RAY ANALYSIS OF A CLASS OF NONDEVELOPABLE SATELLITE LAUNCH VEHICLES

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An efficient algorithm based on Hertz's principle is presented for ray analysis of nondevelopable hybrid quadric surfaces of revolution. This quasi-analytical method of determining ray geometric parameters as required in high-frequency mutual coupling calculations has wide applications for conformal antennas, including those on aerospace vehicles.

Introduction: Satellite launch vehicles (SLVs) and subsonic missiles can often be accurately modelled by a combination of a right circular cylinder and a general paraboloid of revolution (GPOR). These structures are examples of hybrid quadric surfaces of revolution (h-QUASORs) and are of considerable interest in aerospace engineering. With increasing use of higher frequencies for conformal antennas on SLVs, the application of ray-theoretic methods to the calculation of mutual coupling has become a feasible proposition.

To be able to use the ray-theoretic methods, one must know all the ray geometric parameters *a priori*. In appropriate cases, the developability of a scattering convex surface is made use of in obtaining these parameters in closed form based on the extended Fermat principle. However, it is not possible to do so in case of h-QUASORs in general, where one of the surfaces (for example, the GPOR in the case of SLVs) may not be developable. In this letter, an efficient algorithm based on a more general property of surface-diffracted ray paths, derived from Hertz's principle in particle dynamics, has been presented to determine the ray geometric parameters for h-QUASORs in general. The specific example chosen is that of an SLV modelled by a nondevelopable h-QUASOR structure.

Formulation: The matched GPOR and cylindrical sections (Fig. 1) of an SLV may be described by the two sets of hybrid parametric equations

$$\begin{aligned} x &= au \cos \phi & y &= au \sin \phi \\ z &= -u^2 & \text{with } |u| &\leq u_t \end{aligned} \quad (1)$$

where a is a shaping parameter of the GPOR truncated at $u = u_t$, and

$$\begin{aligned} x &= \rho \cos \phi & y &= \rho \sin \phi \\ z &= z & \text{with } -z_c &\leq z \leq -u_t^2 \end{aligned} \quad (2)$$

where ρ is the radius of the cylinder. Eqns. 1 and 2 are represented in the parabolic and cylindrical co-ordinates, respectively. This, however, does not pose any problems, since both the surfaces have rotational symmetry in the variable ϕ .

Fig. 1 Satellite launch vehicle modelled by a hybrid combination of a general paraboloid of revolution (GPOR) and finite circular cylinder

As a general case, let S be a source point on the cylinder and P an observation point on the GPOR. Let T be a point where an extremal surface ray path from S to P crosses the GPOR-cylinder junction (Fig. 1). T is defined as the junction transition point for the ray, and has a dual co-ordinate representation since the junction is shared by both the surfaces. The extended Fermat principle requires that the surface ray path from $S(z_s, \phi_s)$ to $T(z_t, \phi_t)$ be a geodesic of the cylinder (i.e. a helix), and that from $T(u_t, \phi_t)$ to $P(u_f, \phi_f)$ is a geodesic of the GPOR. Since S and P are known points besides the plane of intersection for the junction, ϕ_t alone needs to be determined in both the co-ordinate systems.

For the developable surfaces, the ray parameters may be obtained in closed form by using the well known results derived from the extended Fermat principle that the geodesics over a ruled surface (e.g. cone, cylinder), when developed, map on to straight lines. This is clearly inadequate for nondevelopable surfaces, for which we invoke a more general property that when an extremal path crosses the junction between two adjacent portions of a surface, the angle with respect to the

junction line is preserved. This follows from Hertz's principle' in particle dynamics and is true even if the two portions are geometrically different and/or meet at an angle, but assumes that the boundary line has a continuous slope at the crossover point. This is illustrated in Fig. 1, by noting that angles λ and μ are equal. Since the angle invariance property is a local one, it does not impose global requirements such as the developability of the surface.

