

# Response of an Excitatory-Inhibitory Neural Network to External Stimulation: An Application to Image Segmentation

Sitabhra Sinha and Jayanta Basak

## Abstract

Neural network models comprising elements which have exclusively excitatory or inhibitory synapses are capable of a wide range of dynamic behavior, including chaos. In this paper, a simple excitatory-inhibitory neural pair, which forms the building block of larger networks, is subjected to external stimulation. The response shows transition between various types of dynamics, depending upon the magnitude of the stimulus. Coupling such pairs over a local neighborhood in a two-dimensional plane, the resultant network can achieve a satisfactory segmentation of an image into "object" and "background".

## 1 Introduction

Dynamical transitions in brain activity, in the presence of an external stimulus, has received considerable attention recently. Most investigations of these phenomena have focussed on the phase synchronization of oscillatory activity in neural assemblies. An example is the detection of synchronization of "40 Hz" oscillations within and between visual areas and between cerebral hemispheres of cats [1] and other animals. Assemblies of neurons have been observed to form and separate depending on the stimulus. This has led to the speculation that, phase synchronization of oscillatory neural activity is the mechanism for "visual binding" - the process by which local stimulus features of an object (e.g. color, motion, shape), after being processed in parallel by different (spatially

separate) regions of the cortex are correctly integrated in higher brain areas, forming a coherent representation ("gestalt").

Sensory segmentation, the ability to pick out certain objects by segregating them from their surroundings, is a prime example of "binding". The problem of segmentation of sensory input is of primary importance in several fields. In the case of visual perception, "object-background" discrimination is the most obvious form of such sensory segmentation: the object to be attended to, is segregated from the surrounding objects in the visual field. Several methods for segmentation, both classical [2] and connectionist are reported in literature.

Most of the studies on segmentation through neural assembly formation has concentrated on networks of oscillators which synchronize when representing the same object. Malsburg and coworkers [3] have sought to explain segmentation through *dynamic link architecture* with synapses that rapidly switch their functional state. Similar approaches using synaptic couplings which change rapidly depending on the stimulus have been used in a neural model for segmentation by Sporns *et al* [4]. Grossberg and Sommers [5] have performed figure-ground separation with a network of oscillators, some of which belong to the "object" and the others to the "background". Oscillations of the former are synchronized, whereas the others have non-oscillatory activity. Han et al [6] have used an oscillatory network for Hopfield-type auto-association in pattern segmentation, using the temporal dynamics of the nonlinear oscillators driven by noise and subthreshold periodic forcing.

Similar segmentation through synchronization of activity among a cluster of neurons have been shown by other groups [7, 8].

In contrast to the above approach, we present a method of utilizing the transition between different types of dynamics (e.g., between fixed-point and periodic behaviors) of the network elements, for performing segmentation tasks. For this purpose, we have used a network of excitatory-inhibitory neuron models, updated in discrete time. The discrete-time evolution allows even this extremely simplified model to possess a rich variety of dynamical behavior, including chaos. As chaotic behavior has been reported in brain activity [9], it is interesting to study models which can show such phenomena, while remaining simple enough to be theoretically tractable. Note that, although most existing neuron models comprise of differential equations, which evolve in continuous time, this is not really a limitation of our model - as any  $N$ -dimensional discrete-time dynamical system may be related to a corresponding  $(N+1)$ -dimensional continuous-time dynamical system through the concept of *Poincare* sections [10]. It follows that the discrete-time model we have studied has a higher dimensional differential equation analogue, which will show qualitatively similar behavior. Several differential equation models already exist which describe the activity of single neurons, with varying degrees of fidelity. But it is difficult to study large networks of such continuously evolving systems in detail. It is for this reason that we have focussed on discrete-time systems.

