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A WAVEGUIDE PROBLEM INVOLVING A THICK IRIS IN THE THEORY OF ELECTROMAGNETISM

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The problem of electromagnetic wave propagation in a rectangular waveguide containing a *thick* iris is considered for its complete solution by reducing it to two suitable integral equations, one of which is of the first kind and the other is of the second kind. These integral equations are solved approximately, by using truncated Fourier series for the unknown functions. The reflection coefficient is computed numerically from the two integral equation approaches, and almost the same numerical results are obtained. This is also depicted graphically against the wave number and compared with *thin* iris results, which are computed by using complementary formulations coupled with Galerkin approximations. While the reflection coefficient for a thin iris steadily increases with the wave number, for a thick iris it fluctuates and zero reflection occurs. The number of zeros of the reflection coefficient for a thick iris increases with the thickness. Thus a thick iris becomes completely transparent for some discrete wave numbers. This phenomenon may be significant in the modelling of rectangular waveguides.

Key Words : Rectangular Waveguide; Iris; Galerkin's Technique; Reflection Coefficient

1. INTRODUCTION

Problems of electromagnetic wave propagation in a waveguide lead to those of determining the solutions of the reduced wave equation or the Helmholtz equation under appropriate boundary conditions (see Jones[1,2]). It has been shown in Jone's books that electromagnetic wave propagation in a waveguide gives rise to a class of interesting boundary value problems which can be handled for solution by reducing them to integral equations. The study of one such boundary value problem involves the determination of the electromagnetic field in a rectangular waveguide containing a thin iris formed by placing a thin sheet of metal parallel to the longer side of the waveguide. This problem has been tackled in Jones books by reducing it to two integral equations of the first kind, one valid in the aperture portion below the iris and the other valid in the perfectly conducting metal portion of the iris. These equations can be solved approximately by employing a truncated Fourier expansion method. Such truncated series methods of solution are particular cases of a general method, known as the Galerkin's technique (see Jones[1] [p 269]).

The present paper is concerned with the problem of electromagnetic wave propagation in a rectangular waveguide, in which is present a *thick* iris i.e., an obstacle in the form of a rectangular thick perfectly conducting metal plate placed parallel to the longer side of the waveguide. This problem generalizes the iris problem of Jones[1, 2] in which the thickness of the iris is zero. This generalization takes care of situations when a discontinuity in the medium consists of a region whose thickness is not negligible compared to other geometrical dimensions describing the physical situation. It may also be emphasized here that in practice, the effect of the thickness of the iris can be of utmost importance. In the context of other problems of scattering of electromagnetic waves, there have been efforts to study the effect of the thickness of the scatterer (see Davis and Leppington[3,4]. Also in the case of scattering of water waves by barriers, the effect of the thickness of the barriers has been the subject of investigations by several workers for quite some time (see Mei and Black[5], Guiney *et al.*[6], Owen and Bhatt[7], Kanoria *et al.*[8]).

It may be noted here that the thick iris can be viewed as a double waveguide discontinuity. There exists a considerable literature on waveguide discontinuities (see Collin[9], De Smedt and Denturck[10]), and various numerical techniques such as scattering matrix techniques, Galerkin or point matching techniques etc. have been employed to tackle them. Here the thick iris problem is solved by reducing it to that of finding the solutions of two independent integral equations, one of the first kind and the other of the second kind. Both these integral equations are solved approximately by truncated Fourier series expansion methods, and the reflection coefficient is evaluated numerically by using both the approaches. It is observed that both the approaches produce almost the same set of numerical results. This gives a check on the correctness of the numerical results. Also, the reflection coefficient is depicted graphically against the wave number and compared with the thin iris results. As the thickness increases, the reflection coefficient is seen to fluctuate against the wave number and zero reflection occurs. Thus a thick iris becomes completely transparent for some discrete frequencies of the incident wave field.

2. Statement of the Problem

A Cartesian co-ordinate system is used here and it is assumed that a rectangular waveguide occupied the region defined by $0 \le x \le a, 0 \le y \le h, -\infty < z < \infty$, except that there is a thick iris present in the middle. The configuration of the thick iris is given by $-b \le z \le b, 0 \le x \le a, d \le y \le h$ with $2b$ representing the thickness of the iris.

