values of $r = 0, 1 \ldots \min\{n_t, n_r\}$ and through straight line interpolation elsewhere. For an example see Fig.1

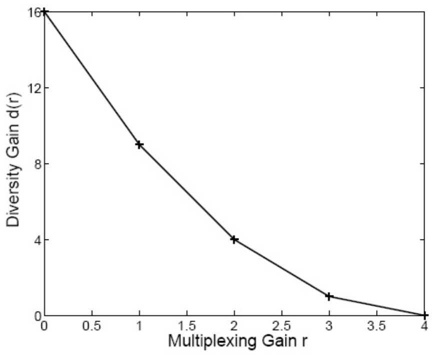


Figure 1: DMT of Rayleigh faded MIMO channel with $n_t = 4$ and $n_r = 4$.

The parallel channel described in (3) can be considered as a set of M links, with the i^{th} link having channel model $\mathbf{y_i} = \mathbf{H_i}\mathbf{x_i} + \mathbf{w_i}$. Let r_i be its multiplexing gain and let $d_i(r_i)$ denote the corresponding DMT. Then, the DMT of the parallel channel is given by [8]

$$d^*_p(r) = \inf_{(r_1, r_2, \cdots, r_M):\ \sum_{i=1}^{M} r_i = r} \sum_{i=1}^{M} d_i(r_i)\ . \tag{12}$$

For example, in the case of a two antenna SISO parallel channel, the

DMT is given by,

$$d_p^*(r) = \inf_{(r_1,r_2):\ r_1+r_2=r} d_1(r_1) + d_2(r_2) \tag{13}$$
$$= \inf_{(r_1,r_2):\ r_1+r_2=r} (1-r_1)^+ + (1-r_2)^+ \tag{14}$$
$$= (2-r)^+ \tag{15}$$

3 Explicit Approximately Universal Codes

A coding scheme is said to be approximately universal (AU), if it achieves the DMT of the channel in (1), irrespective of its fading statistics [7].

Difficulty in modeling the statistics of a channel accurately and good finite SNR performance make AU codes practically appealing. Moreover, we will see later that AU codes can be readily used to construct coding schemes for the relay networks. We will discuss the sufficient condition for a coding scheme to be approximately universal and describe explicit minimum delay codes which are approximately universal.

Coding scheme: Consider the MIMO channel model described in (1). Let $n_r = n_t = n$ for simplicity. Let $Z = \theta X$, where X denotes the $n \times n$ codeword matrices[1,2] of the scheme $\{C(\rho)\}$. θ is chosen to meet the energy constraint $\|\theta X\|_F^2 \leq n\rho$. Let $d(r)$ be the diversity gain of the scheme at multiplexing gain r. We will assume that every codeword has n^2 information symbols, each drawn independently from an M^2-QAM constellation. Since $R = r\log\rho$ we get, $M^2 = \rho^{\frac{r}{n}}$ and $\theta^2 \doteq \rho^{1-\frac{r}{n}}$. Let Z_1 and Z_2 be two codewords and $\Delta Z = Z_1 - Z_2$, $\Delta X = (X_1 - X_2)$. Let $\{\ell_i\}$ be the eigenvalues of $\Delta X \Delta X^\dagger$ and $\{\lambda_i\}$ be the eigenvalues of $HH^\dagger$. Let $\lambda_i = \rho^{-\alpha_i}$. Let $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $\ell_1 \geq \ell_2 \geq \cdots \geq \ell_n$. The probability of error of the coding scheme is given by

$$\begin{aligned} P_e(r,\rho) &= P(\text{error, outage}) + P(\text{error, no outage}) \\ &\leq P_{\text{out}}(r\log\rho) + P(\text{error}|\text{no outage}) \\ &\dot{\leq} \rho^{-d_{\text{out}}(r)} + \rho^{rT}\text{PEP}(Z_1 \to Z_2|\text{no outage}) \ , \end{aligned}$$

[1]We slightly defer from (1) in the sense that, here Z is the actual codeword transmitted while in (1) it is X. Both models will be used in this article; the exact model used in a section would be apparent based on the power constraint described therein.

[2]Though we discuss only AU codes of size $n \times n$ here, an AU rectangular code of size $n \times T, T > n$, can be obtained by deleting any particular set of $T - n$ rows from each of the code matrices of a $T \times T$ AU code.

where we have applied the union bound. Now, if the pairwise error probability PEP($Z_1 \to Z_2$) decays exponentially with SNR for all pair of codewords, whenever the channel is not in outage, then $P_e(r,\rho)\dot{\leq}\rho^{-d_{\rm out}(r)}$ and combining with (10) we get $d(r) = d_{\rm out}(r) = d^*(r)$. Under an ML decoder, we have

$$PEP(Z_1 \to Z_2) \quad \leq \quad \exp\left(-\frac{\|H(Z_1 - Z_2)\|_F^2}{4}\right) . \tag{16}$$

Let $d_E^2 = \|H(Z_1 - Z_2)\|_F^2$. Then,

$$d_E^2 \quad = \quad \theta^2 {\rm Tr}(H\Delta X \Delta X^\dagger H^\dagger) \geq \theta^2 \sum_{i=1}^{n} \lambda_i \ell_i \ . \tag{17}$$

The above lower bound, termed the mismatched eigenvalue bound in [6] was first noted in [9]. One can make use of the AM-GM inequality to put this into a form that aids in signal design. Let $\min_{X \in C} \det(\Delta X \Delta X^\dagger) \doteq \rho^\delta$. From (17) we have

$$\begin{aligned}
d_E^2 \quad &\geq \quad \theta^2 \sum_{i=1}^{k} \lambda_{n+1-i} \ell_{n+1-i} \quad 1 \leq k \leq n & (18)\\
&\geq \quad \theta^2 \left[\prod_{i=1}^{k} \lambda_{n+1-i}\right]^{\frac{1}{k}} \left[\prod_{i=1}^{k} \ell_{n+1-i}\right]^{\frac{1}{k}} & (19)\\
&= \quad \theta^2 \left[\prod_{i=1}^{k} \lambda_{n+1-i}\right]^{\frac{1}{k}} \left[\frac{\prod_{i=1}^{n} \ell_i}{\prod_{i=1}^{n-k} \ell_i}\right]^{\frac{1}{k}} & (20)\\
&\dot{\geq} \quad \rho^{\frac{1}{k}\{-\sum_{i=1}^{k} \alpha_{n+1-i} + k - r + \delta\}} & (21)\\
&:= \quad \rho^{\frac{d_k}{k}} \ , & (22)
\end{aligned}$$

where we have used the fact that $\theta^2 \ell_i \ \dot{\leq}\ \rho$. When the channel is not in outage for a rate $(r + \epsilon) \log \rho$, we have [1]

$$\sum_{i=1}^{n} (1 - \alpha_i)^+ \geq r + \epsilon \tag{23}$$

where $(1 - \alpha_i)^+ = \max(\alpha_i, 0)$. Clearly for some i, $\alpha_i < 1$. Now since $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$ assume $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{n-k} \geq 1$ and $\alpha_i < 1$ for

$i \geq n-k+1$. Then, we have

$$\sum_{i=n-k+1}^{n} (1-\alpha_i) \geq r + \epsilon \ . \tag{24}$$

Substituting the above no outage conditions in (22), we get $d_k \geq \epsilon + \delta$ and hence, $\text{PEP}(Z_1 \rightarrow Z_2) \dot{\geq} \exp(-\rho^{\frac{\epsilon+\delta}{k}})$. Now, if $\delta \geq 0$, then the pairwise probability of error decays exponentially with ρ and the coding scheme will be approximately universal. The property $\delta \geq 0$ of the code is termed as the Non Vanishing Determinant (NVD) property [10]. More formally,

NVD property: A coding scheme is said to have NVD property if

$$\det(\Delta Z \Delta Z^{\dagger}) \dot{>} \rho^{n-r} \tag{25}$$

for all nonzero difference codewords ΔZ.

It follows from our discussion above, that a coding scheme possessing the NVD property is AU. The converse can be shown to be true in the sense that if a coding scheme is AU for any number n_r of receive antennas for the channel model in (1), then it must necessarily have the NVD property.

In [7], a necessary and sufficient condition for any coding scheme to be AU was provided. Explicit AU codes for any value of n_r and n_t were first constructed in [6].

We will now describe explicit codes that are approximately universal. These codes are based on cyclic division algebras (CDA). CDA-based codes were first explored in [11], [10], [12] and constructions of CDA-based NVD codes can be found in [10], [13] and [6]. In [14], [15] codes (not based on CDA) that are DMT optimal for the Rayleigh fading channel can be found, but are not discussed here for lack of space.

3.1 CDA based NVD codes

Detailed introduction to CDAs could be found in [12] and [6]; Appendix D provides a quick overview. All code matrices in a CDA-based code represent elements belonging to a CDA in much the same way as one would represent the linear transformation carried out by the complex number $a + \imath b$ in multiplying the complex number $x + \imath y$ by the 2×2 matrix appearing below.

$$(a+\imath b)(x+\imath y) \Leftrightarrow \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{26}$$

.

For example, a (3×3) codeword based on CDA would look like

$$X = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_2) & \gamma\sigma^2(\ell_1) \\ \ell_1 & \sigma(\ell_0) & \gamma\sigma^2(\ell_2) \\ \ell_2 & \sigma(\ell_1) & \sigma^2(\ell_0) \end{bmatrix}, \tag{27}$$

where the information-bearing symbols ℓ_i are derived from QAM message symbols as shown below.

$$\overbrace{\begin{bmatrix} \ell_0 & \ell_1 & \ell_2 \end{bmatrix}}^{\text{information-bearing symbols}} = \overbrace{\begin{bmatrix} \beta_0 & \beta_1 & \beta_2 \end{bmatrix}}^{\text{basis vectors}} \overbrace{\begin{bmatrix} \ell_{0,0} & \ell_{1,0} & \ell_{2,0} \\ \ell_{0,1} & \ell_{1,1} & \ell_{2,1} \\ \ell_{0,2} & \ell_{1,2} & \ell_{2,2} \end{bmatrix}}^{\text{QAM message symbols}}. \tag{28}$$

The $\{\beta_i\}$ are a basis for a larger field $\mathbb{L}$ containing the field $\mathbb{F} = \mathbb{Q}(\imath)$ that forms a vector space over $\mathbb{Q}(\imath)$ of dimension 3. Also, $\mathbb{L}$ is a cyclic Galois extension field of $\mathbb{F}$ and σ is the generator of the cyclic Galois group $\mathrm{Gal}(\mathbb{L}/\mathbb{F})$. The element γ appearing in the codeword matrix belongs to $\mathbb{F}\backslash\{0\}$

A key property of CDAs is that the determinant of every one of these code matrices lies in $\mathbb{F}$. If the element γ is chosen to be a non-norm element, then the determinant is non zero for $X \neq [0]$. Now, if the element γ and $\{\beta_i\}$ are chosen to be algebraic integers of $\mathbb{F}$ and $\mathbb{L}$ respectively (the set of all algebraic integers in a number field forms a subring of the number field), then the determinant is also an algebraic integer of $\mathbb{F}$. It is known that the set of algebraic integers of $\mathbb{F}(=\mathbb{Q}(\imath))$ is $\mathbb{Z}[\imath]$ and hence the determinant lies in $\mathbb{Z}[\imath]$. Since any non-zero element in $\mathbb{Z}[\imath]$ has magnitude at least one, independent of SNR (i.e. $\delta = 0$), such codes will have the NVD property.

