# STUDY OF TORSIONAL INTERACTIONS IN MULTI-TERMINAL DC SYSTEMS THROUGH SMALL SIGNAL STABILITY ANALYSIS

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## INTRODUCTION

Turbo-generators can be subjected to negatively damped subsynchronous frequency oscillations caused by the interactions between the generator and the external network. This phenomena is termed as subsynchronous resonance(SSR) and it is well known that series compensated ac lines are the major contributors. In recent years, it has been established that HVDC systems with converter controllers can also cause unfavourable torsional interactions. The first experience of such an interaction occured at Square Butte(1).

There is a need to analyse this phenomenon for the planning and design of HVDC systems. The analysis can be based on the calculation of electrical damping torques (in the frequency domain) (1,2) or eigenvalues based on the linearized state space models (3). While it is possible to study the torsional interactions using simulation (4,5), the analytical approach using small signal stability analysis can give an insight into the nature of the problem and its solutions. This paper reports the development of the mathematical model based on small signal stability analysis for the study of torsional interactions in HVDC systems. The formulation is general enough to include multi-terminal DC (MTDC) systems. The case study of two terminal and three terminal systems is presented to illustrate the effect of converter controller gains on the damping of subsynchronous frequency oscillations.

## SYSTEM MODEL

The system consists of generators, AC network and MTDC system (including converters, DC network and controllers). The methodology used is to develop linearized state space models of individual components and integrate them using the interconnection pattern. For example, a generator is viewed as a current source injected at its terminal bus in the AC network. Similarly the MTDC system is also viewed as a set of current sources injected at the converter buses. It is to be noted that the transients in AC, DC networks and generator stator are considered in the model.

### Generator Model

Using Park's equations, the detailed mathematical model of the synchronous generator can be derived and coupled with the excitation system model. The turbine-governor can also be included if relevent. The linearized equations are

$$\dot{x}_G = [A_G]\, x_G + [B_G]\, u_G \qquad (1)$$

$$y_G = [C_G]\, x_G \qquad (2)$$

where $u_G^t = [\Delta V_{GD}\ \Delta V_{GQ}]$, $y_G^t = [\Delta I_{GD}\ \Delta I_{GQ}]$ $\Delta V_{GD}$, $\Delta V_{GQ}$ are the D-axis and Q-axis components of the terminal voltaae (with respect to a common synchronously-rotating reference frame). $\Delta I_{GD}$ and $\Delta I_{GQ}$ are the D-and Q-axes components of the armature current which are related to the state variables from equation (2). It is to be noted that if more than one generator are to be considered, the matrices $A_G, B_G$ and $C_G$ are block diagonal. These matrices are functions of the operating point.

### AC Network Model

It is assumed that the AC network is linear, symmetric and the voltages and currents are balanced. Under such conditions, it is possible to obtain the network equations using the D- and Q- axes components of the state variables (chosen as inductor currents and capacitor voltages). The linearized equations are independent of the operating point and can be expressed as

$$\dot{\underline{x}}_{ND} = [A]\underline{x}_{ND} - w_B \underline{x}_{NQ} + [B]\underline{u}_{ND} \qquad (3)$$

$$\dot{\underline{x}}_{NQ} = [A]\underline{x}_{NQ} + w_B \underline{x}_{ND} + [B]\underline{u}_{NQ} \qquad (4)$$

where $\underline{x}_{ND}$ and $\underline{u}_{ND}$ are D-axis components of the state variables and input variables (current and voltage sources); $\underline{x}_{NQ}$ and $\underline{u}_{NQ}$ are the Q- axis components of the respective variables. $w_B$ is the base (operating) frequency. The equations (3) and (4) can be combined as

$$\dot{\underline{x}}_N = [A_N]\underline{x}_N + [B_N]\underline{u}_N \qquad (5)$$

where $\underline{x}_N^t = [\underline{x}_{ND}^t\ \underline{x}_{NQ}^t]$, $\underline{u}_N^t = [u_{ND}^t\ u_{NQ}^t]$

The generator terminal voltage components can be expressed as

$$y_{NG} \stackrel{\Delta}{=} [\Delta V_{GD}\ \Delta V_{GQ}]^t = [C_{NG}]\underline{x}_N + [D_{NG}]\dot{\underline{u}}_N \qquad (6)$$