The problem here is to determine the electromagnetic field scattered by the iris when a time-harmonic field of known wavelength from the direction of $z = +\infty$ is incident on the iris. Then the field is partially reflected and transmitted by the iris. For simiplicity, the incident field is considered to be of the transverse electric type with the magnetic field having a component along the x-direction, which is represented by $Re\left\{\phi^{inc}(y,z)\sin\frac{\pi x}{a}e^{-i\omega t}\right\}$ where

$$\phi^{inc}(y,z) = 2e^{-i\lambda(z-b)}. \qquad \text{... (2.1)}$$

Here λ represents the wave number of the incident field, determined by $\lambda^2 = k^2 - \frac{\pi^2}{a^2} > 0$ in which $k = \omega/(\varepsilon\mu)^{1/2}$, ε, μ being the dielectric constant and permeability, respectively, of the interior of the waveguide. Assuming that the boundaries of the waveguide as well as the iris are perfectly conducting, the boundary value problem involves the determination of the total field represented by $Re\left\{\phi(y,z)\sin\frac{\pi x}{a}e^{-i\omega t}\right\}$ where the function $\phi(y,z)$ satisfies

$$(\nabla^2 + \lambda^2)\phi = 0 \text{ for } b < |z| < \infty, 0 < y < h, \text{ and } |z| < b, 0 < y < d, \qquad \text{... (2.2)}$$

$$\phi_y(0,z) = 0 \text{ for } -\infty < z < \infty, \qquad \text{... (2.3)}$$

$$\phi_y = 0 \text{ on } y = h \text{ for } |z| > b \text{ and on } y = d \text{ for } |z| < b, \qquad \text{... (2.4)}$$

$$\phi_z(y, \pm b) = 0 \text{ for } d < y < h, \qquad \text{... (2.5)}$$

$$\phi(y, \pm b+0) = \phi(y, \pm b-0) \text{ for } 0<y<d, \qquad \text{... (2.6)}$$

$$\phi_z(y, \pm b+0) = \phi_z(y, \pm b-0) \text{ for } 0<y<d, \qquad \text{... (2.7)}$$

$$r^{1/3}\, \nabla\phi \text{ is bounded as } r \to 0, \qquad \text{...(2.8)}$$

where r denotes the distance from the corner points $(d, \pm b)$,

$$\phi(y,z) \sim \begin{cases} \phi^{inc}(y,z) + R\,\phi^{inc}(y,-z) & as \quad z\to\infty, \\ T\,\phi^{inc}(y,z) & as \quad z \to -\infty. \end{cases} \qquad \text{... (2.9)}$$

where R and T are respectively the *reflection* and *transmission* coefficients (complex) to be determined in the course of the mathematical analysis.

3. Solution of the Problem

Due to the geometrical symmetry of the *thick* iris about the centre plane $z = 0$, it is convenient to split $\phi(y,z)$ into its symmetric and antisymmetric parts $\phi^s(y,z)$ and $\phi^a(y,z)$ respectively so that

$$\phi(y,z) = \phi^s(y,z) + \phi^a(y,z) \qquad \text{... (3.1)}$$

where
$$\phi^s(y,-z) = \phi^s(y,z), \quad \phi^a(y,-z) = -\phi^a(y,z). \qquad \text{... (3.2)}$$

Thus the analysis here may be restricted to the region $z \geq 0$ only. Now $\phi^{s,a}(y,z)$ satisfy the eq. (2.2) and the conditions (2.3) to (2.8) together with the conditions

$$\phi_z^s(y,0) = 0, \phi^a(y,0) = 0 \text{ for } 0<y<d. \qquad \text{.. (3.3)}$$

Let the behaviour of $\phi^{s,a}(y,z)$ for large z be represented by

$$\phi^{s,a}(y,z) \sim e^{-i\lambda(z-b)} + R^{s,a}\, e^{i\lambda(z-b)} \text{ as } z\to\infty \qquad \text{... (3.4)}$$

where R^s and R^a are unknown constants. By using the conditions (2.9) it is found that these constants are related to R and T by the relations

$$R, T = \frac{1}{2}(R^s \pm R^a)^{-2i\lambda b}. \qquad \text{... (3.5)}$$

Now, the eigenfunction expansions of $\phi^{s,a}(y,z)$ satisfying the eq. (2.2) and the conditions (2.3), (2.4), (3.3) and (3.4) (for $z > b$) in the two regions $z > b$, $0 < y < h$ and $0<z<b$, $0<y<d$ are given below :

$$\phi^{s,a}(y,z) = e^{-i\lambda(z-b)} + R^{s,a}\, e^{i\lambda(z-b)} + \sum_{n=1}^{\infty} A_n^{s,a}\, e^{-\mu_n(z-b)} \cos\frac{n\pi y}{h}$$

$$\text{for } z > b, \; 0<y<h \qquad \text{... (3.6)}$$

where $A_n^{s,a}$ $(n = 1, 2, ...)$ are unknown constants, and

$$\begin{pmatrix} \phi^s(y,z) \\ \phi^a(y,z) \end{pmatrix} = \begin{pmatrix} B_0^s \cos \lambda z \\ i B_0^a \sin \lambda z \end{pmatrix} + \sum_{n=1}^{\infty} \begin{pmatrix} B_n^s \cosh \xi_n z \\ i B_n^a \sinh \xi_n z \end{pmatrix}$$

$$\cos \frac{n\pi y}{d} \text{ for } 0<z<b, 0<y<d, \qquad \text{... (3.7)}$$

with

$$\xi_n = \left(\frac{n^2 \pi^2}{d^2} - \lambda^2 \right)^{1/2} > 0, \; n = 1, 2....$$

and $B_n^{s,a}$ $(n = 0, 1, 2, ...)$ being unknown constants.