In this paper, we investigate the dynamical response of an excitatory-inhibitory network model (with autonomous chaotic behavior) to external stimulation of constant intensity  $I$  (in time). We focus on how the behavior of an individual element within the network changes with  $I$ . A theoretical analysis has been presented for the transition from period-2 cycles to fixed-point behavior for an isolated excitatory-inhibitory pair (i.e., not coupled to any other element). Simulation results for the spatially interacting coupled network are presented. The application of the system for segmenting gray-level images is studied. Finally, the possible improvements of the proposed method are discussed.

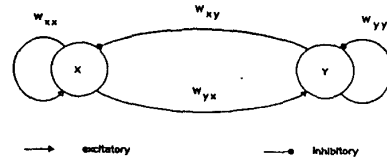


Figure 1: The basic excitatory ( $x$ )-inhibitory ( $y$ ) neural module.

## 2 The Excitatory-Inhibitory Network Model

The model, we have based our investigations, comprises excitatory and inhibitory neurons, coupled to each other over a local neighborhood. As we are updating the model only at discrete time intervals, it is the time-averaged activity of the neurons that is being considered. Therefore, the “neuronal activity” can be represented as a continuous function, saturating to some maximum value, inversely related to the absolute refractory period. This is arbitrarily taken to be 1, so that we can choose the following sigmoid-type function to be the neural activation function for our model:

$$F_{\mu}(z) = \begin{cases} 1 - \exp(-\mu z), & \text{if } z \geq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The basic module of the proposed network is a pair of excitatory and inhibitory neurons coupled to each other (Fig. 1).

If  $x$  and  $y$  be the activities of the excitatory and the inhibitory elements respectively, then they evolve in time according to:

$$\begin{aligned} x_{n+1} &= F_a(w_{xx}x_n - w_{xy}y_n + I_n) \\ y_{n+1} &= F_b(w_{yx}x_n - w_{yy}y_n + I'_n), \end{aligned} \quad (2)$$

where,  $w_{ij}$  is the weight of synaptic coupling from element  $j$  to element  $i$ ,  $F$  is the activation function defined by (1) and  $I, I'$  are external stimuli. By imposing the following restriction on the values of the synaptic weights:

$$\frac{w_{xy}}{w_{xx}} = \frac{w_{yy}}{w_{yx}} = k,$$

and absorbing  $w_x$  and  $w_{yy}$  within  $a$  and  $b$  (respectively), we can simplify the dynam-

ics to that of the following one-dimensional difference equation or “map”:

$$z_{n+1} = F_a(z_n + I_n) - kF_b(z_n + I_n). \quad (3)$$

Without loss of generality, we can take  $k = 1$ . In the following account we will be considering only time-invariant external stimuli, so that, for our purposes:

$$I_t = I, \quad I = I.$$

The resultant neural map, exhibits a wide range of dynamics (fixed point, periodic and chaotic), despite the simplicity of the model [11, 12].

## 2.1 Dynamics of a single excitatory-inhibitory neural pair

The autonomous behavior (i.e.,  $I, I' = 0$ ) of an isolated pair of excitatory-inhibitory neurons show a transition from fixed point to periodic behavior and chaos with the variation of the parameters  $a, b$ , following the ‘period-doubling’ route, universal to all unimodal maps. The map

$$z_{n+1} = F(z_n) = F_a(z_n) - F_b(z_n), \quad (4)$$

describing the dynamics of the pair (Fig. 2), has two fixed points, at  $z_1^* = 0$  and  $z_2^*$  (which is the solution of the transcendental equation  $z = \exp(-bz) - \exp(-az)$ ). The fixed point,  $z_1^*$  is stable if the local slope ( $\approx a - b$ ) is less than 1.

For

$$a > \frac{1}{1 - \mu},$$

where  $\mu = \frac{b}{a}$ , this condition no longer holds and  $z_1^*$  loses stability while  $z_2^*$  becomes stable, by a transcritical bifurcation. On further increase of  $a$  (say), this fixed point also loses stability, with the local slope becoming less than -1, and a 2-period cycle occurs. Increasing  $a$  leads to a sequence of period-doublings ultimately giving rise to chaos.