If there is a shunt capacitor connected at the generator terminal, $[D_{NG}]$ is identically equal to zero. The converter bus AC voltage components can be expressed as

$$\underline{y}_{ND} \stackrel{\Delta}{=} [\Delta V_{CD}\ \Delta V_{CQ}]^t = [C_{ND}]\underline{x}_N \qquad (7)$$

It is assumed that there are shunt capacitors connected at the converter buses. It is adequate to represent the AC filters as equivalent shunt capacitors for the study of torsional interactions.

### DC System Model

The DC system consists of converters, smoothing reactors, DC filters and the DC line. The DC line can be modelled by π equivalents (one section is adequate). A 12 pulse converter is modelled as a controllable voltage source in series with an equivalent impedance as shown in Figure 1. u is the overlap angle in degrees. The angle $\theta$ is $\alpha$ for the rectifier and $\beta$ for the inverter. The voltage setting terminal is assumed to be on (DC) voltage control and the other terminals on current control. The current or voltage controller is represented by the block diagram shown in Figure 2. The switch S is assumed to be closed for pure EPC (Equidistant Pulse Control) scheme and open for IPC (Individual Phase control). The positive sign is to be taken for the rectifier and the negative sign for the inverter. Also, K is positive for current control at the rectifier and negative for the current control at the inverter.

The linearized DC system state equations can be expressed as

$$\dot{x}_D = [A_D]x_D+[B_D]u_D \quad (8)$$

$$y_D = [C_D]x_D+[D_D]u_D \quad (9)$$

where $u_D^t = [\Delta V_{CD}^t \; \Delta V_{CQ}^t]$, $y_D^t = [\Delta I_{CD}^t \; \Delta I_{CQ}^t]$

The output of the DC system are the D- and Q-axes components of the currents injected at the converter buses.

## Interconnection of Component Models

The three component models described earlier are interconnected by the following interconnection equations

$$u_G = y_{NG} \quad (10)$$

$$u_N^t = [u_{NG}^t \; u_{ND}^t] = [y_G^t \; y_D^t] \quad (11)$$

$$u_D = y_{ND} \quad (12)$$

It is to be noted that $u_N$ is partitioned into two components. One of them is equal to $y_G$ and the other equal to y Substituting for the RHS of the above equations from equations (2),(6),(7) and (9) we can solve for u as

$$u = [F]x \quad (13)$$

where $u^t = [u_G^t \; u_N^t \; u_D^t]$, $x^t = [x_G^t \; x_N^t \; x_D^t]$

Using equation (13) it is possible to get the overall system model described by

$$\dot{x} = [A_T]x_T \quad (14)$$

where

$$[A_T] = \text{diag}[A_G \; A_N \; A_D] + \text{diag}[B_G \; B_N \; B_D][F] \quad (15)$$

The small signal stability analysis of the system involves the computation of the eigenvalues of the matrix $[A_T]$. The locations of eigenvalues on the right half plane (in the 's' plane) indicates instability.

## CASE STUDY

The examples of a two and a three terminal HVDC systems are considered here for the study. The single line diagram of the three terminal system is shown in Figure 3. The inverter 1 is the small inverter of rating $1/4^{th}$ of the rating of the rectifier and is situated at the midpoint of the DC line. The inverter 1 is on current control and inverter 2 is on voltage control. The system data is adapted from (5). For the two terminal case study, inverter 1 is not considered and generator is considered to be connected on the rectifier side (in place of the ac system shown in Figure 3).

### Two Terminal System

Figure 4 shows the eigenvalue loci of the four torsional modes as the current controller gain is increased from 10 to 380 (rad/unit). The other torsional mode (frequency of 298 rad/sec) was not affected and is not shown here. Figure 4 shows that three modes can be destabilized due to the current controller and the first mode corresponding to frequency of 98.4 rad/sec is the most affected. It is also interesting to observe that as the gain is increased beyond a certain limit the negative damping decreases. Figure 5 shows the eigenvalue loci with the individual phase control. The trends are similar as shown in Figure 4,but the severity of the interactions are reduced. In this example, the impedance on the inverter side was neglected.

