RAY ANALYSIS OF A CLASS OF HYBRID CYLINDRICAL AIRCRAFT WINGS

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A new approach to the modelling of aircraft wings, based on the combination of hybrid quadric (parabolic and circular) cylinders, has been presented for electromagnetic applications. Closed-form expressions have been obtained for ray parameters required in the high-frequency mutual coupling computation of antenna pairs located arbitrarily on an aircraft wing.

Introduction: Wings are perhaps the most dominant structures of an aircraft, often constituting over 50% of the total surface area. Aircraft wings are also often used for locating antennas. With the growing trend towards the utilisation of higher frequencies of the electromagnetic spectrum, aircraft wings have become electrically large scatterers, and their curvatures and thickness can no longer be ignored in the mutual coupling calculations between antennas located over them. For such cases, an aircraft wing may be satisfactorily modelled as hybrid sections consisting of quadric cylinders (h-QUACYL), where a (truncated) parabolic cylinder constitutes the trailing edge of the airfoil while a (semi-) circular cylinder corresponds to the leading edge. In this letter, ray analysis of an aircraft wing has been performed to obtain ray geometric parameters required in the high-frequency formulations, such as the uniform theory of diffraction (UTD),' for mutual coupling computations. All the ray parameters obtained here are in the closed form.

Formulation; The parabolic and circular cylindrical sections (Fig. 1) of an aircraft wing may be described by the two sets of hybrid parametric equations

$$x = au \qquad y = u^2$$

and

$$z = z \qquad \text{with } |u| \le u_t \tag{1}$$

and

$$x = \rho\cos\phi \qquad y = L + \rho\sin\phi$$

and

$$z = z \qquad \text{with } \rho = au_t \quad \text{and} \quad 0 \le \phi \le \pi \tag{2}$$

where **a** is a shaping parameter for the parabolic cylinder and ρ is the radius of a circular cylinder whose origin has been shifted in the y-direction by a known distance $L = u_t^2$. While the two sections are described in two different co-ordinate systems, uniformity is achieved by using Cartesian co-ordinates for the ray analysis. The hybrid structure is formed along the common parameter z. **As** a general case, let $S(u_s, z_s)$ be a source point on the parabolic cylinder and $P(\phi_f, z_f)$ an observation point on the circular cylinder. Let T be defined as the transition point where a ray path from **S** to P crosses the junction (Fig. 1). T has a dual co-ordinate representation as $T(u_t, z_t)$ and $T(\phi_t, z_t)$, since it is common to both the cylinders. It is noted that $\phi_t = 0$, without loss of generality, and $u_t = L^{1/2}$, leaving only z_t to be determined. Ray-theoretic methods assume that the surface ray paths are extremal. Hence the ray path from **S** to T is a geodesic on the parabolic cylinder and from T to P is a helix, a geodesic on the circular cylinder. The extended Fermat principle asserts that when the surface is developed, the entire ray path becomes a straight line. Using the property of proportional triangles. we now obtain the z co-ordinate **of** the transition point as

$$z_t = z_f - \{\rho\phi_f(z_f - z_s)/(\rho\phi_f + \overline{SS}_1)\} \tag{3}$$

where

$$\overline{SS}_1 = 0{\cdot}5(u_t\surd(a^2 + 4u_t^2) - u_s\surd(a^2 + 4u_s^2)) + 0{\cdot}25a^2 \ln\{(2u_t + \surd(a^2 + 4u_t^2))/(2u_s + \surd(a^2 + 4u_s^2))\}$$

At this point, the transition point has been completely determined in both co-ordinate systems. To trace the complete ray path from S to P, we consider the arc segments ST on the parabolic cylinder and TP on the circular cylinder.

In the case of the parabolic cylinder the geodesic equation is expressed as

$$dz/du = h\surd(a^2 + 4u^2)/\surd(1 - h^2) \tag{4}$$

which may be integrated as

$$z(u) = 0{\cdot}25h(1 - h^2)^{-1/2}\{2u\surd(a^2 + 4u^2) + a^2 \ln(2u + \surd(a^2 + 4u^2))\} + \beta \tag{5}$$

in which h and β are the two constants of integration and may be obtained in closed form in terms of $S(u_s, z_s)$ and $T(u_t, z_t)$, which are known. Substitution of eqn. 5 in eqn. 1 gives the one-parameter form (i.e. in u) for the geodesic on the parabolic cylinder. All the ray geometric parameters can now be obtained using definitions in Reference 1 and the derivation procedure as in Reference 2. For example, the arc length is obtained by integrating the metric with respect to u between the limits S and T:

$$s(u) = 0{\cdot}25(1 - h^2)^{-1/2}\{2u\surd(a^2 + 4u^2) + a^2\ln(2u + \surd(a^2 + 4u^2))\}\Big|_S^T \tag{6}$$

Similarly, the radius of curvature ρ_g and the generalised Fock parameter $\xi(u)$ are found to be

$$\rho_g = -(a^2 + 4u^2)^{3/2}/2a(1 - h^2) \tag{7}$$

and

$$\xi(u) = \int_S^T (\pi/\rho_g^2)^{1/3}\, ds = 0{\cdot}5(4\pi a^2)^{1/3}(1 - h^2)^{1/6} \times \{\ln(2u + \surd(a^2 + 4u^2))\}\Big|_S^T \tag{8}$$

All the ray geometric parameters can be derived for the circular cylinder once the one-parameter form of the geodesic between T and P is obtained after translating the origin by a known distance L along the y-direction. The ray parameters for a circular cylinder are well known.'

The mutual coupling may be then obtained by substituting the ray geometric parameters in the UTD formulation with slight modifications. For example, $s(u)$ and $\xi(u)$ in such an expression would refer to the sum of the arc lengths and Fock parameters on the parabolic and circular cylinder, respectively. Similarly, the tangent, normal and binormal vectors obtained at the source (on the parabolic cylinder) and observation point (on the circular cylinders) now appear as dyadic

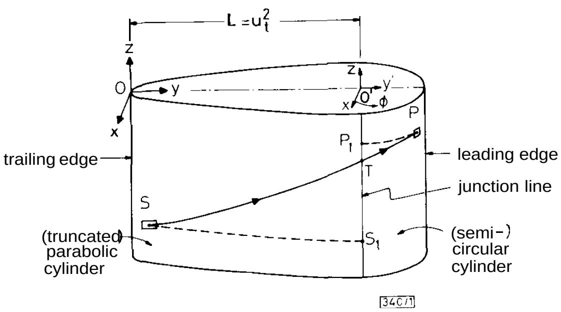

Fig. 1 *Section of an aircraft wing modelled by a hybrid combination of (truncated) parabolic cylinder and (semi-)circular cylinder*