The condition (2.5) gives rise to the relations

$$i\lambda(1 - R^{s,a}) + \sum_{n=1}^{\infty} \mu_n A_n^{s,a} \cos \frac{n\pi y}{h} = 0, d<y<h. \qquad \text{... (3.8)}$$

Again, the conditions (2.6) and (2.7) give rise to the relations

$$1 + R^{s,a} + \sum_{n=1}^{\infty} A_n^{s,a} \cos \frac{n\pi y}{h} = \sum_{n=0}^{\infty} B_n^{s,a} u_n^{s,a} \cos \frac{n\pi y}{d}, 0<y<d \qquad \text{... (3.9)}$$

and

$$-i\lambda(1 - R^{s,a}) - \sum_{n=1}^{\infty} \mu_n A_n^{s,a} \cos \frac{n\pi y}{h} = \sum_{n=0}^{\infty} B_n^{s,a} v_n^{s,a} \cos \frac{n\pi y}{d}, 0<y<d \qquad \text{... (3.10)}$$

where

$$\begin{pmatrix} u_0^s \\ u_0^a \end{pmatrix} = \begin{pmatrix} \cos \lambda b \\ i \sin \lambda b \end{pmatrix}, \begin{pmatrix} u_n^s \\ u_n^a \end{pmatrix} = \begin{pmatrix} \cosh \xi_n b \\ \sinh \xi_n b \end{pmatrix}, n>0 \qquad \text{... (3.11)}$$

and

$$\begin{pmatrix} v_0^s \\ v_0^a \end{pmatrix} = \begin{pmatrix} -\lambda \sin \lambda b \\ i\lambda \cos \lambda b \end{pmatrix}, \begin{pmatrix} v_n^s \\ v_n^a \end{pmatrix} = \begin{pmatrix} \xi_n \sinh \xi_n b \\ \xi_n \cosh \xi_n b \end{pmatrix}, n>0. \qquad \text{... (3.12)}$$

Two approaches are now used to reduce the series relations (3.8)-(3.10) to integral equations of first and second kind.

Approach 1

Let $A_n^{s,a}$ $(n = 1, 2, ...)$ be represented by

$$A_n^{s,a} = -\frac{2}{\mu_n h} \int_0^d f^{s,a}(t) \frac{n\pi t}{h} dt, n \geq 1, \qquad \text{... (3.13)}$$

then using the generalized identity

$$\delta(y) = \frac{1}{2h} + \frac{1}{h} \sum_{n=1}^{\infty} \cos \frac{n \pi y}{h}, 0 < y < h, \qquad \text{... (3.14)}$$

it is found that the relations (3.9) and (3.10) reduce to integral equations of the first kind in $f^{s,a}(t)$, as given by

$$\int_0^d F^{s,a}(t) M^{s,a}(y, t)\, dt = 1 \text{ for } 0 < y < d, \qquad \text{... (3.15)}$$

where
$$F^{s,a}(t) = \frac{f^{s,a}(t)}{1 + R^{s,a}}, 0 < t < d \qquad \text{... (3.16)}$$

and
$$\begin{pmatrix} M^s(y, t) \\ M^a(y, t) \end{pmatrix} = \frac{1}{\lambda d} \begin{pmatrix} -\cot \lambda b \\ \tan \lambda b \end{pmatrix}$$

$$+ 2 \sum_{r=1}^{\infty} \left[\frac{1}{\xi_r d} \begin{pmatrix} \coth \xi_r b \\ \tanh \xi_r b \end{pmatrix} \cos \frac{r \pi y}{d} \cos \frac{r \pi t}{d} + \frac{1}{\mu_r h} \cos \frac{r \pi y}{h} \cos \frac{r \pi t}{h} \right]. \qquad \text{... (3.17)}$$

Also, the relations (3.8) give rise to the relations

$$\int_0^d F^{s,a}(t)\, dt = C^{s,a} \qquad \text{... (3.18)}$$

where
$$C^{s,a} = - i \lambda h \frac{1 - R^{s,a}}{1 + R^{s,a}}. \qquad \text{... (3.19)}$$

It is important to note that, since $M^{s,a}(y, t)$ are real, the functions $F^{s,a}(t)$ and the constants $C^{s,a}$ are all real quantities. The solution of the integral equations (3.15) can be utilized to obtain $C^{s,a}$ from the relations (3.18), and these in turn produce the actual reflection and transmission coefficients $|R|$ and $|T|$, respectively, from the relations

$$|R| = \frac{|(\lambda h)^2 + C^s C^a|}{\Delta}, |T| = \frac{\lambda h |C^s - C^a|}{\Delta}$$

with
$$\Delta = \left[(C^s C^a)^2 + (\lambda h)^2 \left\{ (C^s)^2 + (C^a)^2 \right\} + (\lambda h)^4 \right]^{1/2}, \qquad \text{... (3.20)}$$

obtained by using the relations (3.5) and (3.19). The relations (3.20) also produce the physical equality $|R|^2 + |T|^2 = 1$.