### Three Terminal System

The operating condition selected was a full load condition. The generator output was comparable to the power output of the inverter 1. The eigenvalue loci were obtained for variations of current controllers gains at rectifier and inverter 1 and voltage controller gain at inverter 2. It was observed that the torsional modes are most sensitive to the variation in the current controller gain at inverter 1. Even here, it was found that only the first torsional mode and the zeroth mode (of frequency 9.5 rad/sec) were most affected. The variation in the real part of the eigenvalues for these modes are shown in Figure 6. It is interesting to observe in this case that the IPC controller results in less damping compared to the EPC controller. While the zeroth mode remains always unstable with IPC control, it is stable with EPC control for gains less than 110. It was assumed in this study that the generator is provided with a high gain AVR with static exciter and PSS (Power system Stabilizer) was not considered. Generally, it should be possible to damp the zeroth mode with well designed PSS.

## CONCLUSIONS

In this paper a general formulation for the small signal stability analysis, is presented which is suitable for the study of torsional interactions. The system is modelled in detail and eigenvalue analysis is used for the study. From the results of the case studies, the following conclusions can be drawn.

(1) The current controller at the terminal where the turbo-generator is connected is the major contributor for the torsional interactions.
(2) The interaction is severe if the generator is connected to the rectifier terminal as opposed to the inverter terminal.
(3) For the generator connected at rectifier terminal, the interaction is more severe with EPC control than with IPC. However, the reverse is true when the generator is

connected at the inverter terminal.

(4) Eigenvalue loci can be used to identify stable ranges of the current controller gains from the point of view of torsional interactions: It needs to be checked whether these gains are satisfactory from other considerations. If not, it may be necessary to provide supplementary damping controller(4).


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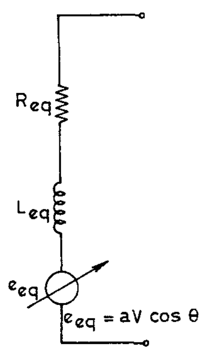


Figure 1. Equivalent circuit of the converter

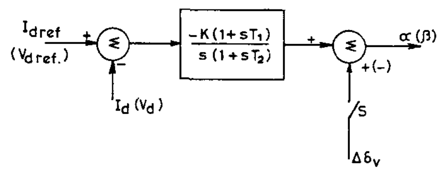


Figure 2. Block diagram of the converter controller

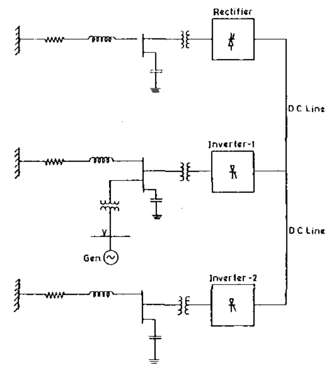


Figure 3. Single line diagram of a three terminal system

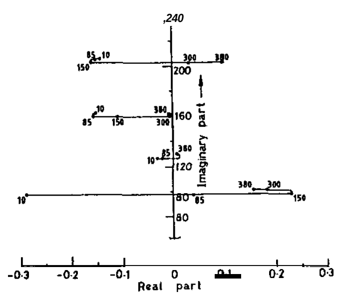


Figure 4. Eigenvalue loci of torsional modes (EPC) (Two terminal system)

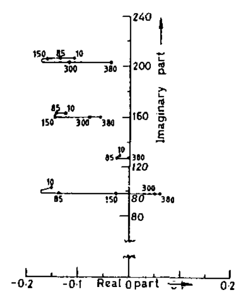


Figure 5. Eigenvalue loci of torsional modes (IPC) (Two terminal system)

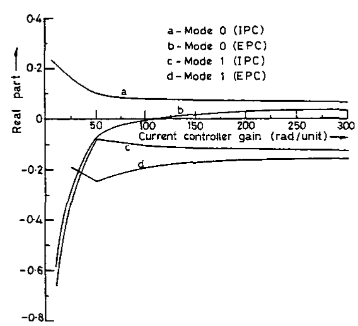


Figure 6. Variation of damping with controller gain (Three terminal system)