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Capacitive-resistive transients in terms of field quantities

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Abstract

Capacitive–resistive transients in extended media are discussed in terms of electric field quantities. Obviously, in these problems, the contribution of the magnetic field to the electric field is deemed negligible. For a simple illustrative example, the field solution is compared with the circuit-theoretical result for the voltage and current. An algorithm for solving such transients in space and time domain with the help of a Laplace solver is presented. Any other Laplace solver can also be used for this purpose. Its applicability is demonstrated with three examples, one of which is chosen to have a circuit-theoretical solution.

Keywords: Capacitive–resistive transients, boundary condition, electromagnetic transients.

1. Introduction

Capacitive–resistive transients occur very frequently in electrical engineering. Such transients prevail in all cases where contribution to the electric field from the time-varying magnetic field is negligible. Some examples are outdoor insulations such as polluted bushings, noninductive high-voltage resistors used in impulse generators and practically all insulation in HVDC systems. For design, analysis and performance studies, it is essential to know the temporal and spatial distribution of the field quantities. Solution for most of such problems has been attempted in the past using ladder network with lumped circuit elements. However, in a class of problems involving extended media or nonuniformities, lumped circuit approach will be of inadequate utility; in fact, it would almost be impossible to arrive at meaningful equivalent circuits. The field-theoretical approach discussed in the present work would be appropriate in such cases.

A number of researchers[2–7] have studied capacitive–resistive fields. Some of them[2–4] have considered only the steady-state distribution for the sinusoidal excitation, whereas others[5–7] have considered the transient response also. Singer[5] used Fourier analysis to split the impulse voltage waveform into component sinusoidal voltages. Then, by solving the Laplace equation for a selected number of these sinusoidal voltages, the response for an impulse voltage was computed. The overall computation and storage space demanded by this indirect method is larger than the direct time domain solution. Takuma and Kawamoto[6] and Chakravorti and Mukherjee[7] have computed the capacitive–resistive transient field in the time domain. However, they have considered transient phenomena in an axisymmetric problem with conductivity present only at the interface. For a better understanding of this transient process from the field point of view and for possible computations of such transients in the time domain, it is necessary to use the general boundary condition given by Woodson and Melcher[8]. It must be noted that the contribution of the magnetic field to the electric field must be negligible for a problem to be treated as a capacitive–resistive transient.

2. Transients in terms of electric field parameters

In any material conductivity κ and permittivity ε, both conduction current κE and displacement current $\partial \boldsymbol{D}/\partial t$ exist simultaneously. Assuming that there is no volume charge distribution, for the interior of any domain (*i.e.*, excluding behaviour at the interfaces)

$$\nabla^2 \phi = 0. \tag{1}$$

If only one material is present, there will be no transient in the potential distribution. This situation with a single material is analogous to RC parallel network connected to an ideal voltage source. The situation changes if there exist at least two materials between the electrodes with different properties. Within each material, $\nabla^2 \phi = 0$ still holds, but at the interface, due to discontinuity in the field[1], it is not applicable. At the material interface, the field (and hence the potential) has to satisfy, at every instant, the conditions $E_{1t} = E_{2t}$ and $D_{1n} - D_{2n} = \sigma_s$ and the condition given in Woodson and Melcher[8], namely,

$$J_{1n} - J_{2n} + \nabla_s \cdot \boldsymbol{J}_s = -\frac{\partial \sigma_s}{\partial t}, \tag{2}$$

where J_{1n} and J_{2n} are the components of the volume conduction current densities normal to the interface of the two media, $\boldsymbol{J}_s$ is the surface current density, σ_s, the surface charge density, and $\nabla_s \cdot$ the divergence operator defined over the surface representing the interface. For isotropic materials, (2) can also be written as

$$\kappa_1 E_{1n} - \kappa_2 E_{2n} + \kappa_s \nabla_s \cdot E_s = -\frac{\partial \sigma_s}{\partial t}, \tag{3}$$

where κ_1 and κ_2 are the conductivities, and ε_1 and ε_2, the permittivities of the interfacing media. κ_s is the conductivity of the interface.