The functions $F^{s,a}(t) = (0, < t < d)$ are now approximated by truncated Fourier series expansions in the form

$$F^{s,a}(t) \approx \sum_{q=0}^{N} a_q^{s,a} \cos \frac{q \pi t}{d}, 0 < t < d \qquad \text{... (3.21)}$$

where $a_q^{s,a}$ $(q = 0, 1, 2, \dots N)$ are unknown constants satisfying the linear systems

$$\sum_{q=0}^{N} a_q^{s,a} K_{mq}^{s,a} = d\,\delta_{0m}, \quad m = 0, 1, 2, \dots, N \qquad \dots (3.22)$$

where
$$\begin{pmatrix} K_{mq}^{s} \\ K_{mq}^{a} \end{pmatrix} = \frac{d}{\lambda}\begin{pmatrix} -\cot\lambda b \\ \tan\lambda b \end{pmatrix}\delta_{0q}\,\delta_{0m} + \sum_{r=1}^{\infty}\left[\frac{d}{2\xi_r}\begin{pmatrix} \coth\xi_r b \\ \tanh\xi_r b \end{pmatrix}\delta_{rm}\,\delta_{rq} + \frac{2}{\mu_r h}P_{rmq}\right] \qquad \dots (3.23)$$

with P_{rmq} being given by

$$P_{rmq} = \begin{cases} (-1)^{m+q}\left(\dfrac{rd^2}{\pi h}\right)^2 \dfrac{\sin^2\left(\dfrac{rd\pi}{h}\right)}{\left\{m^2 - \left(\dfrac{rd}{h}\right)^2\right\}\left\{q^2 - \left(\dfrac{rd}{h}\right)^2\right\}} & \text{for } \dfrac{rd}{h} \neq m \text{ or } q, \\[2ex] \dfrac{d^2}{4} & \text{for } \dfrac{rd}{h} = m = q \\ 0 & \text{otherwise.} \end{cases}$$

and
$$d_m = d\,\delta_{0m}. \qquad \dots (3.24)$$

After finding the unknowns $a_q^{s\,a}$ $(q = 0, 1, \dots N)$ by solving the linear systems (3.22), the constants $C^{s,a}$ are determined approximately from the relations

$$C^{s,a} = \sum_{q=0}^{N} a_0^{s,a}\, d_q \qquad \dots (3.25)$$

obtained by using the approximations (3.21) in the relations (3.18). The reflection and transmission coefficients $|R|$ and $|T|$ are then computed by using the relations in (3.20). This completes the method used in the first approach.

Approach II

In this approach the series relations (3.8), (3.9) and (3.10) are reduced to a set of second ind integral equations, which are then solved approximately by the truncated Fourier series expansions.

Let $A_n^{s,a}$ $(n = 1, 2, \dots)$ have another representation as given by

$$A_n^{s,a} = \frac{1}{h}\left[\int_d^h g_1^{s,a}(t)\cos\frac{n\pi t}{h}\,dt - \frac{1}{\mu_n h}\int_d^h g_2^{s,a}(t)\cos\frac{n\pi t}{h}\,dt\right], \qquad \dots (3.26)$$

where the functions $g_1^{s,a}(t)$ and $g_2^{s,a}(t)$ $(d < t < h)$ are unknown, then the series relations (3.8) become

$$-i\lambda(1-R^{s,a})-\frac{1}{2h^2}\int_d^h g_2^{s,a}(t)\,dt+\frac{1}{h}\int_d^h g_1^{s,a}(t)\,U(y,t)\,dt$$

$$+\frac{1}{2h}g_2^{s,a}(y)=0,\ d<y<h \qquad \dots (3.27)$$

where
$$U(y,t)=-\lim_{\varepsilon\to+0}\sum_{n=1}^{\infty}e^{-\varepsilon n}\mu_n\cos\frac{n\pi y}{h}\cos\frac{n\pi t}{h}. \qquad \dots (3.28)$$

Substitution of the integral representations (3.26) for $A_n^{s,a}$ in the series relations (3.9) and (3.10) produces two expressions for each of the constants $B_n^{s,a}$ $(n=0,1,\ldots)$ in terms of the unknown functions $g_1^{s,a}(t)$ and $g_2^{s,a}(t)$ by using the results of Fourier series expansions.