The introduction of an external stimulus of magnitude  $I$  has the effect of horizontally displacing the map, Eqn. (4), to the left by  $I$ . This implies that  $z_1^* = 0$  is no longer a fixed point, while the other fixed point  $z_2^*$  is now a solution of the equation  $z = \exp(-b(z + I)) - \exp(-a(z + I))$ . The slope at  $z_2^*$  decreases with increasing  $I$  - giving rise to a reverse period-doubling transition - chaos to periodic cycles to finally, fixed-, it, behavior.

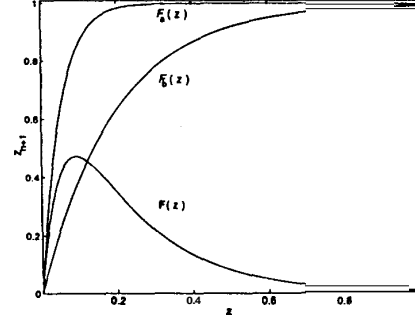


Figure 2: The 1-dimensional neural map,  $F$  (Eqn. (4)), with the activation functions for the constituent excitatory (slope,  $a = 20$ ) and inhibitory ( $b = 5$ ) neurons.

## 2.2 Analysis of response to constant magnitude external stimulus

We shall now consider how the dynamics of the excitatory-inhibitory pair changes in response to external stimulus. Let us consider the isolated neural pair, whose time evolution is given by Eqn. (3). On replacing the expression of the transfer function from (1), we get

$$z_{n+1} = \exp(-b(z_n + I)) - \exp(-a(z_n + I)). \quad (5)$$

Now,

$$z_{n+1} = z_n = z^*,$$

for a fixed point. It is stable if

$$\frac{dz_{n+1}}{dz_n} \geq -1,$$

i.e.,

$$(a - b)\exp(-a(z^* + I)) - bz_{n+1} \geq -1.$$

Therefore, for the fixed point to be marginally stable (i.e.  $\frac{dz_{n+1}}{dz_n} = -1$ ), it must satisfy the following condition:

$$(a - b)\exp(-a(z^* + I_c)) = bz^* - 1, \quad (6)$$

where,  $I_c$  is the critical external stimulus for which  $z^*$  just attains stability.

Let us define a new variable,  $\alpha$ , as

$$\alpha = \frac{bz^* - 1}{a - b} \quad (7)$$

Therefore, from (6), we get

$$\exp(-a(z^* + I_c)) = a. \quad (8)$$

Also from (7),

$$z^* = \frac{1}{b} + \frac{1-\mu}{\mu} \alpha \quad (9)$$

where  $\mu = b/a$ . Now, from (5), a marginally stable fixed point can be expressed as

$$z^* = -\exp(-a(z^* + I_c)) + \exp(-b(z^* + I_c)).$$

Therefore, from (8) and (9), the above expression can be written as

$$\alpha^\mu - a = \frac{1}{b} + \left(\frac{1-\mu}{\mu}\right) a. \quad (10)$$

By simple algebraic manipulation, we get

$$\alpha = \frac{1}{b^{1/\mu}} (1 + a\alpha)^{1/\mu}. \quad (11)$$

Assuming  $a \ll 1$ , we need to consider only the first order terms in  $a$  in the right hand side, so that

$$b^{1/\mu} \alpha = 1 + \frac{a\alpha}{\mu}, \quad (12)$$

which gives the following expression for  $a$ :

$$\alpha = \frac{1}{b^{1/\mu} - \frac{a}{\mu}}. \quad (13)$$

For a real solution of  $z^*$  to exist, we must have  $bz^* - 1 > 0$ , since, otherwise,  $z^*$  will have an imaginary component (from (16)). In other words,  $a > 0$  (from (6)). Therefore, from (13), we must have

$$a < \mu b^{1/\mu}. \quad (14)$$

Since  $b = \mu a$ , we get

$$a > \mu^{\frac{\mu+1}{\mu-1}}. \quad (15)$$

For example, if  $\mu = 0.5$  then  $a > 8$  for  $z^*$  to be real. From (6) we get

$$\exp(aI_c) = \frac{a-b}{bz^* - 1} \exp(-az^*). \quad (16)$$

Taking logarithms on both sides, we have,

$$I_c = -z^* - \frac{1}{a} \log(\alpha).$$

Therefore replacing  $z^*$  from (9),

$$I_c = \left(\frac{\mu-1}{\mu}\right) \alpha - \frac{1}{\mu a} - \frac{1}{a} \log(\alpha). \quad (17)$$