The boundary condition (2) governs the time dependence of the field quantities. It shows that, at the material interface, any difference in the normal component of the conduction current and any divergence in surface current densities will be compensated by the displacement current densities. Therefore, whenever diverging surface currents are present and whenever $\kappa_1/\varepsilon_1 \neq \kappa_2/\varepsilon_2$, there will be a continuous accumulation or depletion of charges at the interface. Assuming κ_1, κ_2 and κ_s are all finite, integrating (3) we get

$$\int_0^t (\kappa_1 E_{1n} - \kappa_2 E_{2n} + \kappa_s \nabla_s \cdot E_s)\, \mathrm{d}t = -\sigma_s|_0^t. \tag{4}$$

Since the electric field is also finite at the interface for a finite excitation (excluding the regions with geometric and physical singularities), the above equation implies that the process of accumulation or depletion of interface charges cannot be instantaneous, but has to be transient. Hence, at $t = 0^+$ the surface charge density at the interface remains at its value at $t = 0^-$. If σ_{s0^-} is the initial charge density, then, at $t = 0^+$, $D_{1n} - D_{2n} = \sigma_{s0^-}$ holds. Therefore, the initial field distribution is governed only by the permittivities of the materials (*i.e.*, field distribution is capacitive) and can be used as the initial condition for the transient field calculation. As the time progresses, the accumulation or depletion of the interfacial charges make the differences in conduction current densities lesser than that at the beginning, and hence field distribution is also dominated by the conductivities of the materials. In case where excitation settles down to a constant value over a period, it can be seen that the final distribution is completely resistive. Therefore, the boundary condition given in eqn (3), which links the conduction current with the displacement current at the material interface, is the key for explaining the capacitive–resistive transients in terms of field quantities.

Consider a simple one-dimensional illustrative case. Two infinite parallel conducting plates exist at $y = a$ and $y = b$. The intervening space is filled with two homogenous and isotropic materials as shown in Fig. 1(a). From $y = y_i$ to $y = b$ material 1 with $\kappa_1 = 1$ and $\varepsilon_1 = 1$ is present. The remaining volume has material 2 with $\kappa_2 = 0$ and $\varepsilon_2 = 2$. At $t = 0^+$, a unit step voltage is applied. Therefore, the excitation boundary conditions for t greater than 0 can be written as

$$V(a) = 0 \quad \text{and} \quad V(b) = 1.$$

In both the media, $\nabla^2\phi = 0$ holds. For this one-dimensional case, it reduces to

$$\frac{d^2\phi}{dy^2} = 0. \tag{5}$$

Therefore, the potentials are given by

$$\begin{aligned} \phi_1(y) &= h_1 + h_2 y, \\ \phi_2(y) &= h_3 + h_4 y, \end{aligned} \tag{6}$$

where h_1, h_2, h_3 and h_4 are time-dependent constants to be determined from the linearly independent initial and boundary conditions. Using the excitation boundary conditions, $h_3 = -h_4 a$ and $h_1 = 1 - h_2 b$. At the interface, $\phi_1(y_i) = \phi_2(y_i)$, which gives

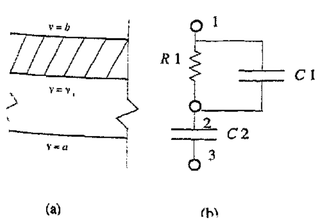

FIG. 1. a. Section of the illustrative problem, b. its circuit equivalent.

$$h_4 = -h_2 \frac{\tilde{b}}{\tilde{a}} + \frac{1}{\tilde{a}}, \tag{7}$$

ıere $\tilde{b} = (b - y_i)$ and $\tilde{a} = (y_i - a)$. By imposing the continuity of potential at the inter-ce, the condition $E_{1t} = E_{2t}$ is automatically satisfied. Consider $t = 0^+$ at the interface at ıich instant, as explained earlier, $D_{1n} = D_{2n}$ holds. Using this with eqn (6),

$$\phi_1(y)|_{t=0^+} = \left(1 - 2\left(\frac{b-y}{2\tilde{b}+\tilde{a}}\right)\right),$$

$$\phi_2(y)|_{t=0^+} = \left(\frac{y-a}{2\tilde{b}+\tilde{a}}\right).$$

he interface voltage at this instant, which is an initial condition for the main problem, is

$$\phi(y_i)|_{t=0^+} = \left(\frac{\tilde{a}}{2\tilde{b}+\tilde{a}}\right). \tag{8}$$

or the present problem, eqn (3) reduces to

$$\kappa_1 E_{1n} + \frac{\partial}{\partial t}(\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n}) = 0.$$