The method described above to obtain NVD codes were first proposed in [10]. Constructing γ to be a non-norm element which is an algebraic integer of $\mathbb{F}$ turns out to be an interesting problem. In [10], such a γ was provided for $n = 2, 3, 4$. In [13], a simplified method to construct such a γ was provided and constructions were given for $n = 2^k, 3.2^k, 2.3^k, q^k \frac{(q-1)}{2}$

where $q = 4k + 3$ is a prime. In [6], constructions were provided for all n.

The CDA based NVD Codes discussed above are approximately universal for all channels that are of the form (1). However, for the parallel channel given by (3), simpler AU codes exist which are derived from CDA based NVD codes of the form (27). These codes known as multi-block codes, entail lesser decoding complexity and lesser delay.

Multi-block codes for Parallel Channel: Let X be a rectangular AU code of size $n \times T, T \geq n$, constructed from an appropriate CDA (See Appendix D). Then, the multi-block code is of the form

$$X_{MB} = \begin{bmatrix} X \\ \phi(X) \\ \vdots \\ \phi^{M-1}(X) \end{bmatrix}, \tag{29}$$

where ϕ is related to the algebraic structure of the CDA. The minimum delay of multi-block codes is n (the delay would have been Mn if construction were done as in the previous section). These multi-block codes were first constructed in [16], [17]. The general construction and the proof of approximate universality were provided in [18].

Perfect Codes: Perfect codes are CDA based NVD codes, which in addition to being AU posses other interesting properties. They are information lossless and have same average energy per transmit antenna. Perfect codes were introduced and constructed for $n = 2, 3, 4, 6$ in [19] and later generalized to any value of n in [20]. The Golden code, which is a perfect code for $n = 2$ is given by the set of matrices of the form

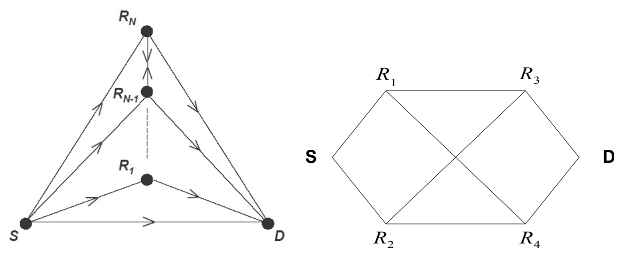
$$X = \begin{bmatrix} \alpha(\ell_{0,0} + \theta\ell_{0,1}) & i\overline{\alpha}(\ell_{1,0} + \overline{\theta}\ell_{1,1}) \\ \alpha(\ell_{1,0} + \theta\ell_{1,1}) & \overline{\alpha}(\ell_{0,0} + \overline{\theta}\ell_{0,1}) \end{bmatrix}, \tag{30}$$

where $\theta = \frac{1+\sqrt{5}}{2}$, $\overline{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + i\overline{\theta}$, $\overline{\alpha} = 1 + i\theta$ and $\ell_{i,j} \in \mathbb{Z}[i]$. A variant of the Golden code is a part of the WiMax Standards.

Recent CDA based NVD Codes: In [21], CDA based NVD codes that employ nested lattice constructions are studied. These codes admit simpler decoding using MMSE-GDFE algorithms and also provide shaping gains. In [22], CDA based NVD codes are constructed from denser MIMO lattices that offer higher rate while maintaining the NVD property.

4 Relay Networks

Consider a ss-ss multi-hop cooperative relay network with m nodes, labeled $\{1 \ldots m\}$, such that node 1 represents the source S and node m, the sink D and where the remaining nodes represent relays. Such a network may be represented by a directed graph, whose vertices represent the nodes and edges the connectivity between the nodes. If a link is bidirectional, we represent it by an undirected edge. Example relay networks are shown in Fig. 2.



(a) A two-hop network (b) A three-hop network

Figure 2: Cooperative relay networks

4.1 Network Model

The channel between any two nodes (where an edge exists between the corresponding vertices in the graphical model of the network) is modeled as a Rayleigh fading channel. In all of the networks considered here, each node is assumed to have a single transmit and a single receive antenna, unless specified otherwise. All the nodes in the network are assumed to operate in half-duplex mode i.e., at a given time instant they can either transmit or receive but not do both. However, it is allowed to remain idle. The signal transmitted by a relay at a particular time instant is permitted to be a function of all the past symbols received by the node. We assume the network to be synchronous i.e., all nodes in the network transmit and receive in synchronous fashion.

4.2 Network Operation

Communication in relay networks requires the specification of a protocol, a schedule and a coding scheme employed at the source and the relays. The protocol at a relay is a reference to the function applied by the relay to its past received symbols. We assume all relays to operate under the same protocol. An example protocol is the Amplify-and-Forward protocol, where a relay is constrained to operate only linearly on the symbols it has received. The schedule adopted by the network identifies for each node, the time slots during which the particular node is permitted (but not required) to transmit. The present paper is oriented towards block codes and hence we will assume throughout that that communication in the network takes place over B blocks, with the bth block comprising of the T consecutive time slots $((b-1)T+1,\ (b-1)T+2, \ldots,\ (b-1)T+T)$ for a total of $n = BT$ time slots. We will also assume that the permission given to each node is also specified in terms of blocks, i.e., the schedule simply identifies the blocks during which the particular node is permitted to transmit. When a node chooses to transmit during a particular block, it must do so for all the time slots within the block.

Encoding at the source: A code at the source consists of a set of M codewords, each of length n. Thus the rate of the code is given by $R = \frac{\log M}{n}$. We impose an average power constraint on the codebook, $\{x(i)\}_{i=1}^{M}$ i.e., $\frac{1}{M}\sum_{i=1}^{M} ||x(i)||^2 \leq n\rho$, where ρ is the average signal-to-noise ratio (SNR) of any link in the network and where $||x(i)||$ denotes the norm of the $(n \times 1)$ vector $x(i)$. Let $\mathcal{W} = \{1, 2, \ldots, , 2^{nR}\}$ be the input alphabet of the source and W represent the input message drawn according to a uniform distribution on $\mathcal{W}$. Then, the source maps each message onto one of the codewords of the code.

Protocols:

Amplify-and-Forward (AF): Under an AF protocol, when a node does transmit during the bth block, it transmits a $(1 \times T)$ vector which is a linear combination of the $(b-1)$ $(1 \times T)$ vectors previously received by the node. Under this protocol, the receiver at the sink is assumed to have knowledge of all channel gains. No channel knowledge is needed however, at the relays.

Decode-and-Forward (DF): Under a DF protocol, when a relay wishes to transmit during say the bth block, it checks to see whether or not the multi-block channel between the source, the relays that have transmitted at any time during the preceding $(b-1)$ blocks and the relay itself is in outage. If this channel is not in outage, then the node will proceed to transmit and will remain silent otherwise. We illustrate with an example of source and three relays shown in Fig. 3. The symbol t_j in the figure indicates that the node at the head of the edge is transmitting during block j. In this example, the relay R_3 wishes to transmit during the 4th block. Let us assume that the source S transmits during all preceding 3 blocks, that relay $R1$ transmits during blocks 2 and 3 and that relay $R2$ transmits only during block 3. Then the multi-block channel takes on the form

$$\begin{aligned}
\mathbf{y}^{(R_3)}(1) &= h_{13}\mathbf{x}^{(S)}(1) + \mathbf{w}(1) \\
\mathbf{y}^{(R_3)}(2) &= h_{13}\mathbf{x}^{(S)}(2) + h_{23}\mathbf{x}^{(R_1)}(2) + \mathbf{w}(2) \\
\mathbf{y}^{(R_3)}(3) &= h_{13}\mathbf{x}^{(S)}(3) + h_{23}\mathbf{x}^{(R_1)}(3) + h_{33}\mathbf{x}^{(R_2)}(3) \\
&\qquad + \mathbf{w}(3)
\end{aligned}$$

which can be put into the form of a parallel channel

$$\begin{bmatrix} \mathbf{y}^{(R_3)}(1) \\ \mathbf{y}^{(R_3)}(2) \\ \mathbf{y}^{(R_3)}(3) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1 & & \\ & \mathbf{H}_2 & \\ & & \mathbf{H}_3 \end{bmatrix}}_{\mathbf{H}_p} \begin{bmatrix} X(1) \\ X(2) \\ X(3) \end{bmatrix} + \begin{bmatrix} \mathbf{w}(1) \\ \mathbf{w}(2) \\ \mathbf{w}(3) \end{bmatrix},$$

where $X(i) = [\mathbf{x}^{(S)^T}(i) \;\; \mathbf{x}^{(R_1)^T}(i) \;\; \mathbf{x}^{(R_2)^T}(i)]^T$ and $\mathbf{H_1} = [h_{13}\ 0\ 0]$, $\mathbf{H_2} = [h_{13}\ h_{23}\ 0]$, $\mathbf{H_3} = [h_{13}\ h_{23}\ h_{33}]$. If the channel $\mathbf{H}_p$ is not in outage for the desired rate R, R_3 decides to transmit in the fourth block; else it remains silent.

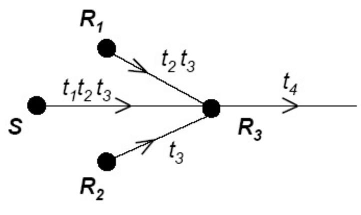


Figure 3: Illustration of DF protocol

Quantize-and-Forward (QF): Under a quantize-and-forward (QF) protocol [23], a relay quantizes the received signal to the noise level, maps it to a codeword and transmits it. More precisely if $\mathbf{v}_b$ is the signal received by a relay during the bth block, and assuming that we have already normalized the received vector by the variance of the additive noise, then the relay computes $([\Re(\mathbf{v_b})], \quad [\Im(\mathbf{v_b})])$ where $[z]$ is the integer part of the real number z. The relay then proceeds to map $([\Re(\mathbf{v_b})], [\Im(\mathbf{v_b})])$ to a codeword from the relay codebook. Note that under this protocol, the rate of the code employed by the relay is a monotonically decreasing function of the noise level at the relay input. Under this protocol, the receiver at the sink is assumed to have knowledge of all channel gains. No channel knowledge is needed at the relays. Each relay is however, assumed to be capable of measuring the variance of the additive-noise component of its received signal.

Remark 1 *As seen above, in the case of the DF and QF protocols, the relays have their own individual codebooks. When in the sequel, we speak of a code for a relay network, we will mean to include the code employed at both source as well as the relays.*

Half-Duplex Constraint on the Schedule: The link from node i to node j is said to be active if node i is transmitting and node j is listening. In a half-duplex network, the set of all edges that can be active at any given time slot is restricted. A set of edges that are active at a given time instant is called an activation set. A schedule spells out which nodes can transmit during a particular block. Clearly these nodes must always be a subset of some activation set. In this way, we can think of a schedule as specifying a sequence of n activations sets with the activation set remaining unchanged for the duration of a block. Let $\{a_i\}_{i=1}^{L}$ be the collection of all activation sets in the network and let $\{f_i\}_{i=1}^{L}$ denote their fractions. A schedule is static, if the fraction $\{f_i\}$ are fixed a priori and dynamic, if they are made to depend on the channel realization.