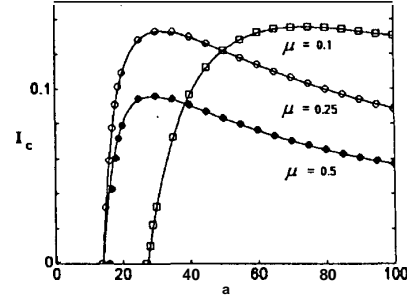


Figure 3: Critical magnitude ( $I_c$ ) of the external stimulus, at which transition from periodic to fixed point behavior occurs. The circles (filled and blank) and squares represent the values obtained exactly through numerical procedures for  $b/a = \mu = 0.5, 0.25$  and  $0.1$ , respectively. The curves indicate the theoretically predicted values.

The equation (17), together with (13), provides the critical value of the external stimulus which leads the oscillatory neuron pair to a fixed stable state, subject to the restriction (15).

This expression can be further simplified. From (11), we can write

$$\mu \log(\alpha) = -\log(b) + \log(1 + a\alpha).$$

As before, assuming  $a \ll 1$ , we need to consider only the first order terms in  $a$  in the right hand side of the logarithmic expansion, which gives us

$$\log(\alpha) = \frac{a\alpha}{\mu} - \frac{1}{\mu} \log(b). \quad (18)$$

From (13), (17), and (18), the critical magnitude of the external stimulus is given as

$$I_c = \frac{1 - \frac{2}{\mu}}{(\mu a)^{1/\mu} - \frac{a}{\mu}} + \frac{1}{\mu a} \log\left(\frac{\mu a}{e}\right), \quad (19)$$

where  $e = \exp(1)$ . Fig. 3 shows the  $a$  vs.  $I_c$  curves for different values of  $\mu$ , viz.  $\mu = 0.1, 0.25$  and  $0.5$ .

### 2.3 Choosing the operating region

To make the network segment regions of different intensities ( $I_1 < I_2$ , say), one can

fix  $\mu$  and choose a suitable  $a$ , such that  $I_1 < \mathbf{I} < I_2$ . So elements, which receive input of intensity  $I_1$ , will undergo oscillatory behavior, while elements receiving input of intensity  $I_2$ , will go to a fixed-point solution. Notice that, the curves obtained from (19) gives two values of  $a$  for the same  $\mathbf{I}$ . This gives rise to an operational question: given a certain  $\mathbf{I}$ , which value of  $a$  is more appropriate? Notice that, the region of the  $a$  vs  $I_c$  curve (Fig. (3)) to the left of the maxima, has a very high gradient. This implies that, in the presence of wide variation in the possible value of  $I_c$ , choice of  $a$  from this region will show very small variation - i.e., the system performance will be robust with respect to uncertainty in the determination of the appropriate value of  $\mathbf{I}$ . This is possible in the case of any gray-level image with a bimodal intensity distribution, having a long, almost uniform valley in between the two maxima.

On the other hand, the region of the curve to the right of the maxima has a very low gradient (almost approaching zero for high values of  $a$ ). This implies structural stability in network performance, as wide variation in choice of  $a$  will give almost identical results. So, choice of  $a$  from this region is going to make the network performance stable against parametric variations. As both robustness against uncertain input and stability against parametric variations are highly desirable properties in network computation, a trade-off seems to be involved here. The nature of the task in hand is going to be the determining factor of which value of  $a$  we should choose for a specific  $\mathbf{I}$ .