'his, with eqn (6), gives

$$\frac{\partial h_2}{\partial t} + \left(\frac{\tilde{a}}{\tilde{a}+2\tilde{b}}\right) h_2 = 0$$

ır

$$h_2 = h_0 \exp\left(\frac{-\tilde{a}}{\tilde{a}+2\tilde{b}} t\right).$$

Therefore, in the process of elimination of the remaining constant h_3, we have ended up with another constant h_0 which is independent of time. Using the initial condition (7),

$$\phi_1(y) = 1 - 2\left(\frac{b-y}{\tilde{a}+2\tilde{b}}\right)\exp\left(\frac{-\tilde{a}}{\tilde{a}+2\tilde{b}} t\right), \tag{9}$$

$$\phi_2(y) = \left(\frac{y-a}{\tilde{a}}\right)\left(1 - \left(\frac{2\tilde{b}}{\tilde{a}+2\tilde{b}}\right)\exp\left(\frac{-\tilde{a}}{a+2\tilde{b}} t\right)\right). \tag{10}$$

From the circuit approach, medium 1 may be considered as a parallel combination of a resistor and a capacitor with resistance per unit area $R1 = \tilde{b}$ and capacitance per unit area $C1 = 1/\tilde{b}$, and medium 2 as a capacitor with capacitance per unit area $C2 = 2/\tilde{a}$. This is schematically shown in Fig. 1b, where node 2 represents the interface of the two media. The step response of this circuit is well known:

$$i(t) = \frac{1}{R}\left(\frac{C2}{C1+C2}\right)^2 \exp\left(\frac{-t}{R1(C1+C2)}\right). \tag{11}$$

The voltage at node 2 is

$$v(t) = 1 - \left(\frac{C2}{C1 + C2}\right) \exp\left(\frac{-t}{R1(C1 + C2)}\right). \tag{12}$$

With the above values of R and C, eqn (12) takes the same form as eqns (9) and (10) when evaluated at the interface. The current per unit area can be computed from both (9) and (10) as follows. The current per unit area in media 1 is

$$J_1 = -\kappa_1 \frac{\mathrm{d}\phi_1}{\mathrm{d}y}\bigg|_{y=b} + \frac{\partial}{\partial t}\left(-\varepsilon_1 \frac{\mathrm{d}\phi_1}{\mathrm{d}y}\bigg|_{y=b}\right)$$

and that in media 2 is

$$J_2 = \frac{\partial}{\partial t}\left(-\varepsilon_2 \frac{\mathrm{d}\phi_2}{\mathrm{d}y}\bigg|_{y=a}\right).$$

After evaluation,

$$J_1 = J_2 = -\frac{1}{\tilde{b}}\left(\frac{2\tilde{b}}{2\tilde{b} + \tilde{a}}\right)^2 \exp\left(\frac{-\tilde{a}}{\tilde{a} + 2\tilde{b}}\, t\right).$$

After substituting for R and C, this equation is the same as eqn (11). In the above illustrative example, the capacitive–resistive transient phenomena has been expressed in terms of field quantities. Now this problem will be discretized in time. The time dependence in this capacitive–resistive problem appears only through the interface condition (3). Using the forward-difference approximation for the time derivative in (4),

$$\sigma_s[t + \Delta t] = \sigma_s[t] - \Delta t\, [\kappa_1 E_{1n}(t) - \kappa_2 E_{2n}(t)]. \tag{13}$$

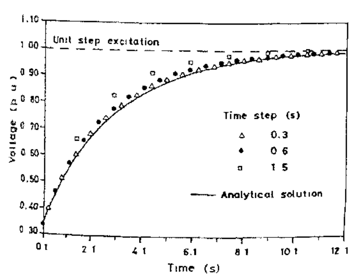

FIG. 2. Voltage at the interface.

herefore, the surface charge at any step will be approximately equal to the charge accu-ıulated up to the previous step plus that due to difference in conduction current between ıe adjacent media during the previous time step. Using this, an algorithm has been evised later in the paper. Figure 2 shows the exact solution and that obtained by the al-orithm for different time steps.