We will now proceed to derive an upper bound to the DMT of a relay network in terms of the DMT of channel across its cuts.

4.3 Cut Set Bounds

Let $\{C(\rho)\}$ be a coding scheme. Let r be its multiplexing gain by which we mean that the rate R of the code at the source is given by $R = r \log \rho$. Let the probability of error at the sink be $P_e(r, \rho)$. For simplicity, we will often write $P(\mathcal{E})$ in place of $P_e(r,\rho)$. The DMT of the network $d^*(r)$ is then the supremum over all schedules, protocols and coding schemes of the achievable diversity gains.

A cut in a network is a partition of the nodes in the network into two sets (Ω, Ω^c), such that the source belongs to Ω and the sink belongs to Ω^c. By abuse of notation, we shall denote a cut (Ω, Ω^c) simply by Ω.

Let $d(r)$ be the diversity gain when the network is operated under a certain schedule, certain protocol and coding scheme $\{C(\rho)\}$. Thus, $P_e(r,\rho) \doteq \rho^{-d(r)}$. Let $\mathbf{H}$ denote the set of all fading coefficients in the network. For the t^{th} time slot, let $X^{(i)}(t)$ and $Y^{(i)}(t),\ i \in \{1,\ldots,m\}$, denote the signals transmitted and received respectively at the m nodes in the network. For the rest of the sequel, we will abbreviate and write $I(W;Y^{\Omega^c}[n]|H)$ in place of $I(W;Y^{\Omega^c}[n]|\mathbf{H}=H)$, $H(W|H)$ in place of $H(W|\mathbf{H}=H)$ etc. We will use $X[k]$ to indicate $\{X(1)\ldots X(k)\}$ and write p_X in place of $p(X^{(1)},\ldots,X^{(m)})$.

4.3.1 Full-duplex cut set bound

Although any schedule that operates upon a half-duplex network must be expressible in terms of activation sets, in the interests of obtaining an upper bound, we will relax this requirement and permit the network to be operated in full-duplex mode.

Theorem 1 *[24] Consider a particular cut Ω in the network. Define the probability of outage of this cut as*

$$P_{out,\Omega}(r\log\rho) := \inf_{p(X^{(1)},\ldots X^{(m)})} P\left\{I(X^{\Omega};Y^{\Omega^c}|X^{\Omega^c},\mathbf{H}=H) < r\log\rho\right\}. \qquad (31)$$

Let $P_{out,\Omega}(r\log\rho) \doteq \rho^{-d_{out,\Omega}(r)}$. Then for every cut Ω, $d(r) \le d_{out,\Omega}(r)$ and thus

$$d(r) \quad \le \quad \min_{\Omega} d_{out,\Omega}(r). \qquad (32)$$

Proof: [3] The rate R of flow of information can be written, following [25], as

$$\begin{aligned}
nR &= H(W) = H(W|H) \\
&= I(W; Y^{\Omega^c}[n]|H) + H(W|Y^{\Omega^c}[n], H) \\
&= \sum_{t=1}^{n} I(W; Y^{\Omega^c}(t)|Y^{\Omega^c}(1), \ldots, Y^{\Omega^c}(t-1), H) \\
&\qquad\qquad + H(W|Y^{\Omega^c}[n], H) \\
&\leq \sum_{t=1}^{n} I(Y^{\Omega^c}(t); X^{\Omega}(t)|X^{\Omega^c}(t), H) \qquad (33) \\
&\qquad\qquad + H(W|Y^{\Omega^c}[n], H) \\
&= nI(Y_Q^{\Omega^c}; X_Q^{\Omega}|X_Q^{\Omega^c}, Q, H) + H(W|Y^{\Omega^c}[n], H) \\
&\leq nI(Y_Q^{\Omega^c}; X_Q^{\Omega}|X_Q^{\Omega^c}, H) + H(W|Y^{\Omega^c}[n], H). \qquad (34)
\end{aligned}$$

where Q is the customary time-sharing parameter and where the distribution $p(X_Q^{(1)} \ldots X_Q^{(m)})$ is obtained from the empirical distributions $p(X^{(1)}(k) \ldots X^{(m)}(k)), 1 \leq k \leq n$ induced by the coding scheme. Combining this with Fano's inequality

$$\begin{aligned}
1 + P(\mathcal{E}|H)nr\log\rho &\geq H(W|Y^{D}[n], H) \qquad (35) \\
&\geq H(W|Y^{\Omega^c}[n], H), \qquad (36)
\end{aligned}$$

we obtain

$$1 + P(\mathcal{E}|H)nr\log\rho \geq n\left(r\log\rho - I(Y_Q^{\Omega^c}; X_Q^{\Omega}|X_Q^{\Omega^c}, H)\right). \qquad (37)$$

Consider the event

$$\mathcal{A}_{\Omega} = \left\{H|I(X_Q^{\Omega}; Y_Q^{\Omega^c}|X_Q^{\Omega^c}, H) < (r-\epsilon)\log\rho\right\}. \qquad (38)$$

Multiplying (37) by the density function $p_{\mathbf{H}}(H)$ and integrating over $\mathcal{A}_{\Omega}$, we obtain

$$\begin{aligned}
&P(H \in \mathcal{A}_{\Omega}) + P(\mathcal{E}, H \in \mathcal{A}_{\Omega})nr\log\rho \\
&\qquad\qquad \geq P(H \in \mathcal{A}_{\Omega})n\epsilon\log\rho, \qquad (39)
\end{aligned}$$

[3] While the result is known from [24], we present a proof here for completeness, as parts of the proof will be called upon during other derivations in the sequel.

which leads to

$$P(\mathcal{E}|H \in \mathcal{A}_\Omega) \geq \frac{\epsilon}{r} - \frac{1}{nr\log\rho}. \tag{40}$$

The probability of error can then be lower bounded as

$$\begin{aligned} P(\mathcal{E}) &\geq P(\mathcal{E}|H \in \mathcal{A}_\Omega)P(H \in \mathcal{A}_\Omega) \\ &\geq \left(\frac{\epsilon}{r} - \frac{1}{nr\log\rho}\right)P(H \in \mathcal{A}_\Omega) \quad (41) \\ &\geq \left(\frac{\epsilon}{r} - \frac{1}{nr\log\rho}\right)P_{\text{out},\Omega}((r-\epsilon)\log\rho), \quad (42) \end{aligned}$$

where (42) follows from the definition of $P_{\text{out},\Omega}(r\log\rho)$ in (31). In the limit $\epsilon \to 0$, we get

$$\begin{aligned} P(\mathcal{E}) &\dot{\geq} P_{\text{out},\Omega}(r\log\rho) \quad \forall \ \Omega \\ \text{which implies that, } P(\mathcal{E}) &\dot{\geq} \max_\Omega P_{\text{out},\Omega}(r\log\rho), \quad (43) \end{aligned}$$

from which (32) follows.

□We shall refer to this upperbound in (32) as the full-duplex cut-set bound.

We next derive a second lower bound on the probability of error of the coding scheme that makes a connection with the recent results in [23]. We begin from equation (34). We have that $\forall\Omega$,

$$\begin{aligned} nR &\leq nI(X_Q^\Omega; Y_Q^{\Omega^c}|X_Q^{\Omega^c}, H) + H(W|Y^{\Omega^c}[n], H) \\ &\leq nI(X_Q^\Omega; Y_Q^{\Omega^c}|X_Q^{\Omega^c}, H) + H(W|Y^D[n], H). \quad (44) \end{aligned}$$

Noting that the distribution $p(X_Q^{(1)} \dots X_Q^{(m)})$ is independent of Ω, and combining with Fano's inequality in (35) we have

$$\begin{aligned} nR \leq & \sup_{p_X}\min_\Omega nI(X^\Omega; Y^{\Omega^c}|X^{\Omega^c}, H) \\ & +1 + P(\mathcal{E}|H)nr\log\rho. \quad (45) \end{aligned}$$

Hence we have,

$$\begin{aligned} & 1 + P(\mathcal{E}|H)nr\log\rho \\ & \quad \geq n\left(r\log\rho - \sup_{p_X}\min_\Omega I(Y^{\Omega^c}; X^\Omega|X^{\Omega^c}, H)\right). \quad (46) \end{aligned}$$

Defining

$$\mathcal{B} = \left\{ H | \sup_{p_X} \min_{\Omega} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) < (r - \epsilon) \log \rho \right\} \tag{47}$$

and following steps similar to those in deriving equations (39) - (41), the probability of error can be lower bounded as

$$\begin{aligned} P(\mathcal{E}) &\geq \left(\frac{\epsilon}{r} - \frac{1}{nr \log \rho} \right) P(H \in \mathcal{B}) \\ \Rightarrow P(\mathcal{E}) &\dot{\geq} P(H \in \mathcal{B}). \end{aligned} \tag{48}$$

It can be shown that the above lower bound coincides with the one in (43) (Please see Appendix A for a proof).

Although we are constrained to use only certain activation sets in a half-duplex network, the DMT of several half-duplex networks has been shown [26], [27], [28] to equal the full-duplex cut-set bound. We will describe these class of networks and the optimal schemes in Section 4.5 and Section 4.6. However, the full duplex cut-set bound cannot be achieved for all networks [29], as we show in the following example.

Example 1 *Consider the network in Fig. 4, where there is only one relay and there is no direct link between the source and the sink. The two acti-*

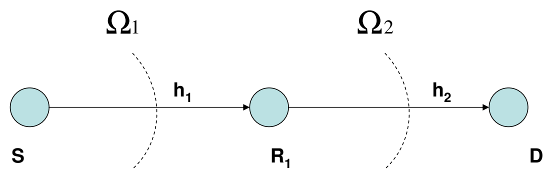


Figure 4: Single relay network

vation sets of this network are the ones in which only the source-relay link is active (a_1) and the other in which only the relay-sink link (a_2) is active. Let us consider first a static schedule. Let $f_1 = f$. Then $f_2 = 1 - f$. Recall that f_1 and f_2 denote the fraction of time for which network operates in activation sets a_1 and a_2 respectively. Since the cut Ω_1 is active only for fraction f of time slots, the maximum mutual information across this cut is $f \log(1 + \rho |h_1|^2)$. Similarly, for the second cut, the maximum mutual *information is* $(1-f)\log(1+\rho|h_2|^2)$. *As we will see later, the probability of error of any coding scheme can be lower bounded as*

$$\begin{aligned} P_e(r\log\rho) \;\; &\dot{\geq} \;\; \inf_f P\left\{\min\left(f\log(1+\rho|h_1|^2),\right.\right. \\ &\left.\left.(1-f)\log(1+\rho|h_2|^2)\right) < r\log\rho\right\}. \end{aligned} \tag{49}$$

It is straightforward to show that the infimum occurs when $f = 0.5$ *and also that* $P_e(r\log\rho)\dot{\geq}\rho^{-(1-2r)^+}$ *i.e.,* $d(r) \leq (1-2r)^+$. *However, the full-duplex cut-set bound is* $d(r) \leq (1-r)^+$. *Thus we see that it is essential to take into account the half-duplex nature of the network while deriving upper bounds.*

4.3.2 Half-duplex cut set bound

We will now derive an upper-bound to the DMT of the half-duplex(HD) network by taking into account the fact that, the activation sets in the case of half-duplex network need not be the complete network[4]. Let $I(X^{\Omega}_{a_j}; Y^{\Omega^c}_{a_j}|X^{\Omega^c}_{a_j}, H)$ denote the mutual information for the cut Ω for the activation set a_j.