If the two expressions for each of $B_n^{s,a}$ $(n=1,2,\ldots)$ are equated, then one obtains for each n $(n=1,2,\ldots)$

$$\int_d^h\left\{g_1^{s,a}(t)\,L_1^{s,a}(t,n)-g_2^{s,a}(t)\,L_2^{s,a}(t,n)\right\}dt=0, \qquad \dots (3.29)$$

which, after summing over n from 1 to ∞, produce the relations

$$\sum_{n=1}^{\infty}\int_d^h\left\{g_1^{s,a}(t)\,L_1^{s,a}(t,n)+g_2^{s,a}(t)\,L_2^{s,a}(t,n)\right\}dt=0, \qquad \dots (3.30)$$

where
$$L_1^{s,a}(t,n)=\frac{1}{h}\frac{u_n^{s,a}}{\upsilon_n^{s,a}}\sum_{r=1}^{\infty}\mu_r\,\alpha_{rn}\cos\frac{r\pi t}{h}, \qquad \dots (3.31)$$

$$L_2^{s,a}(t,n)=\frac{1}{h^2}\sum_{r=1}^{\infty}\frac{\alpha_{rn}}{\mu_r}\cos\frac{r\pi t}{h} \qquad \dots (3.32)$$

with
$$\alpha_{rn}=\begin{cases}(-1)^{n+1}\dfrac{rd^2}{\pi h}\,\dfrac{\sin\left(\dfrac{rd\pi}{h}\right)}{\left(n^2-\left(\dfrac{rd}{h}\right)^2\right)} & \text{for } \dfrac{rd}{h}\neq n,\\[2ex] \dfrac{d}{2} & \text{for } \dfrac{rd}{h}=n.\end{cases} \qquad \dots (3.33)$$

The relations (3.30) are satisfied if one selects

$$g_1^{s,a}(t) = -g_2^{s,a}(t)\,\frac{\sum\limits_{n=1}^{\infty} L_2^{s,a}(t,n)}{\sum\limits_{n=1}^{\infty} L_1^{s,a}(t,n)}. \qquad \text{... (3.34)}$$

Again, equating the two expressions for $B_0^{s;a}$ for the symmetric and antisymmetric cases and using the relations (3.34) between $g_1^{s,a}(t)$ and $g_2^{s,a}(t)$, it is found that the real constants $C^{s,a}$ are related to the unknown functions $g_2^{s,a}(t)$ by the relations

$$\frac{1}{C^{s,a}} = \frac{1}{\lambda h}(-\cot\lambda b, \tan\lambda b) + \int_d^h G_2^{s,a}(t)\, W^{s,a}(t)\, dt \qquad \text{... (3.35)}$$

where
$$G_2^{s,a}(t) = -\frac{g_2^{s,a}(t)}{i\lambda h(1-R^{s,a})},\ d<t<h, \qquad \text{... (3.36)}$$

$$W^{s,a}(t) = L^{s,a}(t) + N^{s,a}(t)\,\frac{\sum\limits_{n+1}^{\infty} L_2^{s,a}(t,n)}{\sum\limits_{n=1}^{\infty} L_1^{s,a}(t,n)} \qquad \text{... (3.37)}$$

with
$$L^{s,a}(t) = \frac{1}{2\lambda h^2}(\cot\lambda b, -\tan\lambda b) + \frac{1}{\pi h d}\sum_{r=1}^{\infty}\frac{1}{r\mu_r}\sin\frac{r\pi d}{h}\cos\frac{r\pi t}{h}, \qquad \text{... (3.38)}$$

$$N^{s,a}(t) = \frac{1}{h}\left[\frac{1}{2} + \frac{h}{\pi\lambda d}(\cot\lambda b, -\tan\lambda b)\sum_{r=1}^{\infty}\frac{\mu_r}{r}\sin\frac{r\pi d}{h}\cos\frac{r\pi t}{h}\right]. \qquad \text{... (3.39)}$$

The relations (3.27), after using (3.34), give rise to integral equations of the second kind for the unknown functions $g_2^{s,a}(t)$ (actually $G_2^{s,a}(t)$), as given by

$$-\frac{G_2^{s,a}(y)}{2} + \int_d^h G_2^{s,a}(t)\left[\frac{1}{2h} - U(y,t)\,\frac{\sum\limits_{n=1}^{\infty} L_2^{s,a}(t,n)}{\sum\limits_{n=1}^{\infty} L_1^{s,a}(t,n)}\right] dt = 1,\ \ d<y<h. \qquad \text{... (3.40)}$$