Now consider the transient due to surface current density alone. The situation is nalogous to *RC* ladder network, with both *R* and *C* being functions of space, *i.e.*, (x, y, z). Of course, there will be no transients if both capacitive and resistive potential istributions are identical. Otherwise, the initial capacitive distribution will force diverg-ıg surface currents. Then there will be accumulation or depletion of the surface charges s described by eqn (4). For the numerical computation, the reduced interface condition vill be discretized in time as follows:

$$\sigma_s[t + \Delta t] = \sigma_s[t] - \Delta t(\nabla_t \cdot J_s). \tag{14}$$

f a cartesian system is chosen for the tangent plane at the point under consideration, then he above equation can be written with the usual notation as

$$\sigma_s[t + \Delta t] = \sigma_s[t] - \Delta t\left(\frac{\partial J_{sx}}{\partial x} + \frac{\partial J_{sy}}{\partial y}\right). \tag{15}$$

The solution of an axisymmetric problem having only the surface conductivity embedded n a nonconducting medium has been considered by other researchers[6,7]. For an axisym-metric case the above equation can be written as

$$\sigma_s[t + \Delta t] = \sigma_s[t] - \Delta t\left(\frac{\partial J_t}{\partial l}\right)$$

where J_t is the current density along the conducting interface (and hence tangential to it) and l, the coordinate along the intersection of the tangent plane and the symmetry plane. Let ρ_s be the surface resistivity. Let $i-1$, i, $i+1$ denote the contour points if CSM is used for the solution, nodes if FEM is used and grid points if FDM is used, as ordered from the top. The tangent plane at i is close to both $i-1$ and $i+1$. The circumference at i is $l_c(i)$, then the second term on the right-hand side of the above is approximately equal to

$$\frac{\Delta t}{\Delta l}\left(\frac{E_i}{\rho_s(i)} - \frac{E_{i+1}}{\rho_s(i+1)}\right),$$

where E_i denotes the field in the interval $(i, i-1)$ and E_{i+1} denotes that in the interval $(i+1, i)$. The potentials at $i-1$, i, $i+1$ are denoted by v_{i-1}, v_i, v_{i+1}, respectively. Then the above term is approximately equal to

$$\left(\frac{\Delta t}{\Delta l \cdot l_c(i)}\right)\left(\frac{v_{i-1} - v_i}{(\rho_s(i)\Delta l_i)/(l_c(i))} - \frac{v_i - v_{i+1}}{(\rho_s(i+1)\Delta l_{i+1}))/(l_c(i))}\right).$$

Therefore,

$$\sigma_s[t+\Delta t] = \sigma_s[t] - \frac{\Delta t}{S(i)}\left(\frac{v_{i-1}-v_i}{R_i} - \frac{v_i - v_{i+1}}{R_{i+1}}\right), \tag{16}$$

where R_l denotes the resistance of the section l and $S(i)$, the surface area attributed to the section i. This is identical to what has been given by Takuma and Kawamoto[6] and used by Chakravorti and Mukherjee[7]. This equation will be used in solving one of the cases below.

3. *Method for solving capacitive–resistive transient problems using* any Laplace solver

The capacitive–resistive transients in any practical system can be solved with the help of any Laplace solver. It may be an analytical or a numerical method like the charge simulation method, the finite difference method, the finite element method, etc. To limit the error growth and to avoid numerical oscillation, electric field at the interface must be computed to a reasonable accuracy. Secondly, to make the algorithm general, one modification is required. For all the media present in the problem both conductivity and permittivity must be specified, whatever may be their relative magnitudes. For good accuracy and temporal resolution it is necessary to have small time steps. This will be at the cost of larger computational time and storage. An approximate idea of time step may be obtained by any of the following considerations. A time step of 1/10 to 1/5, the smallest time constant estimated on the problem, can be taken, or, for a step excitation, a study has to be carried out with one small and one large time step. The signs of the interface charge densities computed are compared. Sign reversal takes place only if the second time step chosen is large. By trial and error, the second time step can be reduced to obtain no sign change, thereby getting an idea of the time step to be taken. After guessing the initial time step required, whenever possible, it may be advantageous to increase the time step size as the solution progresses in time. On the contrary, if the time step chosen were to be large then there may be oscillation picking up slowly as the computation progresses in time. By reducing the time step for further computation, it is possible in some cases to eliminate the oscillations. For some problems, the applied excitation over some time interval may demand a time step less than what has been estimated.