Theorem 2 *Consider a half-duplex network operating under a static schedule. Consider a particular cut* Ω *in the network. Define the probability of outage of the cut as*

$$\begin{aligned} P^{HD}_{out,\Omega}(r\log\rho) \;\; &:= \\ &\inf P\left\{H|\sum_{i=1}^{L} f_i I(X^{\Omega}_{b_i}; Y^{\Omega^c}_{b_i}|X^{\Omega^c}_{b_i}, H) < r\log\rho\right\}, \end{aligned} \tag{50}$$

where the infimum is computed over the set of all distributions $\{p(X^{(j)}_{a_i})\}$, $j \in \{1,\ldots,m\}, i \in \{1,\ldots L\}$. *Let* $P^{HD}_{out,\Omega}(r\log\rho)\dot{=}\rho^{-d^{HD}_{out,\Omega}(r)}$. *Then for every cut* Ω, $d(r) \leq d^{HD}_{out,\Omega}(r)$ *and thus*

$$d(r) \;\; \leq \;\; \min_{\Omega} d^{HD}_{out,\Omega}(r). \tag{51}$$

Proof: The proof is very similar to that done for the full duplex cut set bound in (32) and hence we only outline the first few steps where differences occur due to consideration for activation sets.

[4]We would like to cite [30], where an upperbound to the achievable rates for a half-duplex network, taking the activation sets into consideration, has been derived.

Our starting point is (33). The rate of flow of information can be upper bounded as

$$nR \leq \sum_{t=1}^{n} I(Y^{\Omega^c}(t); X^{\Omega}(t)|X^{\Omega^c}(t), H) + H(W|Y^{\Omega^c}[n], H). \qquad (52)$$

Let T_i denote the time slots where the activation sets a_i are used. Then

$$\begin{aligned} nR &\leq \sum_{i=1}^{L} \sum_{t \in T_i} I(Y_{a_i}^{\Omega^c}(t); X_{a_i}^{\Omega}(t)|X_{a_i}^{\Omega^c}(t), H) \\ &\quad + H(W|Y^{\Omega^c}[n], H) \\ &\leq \sum_{i=1}^{L} |T_i| I(Y_{Q_i}^{\Omega^c}(t); X_{Q_i}^{\Omega}(t)|X_{Q_i}^{\Omega^c}(t), H) \\ &\quad + H(W|Y^{\Omega^c}[n], H), \end{aligned} \qquad (53)$$

where Q_is are time sharing parameters, one for each activation set. Thus

$$\begin{aligned} R \leq \sum_{i=1}^{L} f_i I(Y_{Q_i}^{\Omega^c}(t); X_{Q_i}^{\Omega}(t)|X_{Q_i}^{\Omega^c}(t), H) \\ + \frac{1}{n} H(W|Y^{\Omega^c}[n], H). \end{aligned} \qquad (54)$$

Now, we define the outage event associated with the cut Ω as

$$\mathcal{A}_{\Omega}^{\text{HD}} = \left\{ H | \sum_{i=1}^{L} f_i I(X_{Q_i}^{\Omega}; Y_{Q_i}^{\Omega^c} | X_{Q_i}^{\Omega^c}, H) < (r - \epsilon) \log \rho \right\} \qquad (55)$$

The rest of the proof proceeds exactly as in (39) to (43) to show that

$$\begin{aligned} P(\mathcal{E}) &\dot{\geq} P(\mathcal{A}_{\Omega}^{\text{HD}}) & (56) \\ &\dot{\geq} P_{\text{out},\Omega}^{\text{HD}}(r \log \rho) \ \forall \ \Omega & (57) \\ \text{and hence,} \quad P(\mathcal{E}) &\dot{\geq} \max_{\Omega} P_{\text{out},\Omega}^{\text{HD}}(r \log \rho). & (58) \end{aligned}$$

from which (51) follows. □

The upper bound on the DMT was obtained assuming a fixed schedule. One may further optimize it over all the possible schedules and thus obtain the best possible static schedule which would maximize the DMT. In this case, we have

$$d(r) \quad \leq \quad \sup_{\{f_i\}} \min_{\Omega} d^{\mathrm{HD}}_{\mathrm{out},\Omega}(r). \tag{59}$$

We will refer to this bound as half-duplex cut-set bound.

Theorem 3 *[28],[23] The QF protocol described earlier achieves the half-duplex cut-set bound.*

Proof: Achievability can be shown using a concatenated coding scheme at both the source and the relays. The concatenated coding scheme employs random Gaussian codebooks for both the inner and outer codes. We refer to [28], [23] for the details of the proof. The probability of outage defined in [23] for a static schedule is given by

$$\begin{aligned} P^{\mathrm{HD}}_{\mathrm{out}}(r\log\rho) \quad &:= \\ P\left\{ H | \sup_{p_X^{\mathrm{HD}}} \min_{\Omega} \sum_{i=1}^{L} f_i I(X_{b_i}^{\Omega}; Y_{b_i}^{\Omega^c} | X_{b_i}^{\Omega^c}, H) < r\log\rho \right\}. \end{aligned} \tag{60}$$

However, it can be shown that even with this definition instead of (50), we obtain the same upperbound as in (59).

□

While the above bound to DMT of half-duplex networks is calculated by restricting the schedules to be static, the upperbound could potentially be increased by considering dynamic schedules.

Example 2 *Consider once again the single relay network with no direct link between the source and the sink. The probability of error of any coding scheme under a dynamic schedule can be lower bounded as*

$$\begin{aligned} P_e(r\log\rho) \quad \geq \quad & P\{H| \sup_f \min(f\log(1+\rho|h_1|^2), \\ & (1-f)\log(1+\rho|h_2|^2))\}. \end{aligned} \tag{61}$$

Here, the fraction f depends on the channel realization. Now, for a given channel realization, the supremum is achieved when

$$f = \frac{\log(1+\rho|h_1|^2)\log(1+\rho|h_2|^2)}{\log(1+\rho|h_1|^2)+\log(1+\rho|h_2|^2)}. \tag{62}$$

It can be shown[29] that the upperbound on DMT improves under this dynamic schedule and is given by

$$d(r) = \begin{cases} \frac{1-2r}{1-r} & 0 \leq r \leq \frac{1}{2} \\ 0 & r > \frac{1}{2} \end{cases} \tag{63}$$

but it still does not meet the full duplex cut-set bound. Figure 5 compares the full-duplex and the half -duplex cut-set bounds for the network in this example (the HD-static case is considered in example 1).

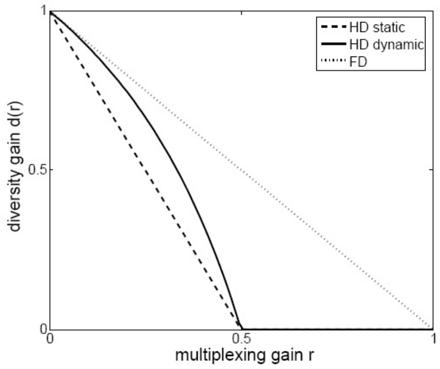


Figure 5: Comparison of the full-duplex and half-duplex cut-set bounds for the source-relay-destination network

Remark 2 *Motivated by the above example, one might want to define the probability of outage as given below, wherein the schedules are assumed to be dynamic. The outage expression could be used in Theorem 2 in place of* (50) *to get a possibly relaxed upper bound to the DMT of half-duplex networks.*

$$P_{out}^{HD}(r \log \rho) := P\left\{ H | \sup_{p_X^{HD}, \{f_i\}} \min_{\Omega} \sum_{i=1}^{L} f_i I(X_{b_i}^{\Omega}; Y_{b_i}^{\Omega^c} | X_{b_i}^{\Omega^c}, H) < r \log \rho \right\}$$

4.4 DMT and Explicit Codes for AF and DF protocols

While the QF protocol achieves the DMT of the half-duplex network under any static schedule, the concatenated coding scheme used in QF protocol employ inner and outer codes of large block length and hence incurs large latency, apart from not being explicit. For certain subclass of half-duplex networks however, there exist explicit low latency coding schemes under an AF protocol.

DMT of AF protocol: Consider a static or dynamic schedule for the network. Under an AF protocol, due to linearity, the relation between the signal transmitted by the source and that received by the sink can be written in the form:

$$\begin{bmatrix} \mathbf{y}(1) \\ \vdots \\ \mathbf{y}(B) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{B1} & \dots & \mathbf{H}_{BB} \end{bmatrix}}_{\mathbf{H}_{\text{blt}}} \begin{bmatrix} \mathbf{x}(1) \\ \vdots \\ \mathbf{x}(B) \end{bmatrix} + \mathbf{W} \ , \tag{64}$$

where $\mathbf{x}^t(b)$ is a vector of size $1 \times T$ transmitted by the source in the b^{th} block and $\mathbf{y}^t(b)$ is the vector received by the sink in the b^{th} block. Here, we have assumed that the source always transmits. The lower triangular form of $\mathbf{H}_{\text{blt}}$ is due to causality. The matrix $\mathbf{H}_{\text{blt}}$ will be referred to as the induced channel for the AF protocol and is of size $BT \times BT$. The DMT of the AF protocol under this schedule, $d_{\text{AF}}(r)$ is then the DMT of $\mathbf{H}_{\text{blt}}$. Although, the noise $\mathbf{W}$ is in general colored, it has been shown in [8] that for purpose of computing the DMT the noise can be assumed to be white. The statistics of $\mathbf{H}_{\text{blt}}$ depend on the schedule employed in the network. It is difficult to compute the DMT of the induced channel matrix for any arbitrary schedule. Hence, we now give a lower bound to the DMT of AF protocol [8] which, as we will see in the next subsection, is tight for many known schedules.

Theorem 4 *Let the l-th sub-diagonal matrix $\mathbf{H}^{(l)} (0 \leq l \leq B-1)$, of the matrix $\mathbf{H}_{blt}$ be defined as the matrix comprising only of the entries $\mathbf{H}_{l1}, \mathbf{H}_{(l+1)2}, \dots, \mathbf{H}_{(l+B-1)B}$ i.e.,*

$$\mathbf{H}^{(l)}_{ij} = \begin{cases} \mathbf{H}_{ij} & i - j = l \\ \mathbf{0} & \textit{elsewhere} \end{cases} . \tag{65}$$

Let the last *sub-diagonal matrix $\mathbf{H}^{(\ell)}$ be defined as the sub-diagonal matrix $\mathbf{H}^{(l)}$ of $\mathbf{H}_{blt}$, where ℓ is the largest integer for which $\mathbf{H}^{(l)}$ is non-zero. The DMT $d_{AF}(r)$ can be lower bounded as*

1. *$d_{AF}(r) \geq d_{H^{(0)}}(r)$.*

2. $d_{AF}(r) \geq d_{H^{(\ell)}}(r)$*.*

3. In addition, if the entries of $H^{(\ell)}$ *are independent of the entries in* $H^{(0)}$*, then* $d_{AF}(r) \geq d_{H^{(0)}}(r) + d_{H^{(\ell)}}(r)$*.*

Distributed code: Now, a CDA code of size $BT \times BT$ achieves the DMT of the AF protocol. In the b^{th} block $b = 1, \ldots B$ the source transmits consecutively the $((b-1)T+1)^{\text{th}}, \ldots, (bT)^{\text{th}}$ rows of the code. Each relay transmits appropriate linear combination of the symbols received in the previous blocks. The blocks at which a relay transmits depend on the schedule. The latency incurred by this code is B^2T^2.