The functions $G_2^{s,a}(y)$ $(d<y<h)$ are determined approximately by using the truncated Fourier series expansions, as given by

$$G_2^{s,a}(y) \approx \sum_{q=0}^{N} b_q^{s,a} \cos \frac{q\pi(h-y)}{h-d}, \; d<y<h \qquad \text{... (3.41)}$$

where $b_q^{s,a}$ $(q = 0, 1, 2, \ldots N)$ are unknown constants satisfying the linear systems

$$\sum_{q=0}^{N} b_{mq}^{s,a} = (h-d) \; \delta_{om}, \; m = 0, 1, 2, ..., N \qquad \text{... (3.42)}$$

$$D_{mq}^{s,a} = \frac{(h-d)^2}{2h} \delta_{0m} \delta_{0q} - \frac{1}{4} \frac{h-d}{h} (1 + \delta_{0m}) \delta_{mq}$$

$$+ \lim_{\varepsilon \to +0} \sum_{r=1}^{\infty} e^{-\varepsilon r} \mu_r \beta_{mr} \int_d^h \cos\left(\frac{q\pi(h-t)}{h-d}\right) \left(\frac{\sum_{n=1}^{\infty} L_2^{s,a}(t,n)}{\sum_{n=1}^{\infty} L_1^{s,a}(t,n)} \right) \cos \frac{r\pi t}{h} dt \qquad \text{... (3.43)}$$

with

$$\beta_{mr} = \begin{cases} (-1)^{m+r+1} \dfrac{r(h-d)^2}{\pi h} \dfrac{\sin\left(\dfrac{r\pi(h-d)}{h}\right)}{m^2 - \left(\dfrac{r(h-d)}{h}\right)^2} & \text{for } \dfrac{r(h-d)}{h} \neq m, \\[2ex] (-1)^r \dfrac{h-d}{2} & \text{for } \dfrac{r(h-d)}{h} = m. \end{cases} \qquad \text{... (3.44)}$$

After obtaining the constants $b_q^{s,a}$ $(q = 0, 1, ..., N)$ by solving the linear systems (3.42), the constants $C^{s,a}$ are determined approximately from the relations

$$\frac{1}{C^{s,a}} = \frac{1}{\lambda h}(-\cot \lambda b, \tan \lambda b) + \sum_{q=0}^{N} b_q^{s,a} \int_d^h W^{s,a}(t) \cos \frac{q\pi(h-t)}{h-d} dt \qquad \text{.... (3.45)}$$

obtained by using the approximations (3.41) in the relations (3.35). The reflection and transmission coefficients $|R|$ and $|T|$ are then obtained by using the relations in (3.20). This completes the method used in the second approach..

4. Numerical Results

Since $|R|^2 + |T|^2 = 1$, it is sufficient to compute the reflection coefficient $|R|$ only for various values of the different parameters. For numerical computation, one has to compute infinite series of the form $K_{mq}^{s,a}$ given by relations (3.23) if the first approach is used and $D_{mq}^{s,a}$ given by the relations (3.43) if the second approach is used. These series are computed numerically by truncation. An accuracy of four figure is achieved by taking two hundred terms in these series.

The Table 1 displays a representative set of numerical estimates of $|R|$ for the *thick* iris problem, computed by using the aforesaid two approaches, for N = 0, 1, 2, 3, 4 and some particular values of the non-dimensional parameters λh, d/h and b/h. It is observed from this table that the numerical estimates of $|R|$ computed by using both the approaches, converge fairly rapidly with N, and for $N \geq 3$, an accuracy of almost three decimal places is achieved.

TABLE 1 : Reflection coefficient $|R|$

$\lambda h = 0.1$	$d/h = 0.512$	$b/h = 0.1$
N	Approach I	Approach II
0	0.0402	0.0315
1	0.0385	0.0377
2	0.0381	0.0382
3	0.0379	0.0384
4	0.0379	0.0384

In Figure 1, $|R|$ for a thick iris, computed through approach I and approach II, is depicted against the wave number λh taking $d/h = 0.512$, $b/h = 0.1$, $N = 4$ in the $(N + 1)$-term Fourier series expansions. It is observed that the two approaches produce almost the same numerical results. However, slight flucturation occurs in the curve of $|R|$ generated by approach II. This may be due to a number of truncations made in various series appearing in the mathematical analysis of the approach II. Also the numerical calculations in the approach II are much more extensive than those in the approach I. In fact, the approach I takes less time to compute the numerical results for $|R|$