3.1. *Algorithm*

The specified excitation, which may be time-varying, and the other specified boundary conditions are to be followed for all the steps below.

1. At zero time step, t_0, the problem is solved for the capacitive field, *i.e.*, by inputting the permittivities and the initial surface charge distribution (if any) to the Laplace solver. The computed interface voltages are used as the boundary conditions at the interfaces for the resistive problem in the time step $t_{1/2}$. Then the resistive problem is solved by inputting the conductivities as the material property to the Laplace solver. Then, by using

$$\sigma_s[t+\Delta t] = \sigma_s[t] - \Delta t[\kappa_1 E_{1n}(t) - \kappa_2 E_{2n}(t) + \nabla_t \cdot J_s], \tag{17}$$

the charge accumulated till t_1 is computed.

2. The charge densities computed for the time steps t_{n+1}, from the time steps $t_{n+1/2}$, are inputted to the capacitive problem at time steps t_{n+1} ($n = 0, 1, 2, \ldots$) and the interface voltages are computed.

3. Using the interface voltages computed at time step t_{n+1}, the resistive problem is solved at time steps $t_{n+3/2}$ and the interface charge densities are computed using (17).

4. Steps 2 and 3 are repeated to cover the desired time duration.

3.2. *Performance of the algorithm*

The difference equation approximations used for eqns (13) and (14) are consistent[8]. If the Laplace solver is also consistent, then the approximation to the problem is consistent. Therefore, we can expect the solution obtained to be an approximation of the actual solution. It is important to know the growth of errors as the time progresses. Whenever the computed surface charge densities converge, they converge to the final steady-state value of the solution, because they have to satisfy (1). This does not rule out the possibility of superimposed oscillations in the computed surface charge densities and potentials when the time step chosen is large. This has been illustrated in one of the case studies given later. Because of the complexity involved, it needs a separate investigation for determining the time step which results in this oscillation and which does not. This study would include either bulk and surface conduction individually or taken together in the general form.

4. Case studies

Using the above algorithm, three illustrative cases are studied. In the first case, only the volume conductivity is present, in the second, only the surface conductivity is present and in the third both volume and surface conductivities are present.

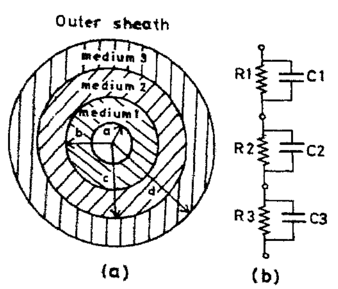

FIG. 3. Section of the cable and its circuit-equivalent. $a = 1.0$ cm, $b = 1.2$ cm, $c = 1.7$ cm, $d = 3.8$ cm. Medium 1: $\kappa_1 = 4.60 \times 10^{-5}\ \Omega^{-1}\ m^{-1}$, $\varepsilon_{r1} = 2.6$; Medium 2: $\kappa_2 = 2.12 \times 10^{-5}\ \Omega^{-1}\ m^{-1}$, $\varepsilon_{r2} = 4.9$; Medium 3: $\kappa_3 = 1.00 \times 10^{-3}\ \Omega^{-1}\ m^{-1}$, $\varepsilon_{r3} = 1.0$.

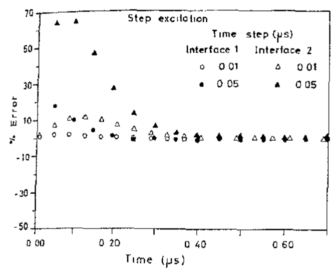

FIG. 4. Error in the computed interface voltages.

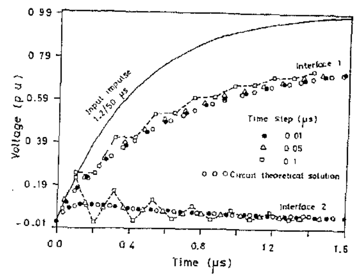

FIG. 5. Variation of the voltage at the interfaces–impulse excitation.