Thus we see here that for any given combination of schedule and AF protocol, there exists a CDA code which achieves the DMT of the network associated with that protocol and schedule.

DMT of DF protocol: Consider a static or dynamic schedule for the network. Under a DF protocol, the signal received by the i^{th} node in the first b blocks $1 \leq b \leq B$, can be written in the form

$$\begin{bmatrix} Y^{(i)}(1) \\ \vdots \\ Y^{(i)}(b) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}^{(i)}(1) & & \\ & \ddots & \\ & & \mathbf{H}^{(i)}(b) \end{bmatrix}}_{\mathbf{H}_p^{(i)}(b)} \begin{bmatrix} X(1) \\ \vdots \\ X(b) \end{bmatrix} + \mathbf{W} \ . \qquad (66)$$

Here, $X^{(i)}(k)$ correspond to the concatenation of the $1 \times T$ vectors of a distributed code, transmitted by all the nodes in block k (except sink) and hence is of size $(m-1) \times T$. A node $i, i \neq \{1, m\}$ is ready to transmit in the bth block if the mutual information corresponding to the matrix $\mathbf{H}_p^{(i)}(b-1)$ is sufficient to support the rate R. The DMT of the DF protocol under this schedule is the DMT of $\mathbf{H}_p^{(m)}(B)$ (recall that m is the sink).

Distributed Code: In this case, the multi-block code introduced in Section 3 is DMT optimal. The matrix X in (29) is chosen to be an appropriate rectangular code of size $(m-1) \times T$. Here, the node $i, 1 \leq i \leq m-1$ is assigned the i^{th} row of $\phi^{b-1}(X)$ in the b^{th} block $b = 1, \ldots B$. A node transmits its assigned row in a particular block only if the schedule permits the node to transmit in that block and is ready to transmit. For example, in a dynamic-DF (DDF) protocol (introduced for two-hop networks in [5]) a relay node listens until it is not in outage and then transmits for the remaining number of blocks. The latency incurred by the distributed code is BT. The coding scheme described above draws from the DDF protocol code presented for two-hop networks in [18].

Summarizing the discussion so far, we have derived the cut-set bounds for general half-duplex relay networks and discussed the DMT of QF, AF and DF protocols. We will now discuss the DMT of certain classes of two-hop and multi-hop networks. Since in all the protocols, the DMT depends on the schedule employed in the network, we will now consider various schedules that have appeared in the literature, apply the different protocols and study their DMT.

4.5 Two-hop Networks

The two-hop relay network with N relays is shown in Fig. 2(a). We will denote these relays by $R_i,\ 1 \leq i \leq N$. Let g_d be the source-sink channel gain and let g_i, h_i be the source-relay and relay-sink channel gains respectively. The network is said to be isolated if no inter-relay link exists and is said to be non-isolated if all the inter-relay links exist. The full-duplex cut-set bound for both isolated and non-isolated two-hop networks can be computed using (32) and shown to be $(N+1)(1-r)$.

4.5.1 Single Relay

Let $\{S, R_1, D\}$ be a single relay network. Consider a static schedule, where in the first block the network is under activation set a_1 and in the second block is under activation set a_2 as shown in Fig. 6. Hence, $f_1 = .5$ and $f_2 = .5$ (termed uniform schedule). The upper bound to the DMT under this schedule can be evaluated using (51). This matches with the full-duplex cut-set bound given by $2(1-r)$ [24][28].

As mentioned earlier, a QF protocol will achieve this bound. Now, if we employ an AF protocol with the uniform schedule, the DMT is given by $d(r) = (1-2r)^+ + (1-r)^+$. This can be shown by using (12) and applying Theorem 4 to the resultant induced channel model given by

$$\mathbf{H} = \begin{bmatrix} g_d & 0 \\ g_1 h_1 & g_d \end{bmatrix}, \tag{67}$$

and from results in [5]. Note that the DMT of the AF protocol under this

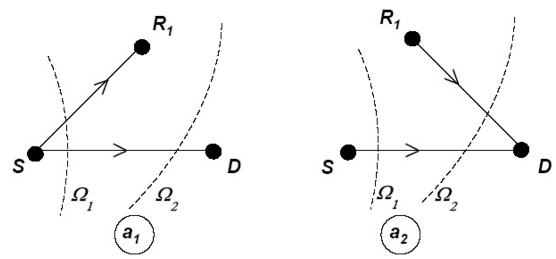


Figure 6: A single relay network with the two cuts

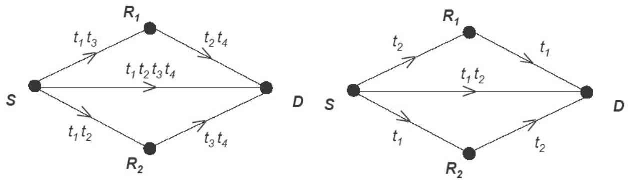
schedule does not match the full-duplex cut-set bound. However, it has been shown that the DMT of the AF protocol cannot be improved by considering any other static schedule [5]. Now, if we employ a DF protocol with the uniform schedule the DMT is same as that under the AF protocol. But, the DMT under a DF protocol can be improved by choosing a static schedule which depends on the multiplexing gain r [31].

4.5.2 N-relay Isolated Network

We will consider two static schedules for the isolated network that have appeared in the literature.

Uniform schedule: Under this schedule, we consider all the activation sets in which at any block, a relay either transmits or receives but does not remain idle and the source transmits always. Each activation set is used for equal fraction of time. There are 2^N activation sets and $f_i = 2^{-N}$ (See Fig. 7(a)). In the figure, all the links that are active at block t_j have a label t_j and these set of active edges specify the activation set at block t_j. Once again, the upper bound to the DMT under this schedule can be evaluated using (51) and it matches the full-duplex cut-set bound given by $(N+1)(1-r)$ [28] and a QF protocol achieves this bound (See Theorem 3). Now, we will describe another schedule which achieves the full-duplex cut-set bound under an AF protocol. We remind the reader that explicit coding schemes can be constructed for any AF protocol.

Round-Robin Schedule: Under this schedule, the source transmits for B blocks. Each relay takes turn in transmitting the source message. The activation set $a_i, i = 1, 2 \dots N$, that occur in this schedule are those in which the relay R_i transmits to sink and relay $R_{(i+1) mod N}$ listens to the source. All



(a) Uniform Schedule (b) Round Robin Schedule

Figure 7: Static Schedules

other relays remain idle. (See Fig. 7(b)) For large B we will have $f_i \approx \frac{1}{N}$.

Now, if we employ an AF protocol (a relay just retransmits its previous received vector) under this schedule, the DMT becomes $d(r) \geq (1-r)^+ + N(1 - \frac{B}{B-1}r)^+$. This can be proved by using (12) and applying Theorem 4 to the induced channel. The DMT equals the full-duplex cut-set bound for very large B. This protocol is known as the slotted AF (SAF) protocol [27].

4.5.3 N-relay Non-isolated Network

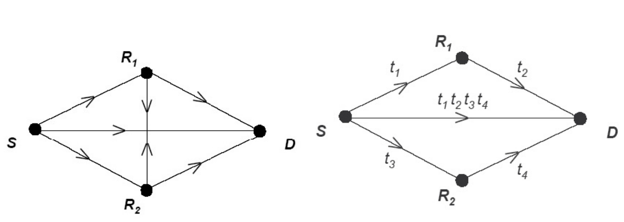
We will now consider a static schedule with AF protocol (known as NAF protocol [5]) and a dynamic schedule with DF protocol (known as DDF protocol [5]) for the non-isolated network. A two relay non-isolated network is shown in Fig. 8(a) where all possible active links are indicated.

In the schedule corresponding to the NAF protocol (termed Uniform Schedule with idle state here), the source transmits for $2N$ blocks. Relay $R_i, i = 1, 2, \ldots N$ listens at block $2i-1$ and retransmits the vector in block $2i$. In every block, only one relay participates and all other relays remain idle. There are $2N$ activation sets in this schedule and $f_i = \frac{1}{2N}$ (See Fig. 8(b)).

The induced channel for NAF protocol is diag $(H_1, \ldots, H_N)$ where

$$\mathbf{H}_i = \begin{bmatrix} g_d & 0 \\ g_i h_i & g_d \end{bmatrix}. \tag{68}$$

By applying Theorem 4 and using (12) and, the DMT of NAF protocol can be shown to be $d(r) \geq (1-r)^+ + N(1-2r)^+$, which is tight from results in [5]. As mentioned earlier, an AU CDA code of size $2N \times 2N$ will be DMT



(a) Non-isolated network (b) Uniform Schedule with idle state

optimal, since this is an AF protocol. Now, since the induced channel is in the parallel channel form, a multi-block code of size $2N \times 2$ is also DMT optimal for this protocol [32].

Now, under the DDF protocol explained earlier, an improved DMT can be obtained for the two-hop network given by [5]

$$d(r) = \begin{cases} (N+1)(1-r), & 0 \leq r \leq \frac{1}{N+1} \\ 1 + \frac{N(1-2r)}{1-r}, & \frac{1}{N+1} \leq r \leq \frac{1}{2} \\ \frac{1-r}{r}, & \frac{1}{2} \leq r \leq 1 \end{cases} . \qquad (69)$$

A detailed treatment of single relay DDF protocol and code constructions can also be found in [33].

4.6 Multi-hop networks

In this section, we will discuss two classes of half-duplex multi-hop networks and then a schedule known as orthogonal schedule and its derived versions. The discussion in this section follows [26]. Protocols and schedules for certain class of multi-hop networks can also be found in [34], [35], [36], [37]. Multi-hop networks can be viewed as a generalization of two-hop networks. For example, we can regard a K relay isolated two-hop network as consisting of either K parallel paths between the source and the sink or as consisting of a single layer of K relays between the source and the sink. These two viewpoints can be generalized to the multi-hop networks as follows:

K-Parallel-Path (KPP) Networks: A K-parallel-path (KPP) network is defined as a union of K node-disjoint parallel paths between the source and the sink, where each path has at least one relay. (See Fig. 8). These parallel paths are denoted by $P_i, i = 1, 2 \ldots K$.