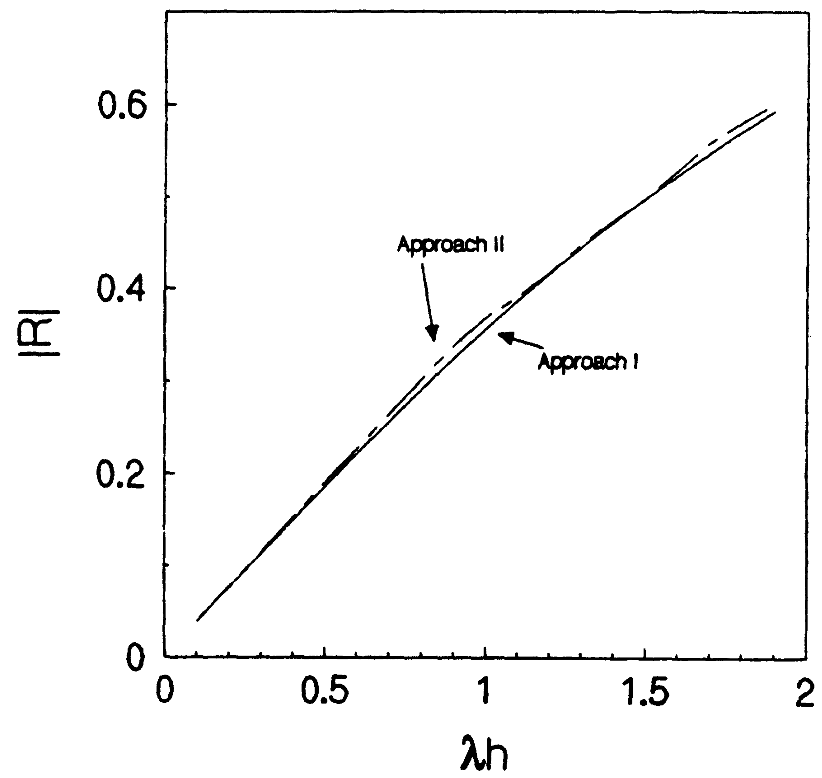

FIG. 1. Reflection coefficient vs wave number for b/h = 0.1, d/h = 0.512

compared to the approach II. However, the approach II provides a very useful check on the correctness of the numerical results. The approach I is used to calculate the reflection coefficient in the thick iris problem for various values of the different parameters to generate the Figures 2 and 3 below.