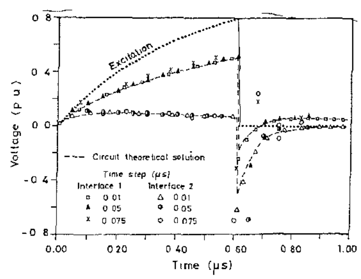

FIG. 6. Variation of the voltage at the interfaces–chopped impulse.

4.1. *Case 1*

In this study, a cable with three concentric real dielectrics (with finite conductivities) is considered. The transient appears essentially because of the finite bulk conductivity. Figure 3 shows the section of the cable with its circuit-equivalent. The results obtained by both the algorithm and the circuit solution are compared. The percentage error for a step excitation for various time steps is presented in Fig. 4. Figures 5 and 6 give the interface voltages for an impulse excitation. The numerical oscillations are evident in the case of a larger time step.

4.2. *Case 2*

Here a high-voltage noninductive tail resistor of an impulse generator is analysed for a standard lightning impulse. A time-to-front of 0.9083 μs was taken, which is within the tolerance limit specified in the standards (1.2 μs ± 30%). Figure 7 shows the dimensions of the resistor studied. It consists of a wire-wound resistance wound on a 4 cm diameter ceramic tube placed in an insulating cylinder containing an insulating oil. The laboratory is approximated to a cylindrical structure of radius 12 m and height 14 m. With this, the problem can be treated as axisymmetric. For the excitation considered, the conduction current in the dielectric is not significant compared to the capacitive currents. Therefore, they are neglected to reduce the computation time. For the solution of the Laplace equation, the thermal module of ANSYS 4.4 was used. The equipotentials for capacitive and resistive (*i.e.*, governed by the resistive surface) fields are shown in Figs 8 and 9. The voltage distribution along the resistive surface, *i.e.*, along the tail resistor at various time steps is shown in Fig. 10.

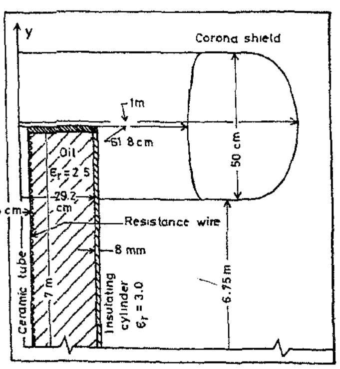

FIG. 7. Section of the tail resistor.

4.3. *Case 3*

In this case, a high-voltage cable (without a metallic sheath) in a tank containing a commercial insulating liquid is studied. The conductor of the cable is of 19 strands, over which there is a solid insulation (Fig. 11). The bulk conductivities of the solid insulation and the insulating liquid are taken arbitrarily as $4.545 \times 10^{-15}\ \Omega^{-1}\ m^{-1}$ and $1.0 \times 10^{-14}\ \Omega^{-1}\ m^{-1}$, respectively. The respective relative permittivities are taken as 4.2 and 2.2. The outer surface of the solid insulation is assumed to have a surface conductivity of $3.571 \times 10^{-12}\ \Omega^{-1}\ m^{-1}$. A step voltage was given to the cable with respect to the tank. For this case also, ANSYS 4.4 was used for the solution of the Laplace equation. The potential distributions at selected time instants are shown in Figs 12–15.

The assumption that there is no volume charge distribution is made to avoid complications associated with the movement of these charges under the influence of the field. In case these charges are fixed or if their behaviour is very close to carriers in a conducting medium, then the above algorithm can easily be extended to include them. The only change required would be to use a Poisson solver instead of a Laplace solver. In addition, $\partial\rho/\partial t$ has to be calculated from the capacitive distribution. Also, in this work, only linear

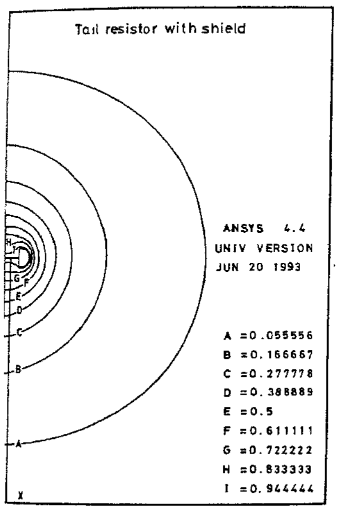

FIG. 8 Capacitive potential distribution (normalized).