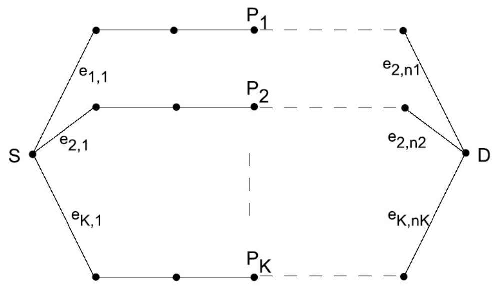


Figure 8: The KPP network. An edge is denoted as $e_{i,j}$

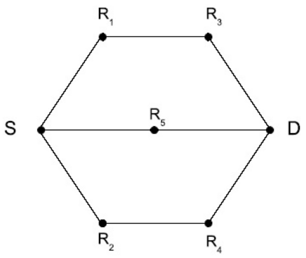
A KPP network in which there is a direct link between the source and the sink is called a KPP(D) network and one in which there are links interconnecting relays in various paths is called a KPP network with interference denoted by KPP(I). A KPP network which is both KPP(I) and KPP(D) is denoted as KPP(I,D) network (See Fig. 9). For a KPP(D), KPP(I) and KPP(I,D) network, a backbone KPP network is a union of some K node disjoint paths in the network connecting the source to the sink. These paths are referred to as backbone paths. In the case of KPP network, the backbone KPP network is same as the network itself.

Layered Networks: A layered network is one in which there are layer of relays between the source and the sink. Here any link is either between the nodes in the adjacent layers or between two nodes of a same layer. If all nodes in any two adjacent layers are connected then it is referred to as a fully connected (fc) layered network (See Fig. 10).

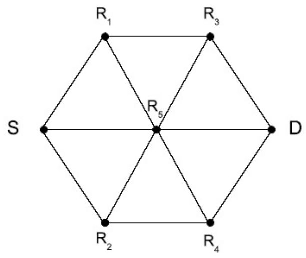
Cut-set bound: Using (32), the full-duplex cut-set bound can be shown to be $K(1-r)$ for KPP and KPP(I) networks $(K+1)(1-r)$ for a KPP(D) network. In the case of fc layered network, the full-duplex cut-set bound is shown to be concave in general.

Orthogonal Schedule: A schedule is called an orthogonal schedule if at any node, at a given time instant only one of the incoming or outgoing links is active and each link in the network is activated an equal number of times.

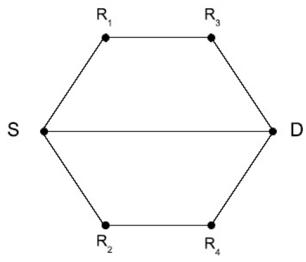
Orthogonal protocol: A protocol in conjunction with an orthogonal schedule is called an orthogonal protocol. In multi-hop networks discussed here always an AF protocol is employed. Hence we will refer to an orthogonal AF protocol as orthogonal protocol.



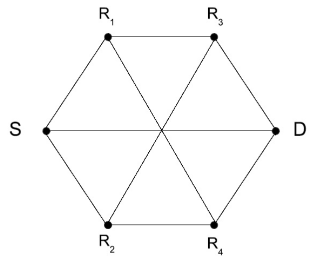
(a) A KPP network with $K = 3$



(b) A KPP(I) network with $K = 3$



(c) A KPP(D) network with $K = 2$



(d) A KPP(I, D) network with $K = 2$

Figure 9: Examples of KPP networks.

4.6.1 DMT of general KPP networks

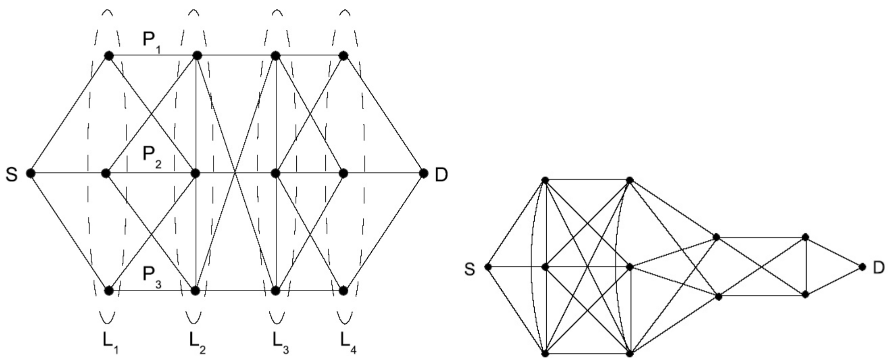
It has been shown that an orthogonal protocol that satisfies some additional constraints (which depend on the type of KPP network) can achieve the cut-set bound for the general KPP network. We will now see what these additional constraints are and for which kind of KPP, KPP(I), KPP(D) networks such protocols can be identified.

KPP network: We will first state a lemma concerning orthogonal protocols.

Lemma 5 *An orthogonal protocol achieves the cut-set bound $K(1-r)$ for the KPP network if it satisfies the following constraints.*

1. *In each time instant at least one of the outgoing links from the source is active.*

2. *In every cycle, the sink receives an equal number of symbols from each of the K backbone paths.*



(a) A layered network with with 4 relaying layers

(b) A fully-connected layered network with intra-layer links

Figure 10: Layered networks

3. *The protocol avoids back-flow i.e., there are at least two inactive edges between any two active edges along a backbone path.*

In this case the induced channel takes the form

$$\begin{bmatrix} \gamma_1 & & & \\ & \gamma_2 & & \\ & & \ddots & \\ & & & \gamma_K \end{bmatrix}, \tag{70}$$

where γ_i is the product of the fading coefficients in the path P_i. The DMT evaluates to $K(1-r)$. An orthogonal protocol that satisfies the constraints in Lemma 5 can be identified for $K \geq 4$. For $K = 3$ an orthogonal protocol can be identified which even in the presence of back-flow achieves the cut-set bound.

KPP(D) network: In the case of KPP(D) networks, an orthogonal protocol satisfying the conditions of Lemma 5, is applied for its backbone KPP network. By introducing suitable delays at the nodes which have a direct link towards sink this protocol can be shown to achieve the cut-set bound. Thus the cut-set bound is achievable for $K \geq 4$.

KPP(I) network: In the case of KPP(I) networks also, an orthogonal protocol satisfying the conditions of Lemma 5, is applied for its backbone KPP network. Now, by introducing suitable delays at some intermediate nodes, this protocol too can be shown to achieve the cut-set bound. Thus the cut-set bound is achievable for $K \geq 4$.

4.6.2 DMT of Layered networks

In the case of KPP networks a DMT optimal protocol was derived from the one for its backbone KPP network. A common feature of the KPP networks was that the backbone network had K node disjoint paths. For the layered networks if we can identify M node disjoint paths then a DMT of at least $M(1-r)^+$ can be achieved by identifying a suitable schedule. For fully connected layered networks it is possible to identify d_{max} node disjoint paths, where d_{max} is the maximum diversity gain and hence a DMT of at least $d_{max}(1-r)^+$ can be achieved.

5 DMT of Multiple Access Channel

Consider the multiple access channel with K users, with each user having n_t transmit antennas and a single receiver having n_r receive antennas. The users are not allowed to cooperate and want to communicate independent messages to the receiver with a common multiplexing gain r.

The channel model for MAC can be written as

$$Y = \sum_{i=1}^{K} H_i X_i + W \ , \tag{71}$$

where H_i is the $n_r \times n_t$ Rayleigh faded MIMO channel between user i and the receiver and W is the noise matrix at the receiver. X_i is the $n_t \times T$ matrix codeword sent by the i^{th} user. We assume a slow fading scenario, where the channel matrices remain constant for a duration of T channel uses and changes from one block to another.

A codebook for each user denoted by $\mathcal{C}_i$ comprises of $\lceil 2^{RT} \rceil$ codeword matrices, where $R = r \log \rho$ is the data rate of each user. Let $\{X_i(j)\}$ denote the codeword matrices of the i^{th} user. Then the codewords for each user satisfy power constraint

$$\frac{1}{|\mathcal{C}_i|} \sum_{j=1}^{|\mathcal{C}_i|} \|X_i(j)\|_F^2 \quad \leq \quad T\rho, \tag{72}$$

where ρ is the signal-to-noise ratio from each user to the reciever.

Let $\mathcal{C}_i(\rho)$ and $P_e^{(i)}$ respectively denote the coding scheme and the probability of error corresponding to the i^{th} user. Let $d(r)$ be the common diversity gain achieved for each user i.e., $P_e^{(i)} \doteq \rho^{-d(r)}$. The optimal decoder that minimizes the probability of error for each user is the individual ML decoder. However, for the purposes of DMT, it is enough to consider the joint ML decoder and hence, the DMT of MAC channel is defined to be the supremum of diversity gains $d(r)$ achieved under the joint ML decoder. The DMT, $d_{\text{MAC}}(r)$, of the MAC channel is then given by [2]

$$d_{\text{MAC}}(r) = \begin{cases} d^*_{n_t,n_r}(r) & r \leq \min(n_t, \frac{n_r}{K+1}) \\ d^*_{Kn_t,n_r}(Kr) & r \in J \end{cases} , \tag{73}$$

where J denotes the open set $(\min(n_t, \frac{n_r}{K+1}) \quad \min(n_t, \frac{n_r}{K}))$ and $d^*_{n_t,n_r}(r)$ is the DMT of the Rayleigh fading MIMO channel with n_t transmit and n_r receive antennas. The achievability in [2] is shown using a random Gaussian code with delay $Kn_t + n_r - 1$. From the above DMT expressions, we see that if $r \leq \min(n_t, \frac{n_r}{K+1})$, the single user DMT is achieved simultaneously for all the users. If $r \geq \min(n_t, \frac{n_r}{K+1})$, the DMT is as if the K users are pooled together into a single user with Kn_t antennas and multiplexing gain Kr. In this case, the performance of each user is affected by the presence of other users.

5.1 DMT Optimal Code

Recently in [38], a DMT optimal code for the MAC channel has been constructed based on CDAs. Let $K_o = K + 1$, if K is even and $K_o = K$, if K is odd. Let $Z_i, i = 1, \ldots, K$ be CDA codes each of size $n_t \times n_t$. The CDA used in the code construction is given in Appendix D. The codeword transmitted by the i^{th} user, X_i, is then given by

$$X_i \quad = \quad \theta[Z_i \ \tau(Z_i) \ \ldots \tau^{K_o-1}(Z_i) \ ] \ , \tag{74}$$

where the parameter τ is related to the structure of the CDA and θ is chosen to satisfy the power constraint. The delay of the code is $K_o n_t$.

6 Impact of Feedback

In this section, we discuss the DMT of a point-to-point MIMO channel in the presence of a noiseless instantaneous feedback link from the receiver to the transmitter. We also discuss DMT optimal codes for the point-to-point MIMO channel with feedback. In a feedback communication, two questions arise: How should the the feedback link be used and given the feedback information should the transmitter do rate adaptation, power control, or just choose the codebook based on the feedback. We will now consider two models for the feedback link and explore the different options available at the transmitter.