Applying the angle invariance property of the geodesic at the transition point T, on the boundary between the GPOR and cylindrical portions of the SLV (Fig. 1),

$$\hat{t}_p \cdot \hat{\phi}_t = \hat{t}_c \cdot \hat{\phi}_t \quad (3)$$

where $\hat{\phi}_t$, $\hat{t}_c$ and $\hat{t}_p = (t_{px}\hat{i} + t_{py}\hat{j} + t_{pz}\hat{k})$ are the unit tangent vectors at T along the junction, the geodesic on the cylinder and the geodesic on the GPOR, respectively, and are defined as

$$\hat{\phi}_t = -\sin \phi_t \hat{i} + \cos \phi_t \hat{j} + 0\hat{k} \quad (4)$$

$$\hat{t}_c = -\sqrt{(1 - h_c^2)} \sin \phi_t \hat{i} + \sqrt{(1 - h_c^2)} \cos \phi_t \hat{j} + h_c \hat{k} \quad (5)$$

(from eqn. 2)

$$t_{px} = \{\sqrt{(a^2 u_t^2 - h_p^2)} a \cos \phi_t - \sqrt{(a^2 + 4u_t^2)} h_p \sin \phi_t\} / \{a u_t \sqrt{(a^2 + 4u_t^2)}\} \quad (6)$$

$$t_{py} = \{\sqrt{(a^2 u_t^2 - h_p^2)} a \sin \phi_t + \sqrt{(a^2 + 4u_t^2)} h_p \cos \phi_t\} / \{a u_t \sqrt{(a^2 + 4u_t^2)}\} \quad (7)$$

$$t_{pz} = \{-2\sqrt{(a^2 u_t^2 - h_p^2)}\} / \{a \sqrt{(a^2 + 4u_t^2)}\} \quad (8)$$

Eqs. 6–8 follow from earlier results.³ h_c and h_p are constants of integration of the geodesic equations on the respective quadric surfaces. Substituting eqns. 4–8 in eqn. 3 gives

$$\sqrt{(1 - h_c^2)} = h_p / a u \quad (9)$$

which is, in fact, an implicit equation in ϕ_t , since h_c and h_p are functions of ϕ_t . While h_c can be expressed explicitly as

$$h_c = (z_t - z_s) / \sqrt{(z_t - z_s)^2 + \rho^2(\phi_t - \phi_s)^2} \quad (10)$$

it is not possible to express h_p in closed form, and which therefore must be determined numerically in terms of the co-ordinates of T and P. It follows from eqn. 9 that ϕ_t also has to be determined numerically. Since the entire range from ϕ_s to ϕ_f must also be searched for locating ϕ_t , and since a search for h_p is involved in each trial for ϕ_t , a dual-parameter search results. To minimise this search effort, a good starting point for ϕ_t is provided here by applying the arc PP, onto the tangent plane of the cylinder at P, :

$$\phi = \phi_s + \{(\phi_f - \phi_s)(z_t - z_s) / (\overline{PP}_1 + z_t - z_s)\} \quad (11)$$

where

$$\begin{aligned} \overline{PP}_1 = & 0.5(u_f \sqrt{(a^2 + 4u_f^2)} - u_t \sqrt{(a^2 + 4u_t^2)}) \\ & + 0.25a^2 \ln \{(2u_f + \sqrt{(a^2 + 4u_f^2)}) / (2u_t \\ & + \sqrt{(a^2 + 4u_t^2)})\} \end{aligned} \quad (12)$$

This leads to a quick convergence of the solution. For the special case where S is away from the junction while P is close to it, the starting point (eqn. 11) may be used to replace ϕ_t for all practical purposes, thereby obviating the need for a numerical search for ϕ_t .

It can easily be shown that even for those h-QUASORs where both the surfaces are developable (e.g. the cone-cylinder model of an SLV), the equivalent of eqn. 3 is nonlinear and ϕ_t still cannot be obtained in closed form. The formulation presented in this letter is applicable even to these cases.

Once the value of ϕ_t has been determined, the co-ordinates of T are precisely known in both the co-ordinate systems. Following this, the ray geometric parameters on the GPOR³ (between T and P) and those for the cylinder³ (between S and T) can be obtained in a straightforward manner. Using these ray parameters in the UTD formulation,¹ the electric field at

any observation point due to an arbitrarily located source point on the SLV can be determined. In the case of finite-dimensional slots, a dual-aperture integration of the electric field would be required for the mutual coupling computations.

In the ray analysis presented here, the edge diffraction at the junction has been ignored since, in practice, the two surfaces usually blend smoothly⁴ and the wedge angles are usually shallow.

Conclusion: The quasi-analytical approach developed in this letter permits derivation of the ray geometric parameters in the more general case of nondevelopable h-QUASORs. It can also be used to treat those combinations of developable surfaces which cannot be solved analytically. This new approach based on Hertz's principle can be efficiently used to obtain the ray parameters required in the mutual coupling analysis of conformal antennas located on aircraft, missiles and satellite launch vehicles.

References

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