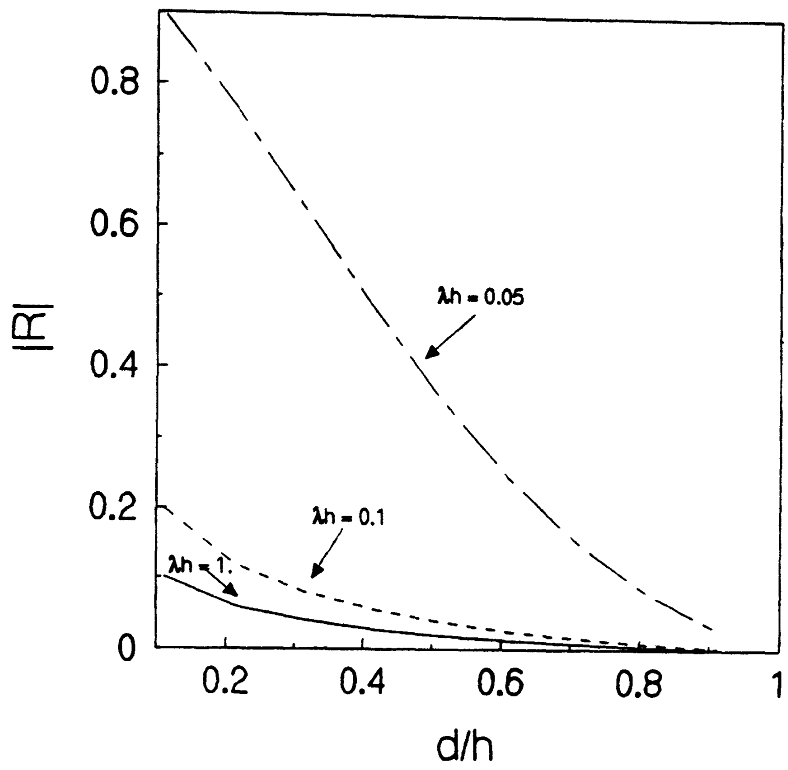

FIG. 2. Reflection coefficient vs d/h = 0.1

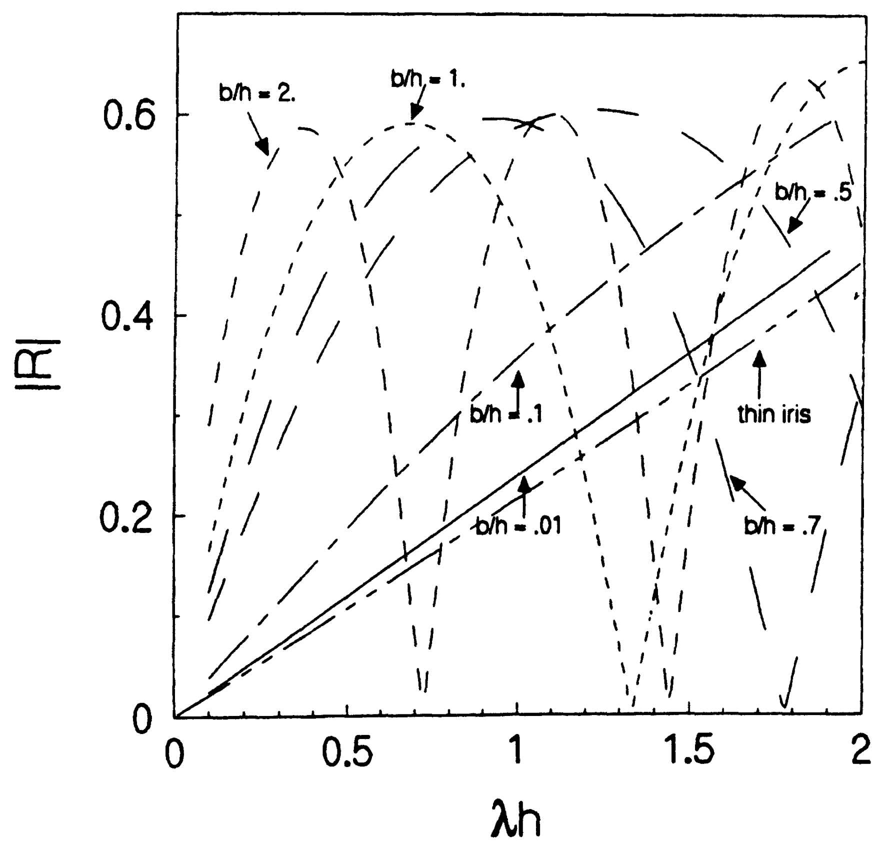

FIG. 3. Reflection coefficient vs wave number for d/h = .511

In Figure 2, $|R|$ for a thick iris is plotted against d/h for different values of λh and the fixed values of the thickness parameter $b/h = 0.1$. It is observed that for a fixed λh, $|R|$ decreases to zero as d/h increases to 1. This is physically plausible since as the height of the gap below the thick iris increases, most of the incident wavefield is transmitted through the gap. It is also observed that more than 80 percent of the energy is transmitted even through a narrow gap when the wavenumber is as low as 0.1 and the thickness of the iris is one-fifth of the height h of the waveguide.

In Figure 3, $|R|$ for a *thick* iris is plotted against the wave number λh taking $d/h = 0.512$ and various vaues of the thickness parameter b / h such as $b / h = 0.01, 0.1, 0.5, 0.7, 1.0, 2.0$. For the purpose of comparison with the thin iris results, $|R|$ for a *thin* iris (obtained by using Jones method) is also plotted against λh taking $d / h = 0.512$. For a *thin* iris, $|R|$ increases steadily with λh. This nature of $|R|$ remains almost the same for a *thick* iris so long as the thickness remains less than one-fifth of the height of the waveguide. However, as the thickness further increases, $|R|$ starts fluctuating and assumes zero values. The frequency of occurrence of zero values of $|R|$ depends on the thickness parameter b/h. The number of zeros of $|R|$ increases with the increase of b/h. This may be attributed due to the interference of incident wave field by the two edges of the thick iris. As the thickness increases, the distance between the two sides increases and this produces multiple reflections of the incident wave field by the two sides, which results in the fluctuation of the reflection coefficient against the wave number.

5. CONCLUSION

Appropriate solutions have been obtained for a rather complicated boundary value problem in the theory of electromagnetism, associated with the propagation of waves of transverse electric type in a rectangular waveguide in the prsence of a *thick* iris. Two approaches, one leading to an integral equation of the first kind and the other to an integral equation of the second kind, have been used to tackle it. The integral equations are solved numerically and numerical estimates of the reflections coefficient are obtained ultimately. The two approaches produce almost the same numerical results. However, the first approach, based on a first kind integral equation, appears to be more economic than the second approach, based on a second kind integral equation, from a computational point of view. Because of this, the curves of the reflection coefficient for the *thick* iris problem, have been generated by using the first approach. One potential advantage of the numerical technique employed here is that it appears to be simple and straightforward, and there is scope for checking the correctness of the numerical results as two different approaches have been utilized to compute these.

The thickness of the iris plays a significant role. So long as the thickness is small compared to the height of the rectangular waveguide, the reflection coefficient $|R|$ increases uniformly with the wave number as in the case of the thin iris problem. However, as the thickness increases, $|R|$ starts fluctuating and assumes zero values for a discrete set of wave numbers. Thus for a number of values of the incident wave frequency, the thick iris becomes completely transparent. This phenomenon may have some significance in the modelling of rectangular waveguides. A somewhat similar phenomenon occurs in the case of water wave scattering by a thick rectangular barrier present in water of uniform finite depth (cf. Mei and Black[6], Kanoria *et al.*[8], Mandal and Kanmoria[11]).

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