### 3 Simulation and Results

The response behavior of the excitatory-inhibitory neural pair, with local couplings, has been utilized in segmenting images and the results are shown in Fig. 4. Both synthetic and "real-life" gray-level images have been used to study the segmentation performance of the network. The initial state of the network is taken to be totally random. The image to be segmented is presented as external input to the network, which undergoes 200 - 300 iterations. Keeping  $a$  fixed, a suitable value of  $\mu$  is chosen from a consideration of the histogram of the intensity distribution of the image. This allows the choice

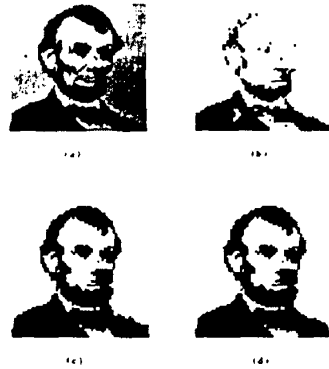


Figure 4: Results of implementing the proposed segmentation method on "Lincoln" image: (a) original image, (b) output by the uncoupled network, (c) output by the coupled network ( $r_{ex} = 1, r_{in} = 2$ ), and (d) output by the coupled network ( $r_{ex} = r_{in} = 2$ ), after 300 iterations ( $a=30, b/a=0.25$  and threshold  $th=0.02$ ).

of a value for the critical intensity ( $\mathbf{I}$ ), such that, the neurons corresponding to the 'object' converge to fixed-point behavior, while those belonging to the 'background' undergo period-1 cycles. In practice, after the termination of the specified number of iterations, the neurons which remain unchanged over successive iterations (within a tolerance value,  $th$ ) are labeled as the "object", the remaining being labeled the "background".

We have considered the 5-bit gray level "Lincoln" image (Fig. 4(a)) as an example of a "real-life" picture. A suitable  $\mathbf{I}$  has been estimated by looking at the histogram of the gray-level values, and taking the trough between two dominating peaks as the required value. As is clear from Fig. 4(b), the isolated neurons perform poorly in identifying the 'background' in the presence of noise. The segmentation performance improves remarkably when spatial interactions are included in the model. The results of including spatial interaction are shown in Figs. 4(c) and (d). The neighborhoods of the excitatory and inhibitory neurons are specified by the radii  $r_{ex}$  and  $r_{in}$ , respectively, of a discrete circular neighborhood [13]. Most of the image has been labeled accurately, except for a few regions (e.g., near

the neck).

Note that, we have considered a single value of  $a$  (and hence  $I_c$ ) for the entire image. This is akin to “global thresholding”. By implementing local thresholding and choice of  $a$  on the basis of local neighborhood information, the performance of the network can be improved.

## 4 Discussion

In the present work, we have assumed constant connection weights over a local neighborhood. However, a gaussian profile of weights may be biologically more realistic. One can also make the critical intensity  $I_c$  proportional to the ambient intensity. This is in tune with how the retina seems to alter its sensitivity to incoming visual stimuli [14]. Finally, the role of variable connection weights, that can be implemented in the present model by changing the value of  $k$  (ratio of the weights), may be investigated. Further, in this paper, we have used only the distinction between fixed-point and period-2 cycles to achieve two-level segmentation. However, this principle can be extended to achieve multi-level segmentation, by distinguishing between neurons with fixed-point behavior, period-2 cycle behavior, period-4 cycle behavior, and so on.

Chaotic neurodynamics has been used to segment images by Hasegawa *et al* [15]. However, their method is based on using chaos to avoid the local minima problem in the variable shape block segmentation method. We have instead concentrated on using stimulus induced transitions in neural network dynamics to segment images. This is closer to the neurobiological reality. As Malsburg [3] has indicated, the reason oscillatory synchronization has been studied so far, as a mean of segmenting sensory stimuli is its relative ease of analysis. However, with the developments in nonlinear dynamics and chaos theory, we can advance to segmentation using more general dynamical behavior.

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