Model 1 *(ARQ):* In an ARQ scheme for the point-to-point MIMO channel [14], the receiver at the end of receiving a codeword (of length T), sends 1 bit of feedback (ACK/NACK) to the transmitter, indicating success or failure in decoding the source message. In an L round ARQ system, if the receiver is unable to decode the source message, the source is allowed to transmit another codeword matrix corresponding to the same message, up to a maximum of L transmissions. For now, assume that there is no power adaptation across retransmissions corresponding to a message. In a long term static channel, where the channel is assumed to remain constant during all the transmission rounds for a given message and varies independently for the next message, the DMT of a Rayleigh fading MIMO channel is given by,

$$d^*_{ls}(r, L) = \begin{cases} d^*_{\text{mimo}}(\frac{r}{L}), & 0 \leq r < \min\{n_r, n_t\} \\ 0, & r \geq \min\{n_r, n_t\} \end{cases} . \tag{75}$$

In a short term static channel, the channel is assumed to vary independently after every transmission round. The DMT in this case can be shown to be

$$d^*_{ls}(r, L) = \begin{cases} Ld^*_{\text{mimo}}(\frac{r}{L}), & 0 \leq r < \min\{n_r, n_t\} \\ 0, & r \geq \min\{n_r, n_t\} \end{cases} . \tag{76}$$

From the DMT expressions we see that there is a tradeoff between the maximum delay L and the diversity gains. Hence this tradeoff is known as diversity-multiplexing-delay (DMD) trade off [14]. The tradeoff with retransmissions could be explained as follows. At high SNRs, retransmissions are rare events and hence loss in rate due to retransmissions vanishes as SNR $\rightarrow \infty$ while diversity gains due to retransmissions is obtained. Fig 11 shows the DMT curves for a 2×1 MIMO system with $L = 3$.

In a long term static channel, one could make the transmit power of a retransmission to be inversely proportional to the probability of the retransmission but ensuring a long term average power constraint. This gives significant improvement over no power adaptation case, especially at the low multiplexing gains. (See Fig 11).

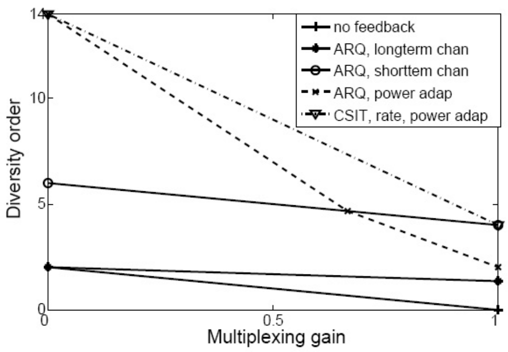


Figure 11: DMT of a MIMO channel with feedback $n_t = 2$, $n_r = 1$, $L = 3$ for feedback.

In [14], it is also shown that, for both long term and short term static channels, finite block length incremental redundancy lattice codes exist that achieve the above DMD tradeoff for the Rayleigh fading channel. However, these codes are not discussed here.

Explicit schemes for ARQ: Explicit code constructions based on CDAs which meet the DMD trade off for the long term static channel, without power adaptation across retransmissions, are provided in [39]. Consider the code $X_{\text{ARQ}} = [X, \phi(X), \ldots \phi^{L-1}(X)]$, where $\phi^i(X)$ is obtained from a multi-block code of size $n_t L \times n_t$. Now the source first transmits X and then in the l^{th} round, it transmits $\phi^{l-1}(X)$. After L rounds, the source proceeds to transmit the codeword corresponding to the next message. The above code is approximately universal for the ARQ channel. The delay (maximum) of the code is $n_t L$ which is not delay optimal in general. However delay-optimal codes which are not AU, but DMD optimal for a wide class of MIMO channels can be obtained whenever $n_r \geq n_t$ and either $n_t | L$ or $L | n_t$. The delay in these cases is $\lceil \frac{n_t}{L} \rceil$.

The code can be described as follows. If $L | n_t$ (note this implies $T = \frac{n_t}{L}$), one starts with an AU code of size $n_t \times n_t$ described in Section 3. In the l^{th} round of transmission of a message, transmit the columns from $(l-1)T+1$ to lT from the $LT \times LT$ matrix corresponding to the message. For example, if $n_t = 4, L = 2$, the codewords for the two rounds would look like

$$X_1 = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_3) \\ \ell_1 & \sigma(\ell_0) \\ \ell_2 & \sigma(\ell_1) \\ \ell_3 & \sigma(\ell_2) \end{bmatrix}, X_2 = \begin{bmatrix} \gamma\sigma^2(\ell_2) & \gamma\sigma^3(\ell_1) \\ \gamma\sigma^2(\ell_3) & \gamma\sigma^3(\ell_2) \\ \sigma^2(\ell_0) & \gamma\sigma^3(\ell_3) \\ \sigma^2(\ell_1) & \sigma^3(\ell_0) \end{bmatrix}.$$

In the case $n_t|L$, $T = 1$. Consider an example with $n_t = 2$, $L = 4$. It is shown that, in round l, transmitting the l^{th} column of the matrix

$$\begin{bmatrix} \ell_0 + \alpha\gamma\sigma^2(\ell_2) & \gamma\sigma(\ell_3) & \gamma\sigma^2(\ell_2) & \gamma\sigma^3(\ell_1) \\ \ell_1 + \alpha\gamma\sigma^2(\ell_3) & \sigma(\ell_0) & \gamma\sigma^2(\ell_3) & \gamma\sigma^3(\ell_2) \end{bmatrix},$$

will achieve the DMD trade off. Here $\{1, \alpha\}$ is an integral basis of any degree 2 extension field of $\mathbb{L}$, the maximal subfield of the division algebra used in the construction.

Model 2: In another model of feedback considered in [40], the receiver quantizes the channel H_k to an index $I_\rho(H_k) \in \{1, 2, \ldots, L\}$, in the k^{th} quasi-static interval; $k = 1, 2, \ldots$ and then sends it back to the transmitter prior to the transmission of the k^{th} codeword. L is called the feedback resolution. The transmitter then chooses both its multiplexing gain, r_k and the power, P_k for the k^{th} codeword, based on the index $I_\rho(H_k)$. In [40], the DMT of the Rayleigh fading MIMO channel is discussed under the constraints of a minimum multiplexing gain, r_{min} (i.e. $r \geq r_{min}$) and a long term average transmit power. The DMT for a 2×1 MIMO system with $L = 3$ and $r_{min} = \frac{r}{3}$ is shown in Fig 11, where r denotes the average multiplexing gain. It turns out that the explicit CDA based NVD codes referred to in Section 3 could be directly used to achieve the DMT bounds. Once r_k and P_k are chosen, the coding scheme corresponding to the rate $r_k \log_2(\rho)$ is used, after appropriately scaling with power P_k.

7 Conclusion

In this paper, an overview of the DMT of the point-to-point MIMO channel, single-source single-sink half-duplex relay networks and the MAC channel is provided. In the case of relay networks, two upper bounds on the DMT of the network, known as full-duplex cut-set bound and half-duplex cut-set bound, are derived and expressed in terms of the DMT of the MIMO channel appearing across its cuts. It turns out that in many half-duplex networks the full-duplex cut-set bound can be achieved while in certain networks it cannot be achieved. The DMT of several two-hop and multi-hop networks are also discussed. DMT optimal codes are provided wherever possible. Also, the impact of feedback on the DMT of a point-to-point MIMO channel is discussed.

A Proof of Equality of Outage Definitions

Let us consider the right hand side of (43). We have

$$\begin{aligned} &\max_{\Omega} P_{\text{out},\Omega}(r\log\rho) \\ &\quad = \max_{\Omega}\inf_{p_X} P(H|I(X^{\Omega};Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho). \end{aligned} \tag{77}$$

Since the channel across any cut is a MIMO channel, infimum over p_X in the right hand side of (77) is achieved by a joint Gaussian distribution on the transmissions by the m_Ω nodes on the source side of the cut with links to nodes on the sink side of the cut. If Σ_Ω is the covariance matrix associated with this joint Gaussian distribution, then we have the power constraint $\text{Tr}(\Sigma_\Omega) \leq m_\Omega\rho$. This follows since for a given channel matrix H, the Gaussian distribution maximizes the mutual information for a given power constraint. Hence,

$$\begin{aligned} &\max_{\Omega} P_{\text{out},\Omega}(r\log\rho) \\ &\quad = \max_{\Omega}\inf_{\Sigma_\Omega} P(H|I(X^{\Omega};Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho) \end{aligned} \tag{78}$$

since a Gaussian distribution is completely specified by Σ_Ω. Further, for Gaussian distributions, it is shown in [1] that

$$\inf_{\Sigma_\Omega} P(H|I(X^\Omega;Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho) \doteq P(H|I(X^\Omega_{\rm iid};Y^{\Omega^c}|X^{\Omega^c}_{\rm iid},H) < r\log\rho) \ \ \forall\ \Omega \tag{79}$$

which implies that,

$$\max_\Omega P_{{\rm out},\Omega}(r\log\rho) \doteq \max_\Omega P(H|I(X^\Omega_{\rm iid};Y^{\Omega^c}|X^{\Omega^c}_{\rm iid},H) < r\log\rho) \tag{80}$$

where iid indicates the i.i.d. Gaussian distribution with covariance matrix $\Sigma_\Omega = \rho I_{m_\Omega}$. Thus we have shown that

$$P(\mathcal{E}) \dot{\geq} \max_\Omega P(H|I(X^\Omega_{\rm iid};Y^{\Omega^c}|X^{\Omega^c}_{\rm iid},H) < r\log\rho). \tag{81}$$

Next we consider the right hand side of (48) as $\epsilon \to 0$, namely

$$P(H \in \mathcal{B}) = P(H|\sup_{p_X}\min_\Omega I(X^\Omega;Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho). \tag{82}$$

Once again, the supremum over p_X is achieved by a joint Gaussian distribution having some covariance matrix Σ_H. Thus we replace sup over p_X by sup over Σ_H. Now, the above expression can be upper and lower bounded as

$$\begin{aligned} P(H|\min_\Omega \sup_{\Sigma_{\Omega,H}} & I(X^\Omega;Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho) \\ &\leq P(H|\sup_{\Sigma_H}\min_\Omega I(X^\Omega;Y^{\Omega^c}|X^{\Omega^c},H) < r\log\rho) \\ &\leq P(H|\min_\Omega I(X^\Omega_{\rm iid};Y^{\Omega^c}|X^{\Omega^c}_{\rm iid},H) < r\log\rho). \end{aligned} \tag{83}$$

We will prove exponential equality of the first and last terms (and thus the equality of all three terms) in the above expression with the aid of the two lemmas stated below whose proofs can be found in Appendix B and Appendix C.

Lemma 6 *[23] Consider any cut Ω in the Gaussian relay network. Then,* *for jointly Gaussian input distributions,*

$$\sup_{\Sigma_{\Omega,H}} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) \leq I(X^{\Omega}_{iid}; Y^{\Omega^c} | X^{\Omega^c}_{iid}, H) + m,$$

where m is the number of nodes in the network.