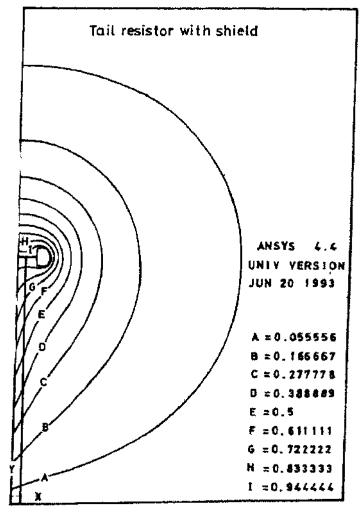

FIG. 9. Resistive potential distribution (normalized)

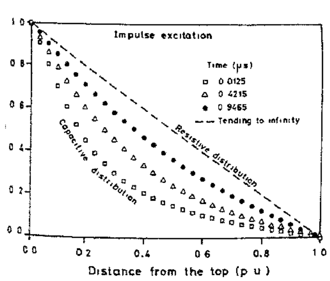

FIG. 10. Voltage distribution in the tail resistor.

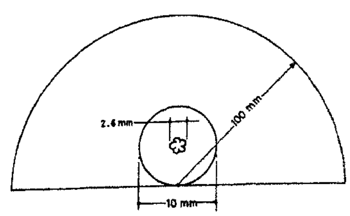

FIG. 11. Schematic diagram of the cable inside tank

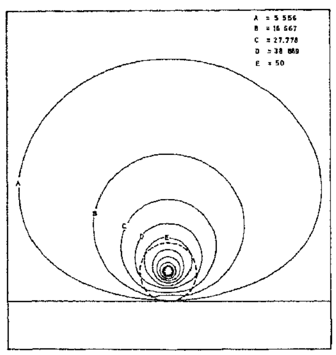

FIG 12 Initial potential distribution

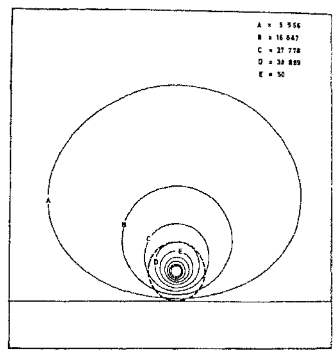

FIG 13 Potential distribution at 75 s

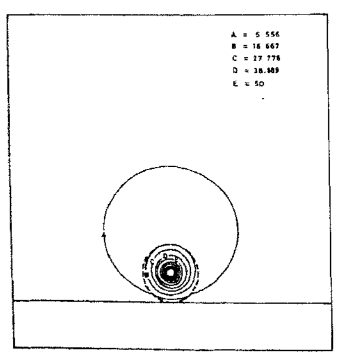

FIG. 14. Potential distribution at 225 s.

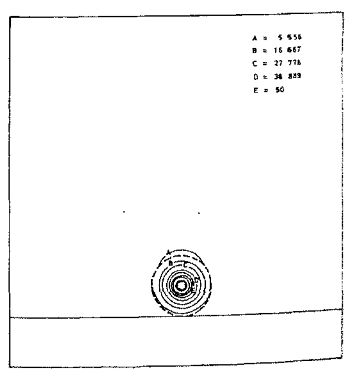

FIG 15. Potential distribution at 457 5 s.

material properties are considered. If the material properties are nonlinear then the only complication that seems to arise is the requirement of a Laplace solver capable of handling it.

5. Conclusions

1. It is shown that the general boundary condition on the current density vector expresses capacitive–resistive transients in terms of field quantities.

2. A general algorithm has been given which solves these transients in terms of field quantities, with the help of any Laplace solver.

3. Three different cases have been illustrated using this algorithm. The first example has only bulk conduction and the solution obtained by the algorithm compares well with the circuit solution for the voltage and current at the interfaces. In the second example, a high-voltage resistor was considered. Voltage distribution along the resistor for an impulse excitation was computed. The third example has both surface and bulk conductions playing a role in determining the field distribution. Potential distributions at four selected instants are given.

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