Lemma 7 *Consider the event*

$$E_{\Omega} = \{H | I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) < R\} \tag{84}$$

corresponding to an arbitrary distribution p_X. Then

$$P(\min_{\Omega} E_{\Omega}) \doteq \max_{\Omega} P(E_{\Omega}). \tag{85}$$

Using Lemma 6 for a given Ω, H, we get

$$\sup_{\Sigma_{\Omega,H}} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) = I(X^{\Omega}_{\text{iid}}; Y^{\Omega^c} | X^{\Omega^c}_{\text{iid}}, H) + m_{\Omega,H}, \tag{86}$$

where $0 \leq m_{\Omega,H} \leq m$. Now

$$\begin{aligned} & P(H | \min_{\Omega} \sup_{\Sigma_{\Omega,H}} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) < r \log \rho) \\ & \quad = \; P(H | \min_{\Omega} \{ I(X^{\Omega}_{\text{iid}}; Y^{\Omega^c} | X^{\Omega^c}_{\text{iid}}, H) + m_{\Omega,H} \} < r \log \rho) \\ & \quad \doteq \; P(H | \min_{\Omega} I(X^{\Omega}_{\text{iid}}; Y^{\Omega^c} | X^{\Omega^c}_{\text{iid}}, H) < r \log \rho), \end{aligned} \tag{87}$$

where the last equality follows by first using (97) and subsequently using Lemma 7. Hence we have shown the exponential equality of all three terms in (83) and so we have

$$\begin{aligned} & P(H \in \mathcal{B}) \\ & \quad = \; P(H | \sup_{\Sigma_H} \min_{\Omega} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}, H) < r \log \rho) \\ & \quad \doteq \; P(H | \min_{\Omega} I(X^{\Omega}_{\text{iid}}; Y^{\Omega^c} | X^{\Omega^c}_{\text{iid}}, H) < r \log \rho) \\ & \quad \doteq \; \max_{\Omega} P(H | I(X^{\Omega}_{\text{iid}}; Y^{\Omega^c} | X^{\Omega^c}_{\text{iid}}, H) < r \log \rho) \end{aligned} \tag{88}$$

where the last equality follows from Lemma 7. Noting that (80) and (88) are the same, we establish the equivalence of the bounds in (43) and (48) i.e.,

$$\lim_{\epsilon\to 0} P(H\in\mathcal{B}) \quad\doteq\quad \max_{\Omega} P_{\text{out},\Omega}(r\log\rho). \tag{89}$$

B Proof Of Lemma 6

In the relay network, the channel across the cut can be written in the form $\mathbf{y} = H\mathbf{x}+\mathbf{w}$, where the size of the matrix H depends on the cut Ω. Let

$$I^{\Omega} \quad=\quad \sup_{p(X^{(1)},\ldots,X^{(m)})} I(X^{\Omega};Y^{\Omega^c}|X^{\Omega^c},H), \tag{90}$$

$$I^{\Omega}_{\text{iid}} \quad=\quad I(X^{\Omega}_{\text{iid}};Y^{\Omega^c}|X^{\Omega^c}_{\text{iid}},H). \tag{91}$$

It can be shown that

$$I^{\Omega} \quad=\quad \sum_{i=1}^{m_\Omega}\log(1+Q_i\lambda_i)\ , \tag{92}$$

where λ_i are the eigenvalues of $HH^{\dagger}$ and $\{Q_i\}$ is chosen to maximize $I(X^{\Omega};Y^{\Omega^c}|X^{\Omega^c},H)$, subject to the constraint $\sum_{i=1}^{m_\Omega} Q_i = m_\Omega\rho$. Now, if the all the input distributions are chosen i.i.d. as $\mathcal{N}(0,\rho)$, we have

$$I^{\Omega}_{\text{iid}} = \sum_{i=1}^{m_\Omega}\log(1+\rho\lambda_i). \tag{93}$$

Thus,

$$I^{\Omega}-I^{\Omega}_{\text{iid}} \quad=\quad \sum_{i=1}^{m_\Omega}\log\left(\frac{1+Q_i\lambda_i}{1+\rho\lambda_i}\right) \tag{94}$$

$$\leq \quad \sum_{i=1}^{m_\Omega}\log\left(1+\frac{Q_i\lambda_i}{\rho\lambda_i}\right) \tag{95}$$

$$\leq \quad \sum_{i=1}^{m_\Omega}\frac{Q_i}{\rho} \leq m_\Omega \leq m. \tag{96}$$

C Proof Of Lemma 7

We have $P(\min_\Omega E_\Omega) = P(\cup_\Omega E_\Omega)$. Also,

$$\max_\Omega P(E_\Omega) \leq P(\cup_\Omega E_\Omega) \leq \sum_\Omega P(E_\Omega) \tag{97}$$

and $\max_\Omega P(E_\Omega) \doteq \sum_\Omega P(E_\Omega)$. Combining the three equations, we get (85).

D Primer on Cyclic Division Algebras

D.0.1 Cyclic Division Algebras

Division algebras are rings with identity, in which every nonzero element has a multiplicative inverse. The center $\mathbb{F}$ of any division algebra D is the set of all elements of D that commute with every element of D. A field $\mathbb{L}$ such that $\mathbb{F} \subset \mathbb{L} \subset D$ and such that no proper subfield of D contains $\mathbb{L}$ is called a *maximal subfield* of D (Fig. 12). Cyclic Division Algebras (CDA) are division algebras in which the center $\mathbb{F}$ and a maximum subfield $\mathbb{L}$ are such that $\mathbb{L}/\mathbb{F}$ is a finite cyclic Galois extension. Let $\mathbb{F}$, $\mathbb{L}$ be number fields(finite field extensions of $\mathbb{Q}$) , with $\mathbb{L}$ a cyclic Galois extension of $\mathbb{F}$ of degree n. Let σ denote the generator of the Galois group $\text{Gal}(\mathbb{L}/\mathbb{F})$, where $\text{Gal}(\mathbb{L}/\mathbb{F})$ is the group of $\mathbb{F}$-automorphisms of $\mathbb{L}$. The norm of an element $u \in \mathbb{L}$ is defined as

$$N_{\mathbb{L}/\mathbb{F}}(u) \quad = \quad \prod_{i=1}^{n} \sigma^i(u).$$

Let z be an indeterminate satisfying

$$\ell z \;=\; z\sigma(\ell) \quad \forall \;\; \ell \in \mathbb{L} \quad \text{and} \quad z^n = \gamma, \tag{98}$$

for some non-norm element $\gamma \in \mathbb{F}^*$, by which we mean some element γ having the property that the smallest positive integer t for which γ^t is the norm $N_{\mathbb{L}/\mathbb{F}}(u)$ of some element $u \in \mathbb{L}^*$, is n. Then, a CDA $D(\mathbb{L}/\mathbb{F}, \sigma, \gamma)$ with index n, center $\mathbb{F}$ and maximal subfield $\mathbb{L}$ is the set of all elements of the form

$$\sum_{i=0}^{n-1} z^i \ell_i, \quad \ell_i \in \mathbb{L}. \tag{99}$$

Moreover, it is known that every CDA has this structure. It can be verified that D is a right vector space over the maximal subfield $\mathbb{L}$.

D.0.2 Space-Time Codes from Cyclic Division Algebras

A space-time code $\mathcal{X}$ can be associated to D by selecting the set of matrices corresponding to the matrix representation of elements of a finite subset of D. All the matrices will be square matrices of size $n \times n$.

The matrix representation of an element $d \in D$ corresponds the matrix of left multiplication by the element d in the division algebra. Let λ_d denote this operation, $\lambda_d : D \to D$, defined by

$$\lambda_d(e) = de, \ \forall \ e \in D. \tag{100}$$

It can be verified that λ_d is a $\mathbb{L}$-linear transformation of D. From (99), a natural choice of basis for the right-vector space D over $\mathbb{L}$ is $\{1, z, z^2, \ldots, z^{n-1}\}$. Under λ_d, the basis elements are mapped to

$$d.z^i \;=\; \sum_{k=0}^{n-i-1} z^{i+k}\sigma^i(\ell_k) + \gamma \sum_{k=n-i}^{n-1} z^{k-n+i}\sigma^i(\ell_k).$$

Thus, the matrix of the linear transformation λ_d under this basis can be written as

$$\begin{bmatrix} \ell_0 & \gamma\sigma(\ell_{n-1}) & \gamma\sigma^2(\ell_{n-2}) & \ldots & \gamma\sigma^{n-1}(\ell_1) \\ \ell_1 & \sigma(\ell_0) & \gamma\sigma^2(\ell_{n-1}) & \ldots & \gamma\sigma^{n-1}(\ell_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \ell_{n-1} & \sigma(\ell_{n-2}) & \sigma^2(\ell_{n-3}) & \ldots & \sigma^{n-1}(\ell_0) \end{bmatrix}. \tag{101}$$

Let $\mathcal{O}_{\mathbb{F}}$ and $\mathcal{O}_{\mathbb{L}}$ denote the ring of algebraic integers in $\mathbb{F}$ and $\mathbb{L}$ respectively. Let $\{\beta_1, \ldots, \beta_n\}$ be an integral basis for $\mathcal{O}_{\mathbb{L}}/\mathcal{O}_{\mathbb{F}}$. Let $\ell_i = \sum_{j=1}^{n} \beta_j \ell_{i,j}$. Now if the information symbols $\{\ell_{i,j}\}$ belong to $\mathcal{O}_{\mathbb{F}}$, then the determinant of the matrix representation of an element also belongs to $\mathcal{O}_{\mathbb{F}}$.

D.0.3 CDA for multi-block codes

Let T be an integer such that $T \geq n_t$. Let $m \geq M$ be the smallest integer such that the $\gcd(m, T) = 1$. Let $\mathbb{K}, \mathbb{M}$ be cyclic Galois extensions of $\mathbb{Q}(\imath)$ of degrees m, T respectively, whose Galois groups are generated by the

$$
\begin{array}{ll}
D & \\
| & \\
\mathbb{L} & \mathcal{O}_{\mathbb{L}} = <\beta_1, \ldots, \beta_n> \\
| & \quad | \\
\mathbb{F} = \mathbb{Q}(\imath) & \mathcal{O}_{\mathbb{F}} = \mathbb{Z}[\imath]
\end{array}
$$

Figure 12: *Structure of Division Algebra.*

automorphisms ϕ_1 and σ_1. Let $\mathbb{L}$ be the composite of $\mathbb{K}$ and $\mathbb{M}$, denoted as $\mathbb{KM}$ (See Fig. 13). It can be shown that $\mathbb{L}$ is a cyclic Galois extension of $\mathbb{K}$ of degree T and σ generates Gal($\mathbb{L}/\mathbb{K}$). Let $D_{\text{mb}}\left(\mathbb{L}/\mathbb{K}, \sigma, \gamma\right)$ be the corresponding CDA, where γ is a non-norm element of $\mathbb{K}$. The matrix X used in the construction of multi-block codes (See equation (29)) is the representation of elements of D_{mb}.

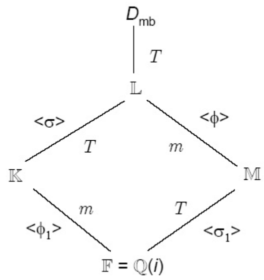


Figure 13: Structure of CDA for multi-block code

D.0.4 CDA for MAC codes

For a K user MAC, let $K_0 = K$ if K is odd and $K_0 = K + 1$ if K is even. Let $\mathbb{L}$ and $\mathbb{K}$ be cyclic Galois extensions of $\mathbb{Q}(\imath)$ of degrees n_t and K_0 respectively, and the generators for the corresponding automorphisms be σ and τ. $\mathbb{E} = \mathbb{KL}$. Consider the CDA $D_{\text{mac}}\left(\mathbb{E}/\mathbb{K}, \sigma, \zeta = \frac{\gamma}{\gamma *}\right)$,where γ is a non-norm element. The matrices Z_i used in the DMT optimal MAC code (See equation (74)) correspond to representation of elements of D_{mac}.

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