i

TRANSVERSE FOLDING ALGORITHM FOR PLAs

Indexing terms: Logic and logic design, Programmable logic arrays, PLA folding

The letter reports a new concept for transverse folding of programmable logic arrays (PLAs). With the new definitions for the compatibility and foldability of rows (columns) of a column-folded (row-folded) PLA, the transverse compatibility-cum-foldability matrix (TCFM) is plotted. From the TCFM, a transverse folding matrix (TFM) is found, from which the row (column) folding pairs and the resulting ordering of rows and columns are obtained.

In this letter we report a new method for transverse folding of PLAs; that is, simple row folding (SRF) after simple column folding (SCF) or vice versa. Hachtel et *al.*[1] have adopted a heuristic approach to this problem, which does not guarantee 'maximum' folding. Our method is based on defining 'compatibility' and 'foldability' of rows (columns) in the light of the constraints imposed by SCF (SRF) previously, and provide maximum folding.

Consider the PLA of Fig. 1, with eight input and six output columns which are all folded. We can arrange the column pairs of a column folded PLA in nonascending order of their folding cuts, as shown in Fig. 2. Each folding cut divides the rows into classes, m cuts forming $m + 1$ classes. In Fig. 2, there are eight classes due to seven cuts. Class 5 has rows 4 and 6 and class 3 has no rows. We see that the rows in the same class cannot be folded except in the last, that is, $(m+1)$th class, but they possibly can be folded with the rows in other classes. Rows 4 and 6 cannot be folded with each other, but they can be folded with row 1 in class 1 or row 3 in class 2.

Here, we shall give only the new definitions, the other definitions being the same as in Reference 2.

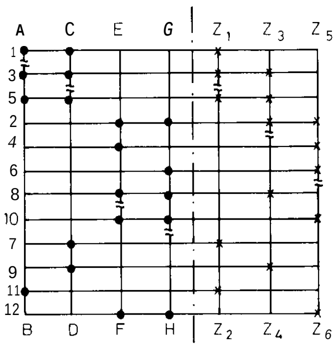

Fig. 1 *Example PLA with SCF*

Defnition 1 (transoersefolding): A simple row folding after a simple column folding (SRF after SCF) or vice versa is called a 'transverse folding' of the PLA.

In this letter, we shall confine our study to SRF and SCF, but our comments are equally valid for SCF after SRF.

Definition 2: The set of columns whose cross points with a row 'r' are personalised, is said to be the set of subsuming columns of r or SSC(r).

Definition 3 (compatibility): In a simple column folded PLA, two rows r_1 and r_2 are said to be compatible if (i) SSC(r_1) $\cap$ SSC(r_2) $= \emptyset$ (null set) and (ii) if $r_1 \in$ SSR(c_i) then $r_2 \notin$ SSR(c_j) and vice versa, for all $(c_i, c_j) \in$ {set of folded column pairs of the PLA}, where (c_i, c_j) is a column folding pair and SSR(c_i) is the set of subsuming rows of the column c_i.[2]

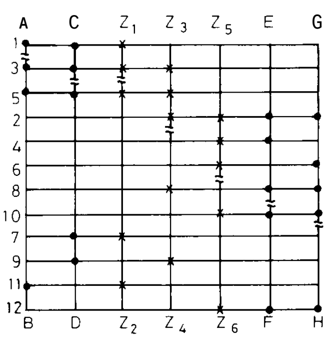

Fig. 2 *Example PLA folding cuts ordered*

We see from Fig. 2 that the rows of class i are bounded by folding cuts in the $(i-1)$th and ith columns reckoned from left. Obviously, a row in SSR of the ith lower column from left is not compatible with a row in class i. **Also,** a row in the ith class will not be in SSR of ith lower column.

Definition 4 (foldability): In a simple column folded PLA with folding cuts in nonascending order of their heights and given $r_1 \in$ class i and $r_2 \in$ class j, the rows r_1 and r_2 are foldable, if (a) r_1 and r_2 are compatible, and (b) $(i = j)$ or (if $i < j$, then r_2 is not in SSR of kth lower column for all k, $k = (i+1), \ldots, (j-1)$, and if $j < i$ then r_1 is not in SSR of kth lower column for all k, $k = (j+1), \ldots, (i-1)$).

Using this definition, we see that row 1 in class 1 is foldable with row 6 in class **5**, since, they are compatible and row 6 has no personalities in the 2nd to 4th lower columns. But row 1 is not foldable with row 8 which is in class 6, since, though row 1 and row 8 are compatible, row 8 has a personality on Z_4, the 4th lower column from left.

Now we form a matrix called the 'transverse compatibility-cum-foldability matrix (TCFM)' of pairwise compatibility and foldability among all the rows of the column folded PLA. Fig. 3 shows TCFM for Fig. 2. A '1' in a cell (r_i, r_j) implies r_i and r_j are compatible, while a '2' implies they are foldable. A blank **or 0** means neither.

	1	2	3	4	5	6	7	8	9	10	11	12
1		2		2		2		1		1		1
2	2						2				2	
3				2		2				1		1
4	2		2		2		2		2		2	
5				2		2				1		1
6	2		2		2		2		2		2	
7		2		2		2		2		2		2
8	1						2				2	
9				2		2				2	2	2
10	1		1		1		2		2		2	
11		2		2		2		2	2	2		2
12	1		1		1		2		2		2	

747/3

Fig. 3 *TCFM ofexample PLA*

From the TCFM, we obtain a matrix called a transverse folding matrix (TFM), as shown in Fig. **4.** It has the following properties:

(i) Its columns and rows are two disjoint subsets of the entire set of rows of the PLA.

(ii) It is an $m \times m$ matrix where $2m \leq n$ and n is the total number of rows of the PLA.

(iii) It has 2s in all the cells along the leading diagonal.

(iv) The cells below the leading diagonal have 1 **or** 2 (1/2).

(v) The cells above the leading diagonal are don't cares

	L_1	L_2	.	.	L_m
R_1	2	Φ	Φ	Φ	Φ
R_2	1/2	2	Φ	Φ	Φ
	1/2	1/2	2	Φ	Φ
	1/2	1/2	1/2	2	Φ
R_m	1/2	1/2	1/2	1/2	2

747/4

Fig. 4 *TFM*

Theorem (SRF after SCF theorem): The $2m$ rows of a column folded PLA with n rows can be folded with m row folding pairs if and only if an $m \times m$ TFM can be derived from the TCFM of the column folded PLA. (For proof see Reference **3.)**

Corollary 1: $((L_1, R_1), \ldots, (L_m, R_m))$ is an implementable row folding set.

Corollary 2: The ordering of the columns of a PLA with SRF after SCF is the ordered set union of SSCs of the columns $(L_1, \ldots, L_m)$ of TFM reckoned from left to right. (For proofs refer to Reference **3.)**

We adapted the 'maximum folding algorithm' of Reference **3** to find a TFM of maximum order that exists. When applied to the PLA of Fig. 2, we obtained a TFM **as** shown in Fig. **5,**

	4	6	10	2	8	12
5	2	2	1			1
3	2	2	1			1
9	2	2	2			2
1	2	2	1	2	1	1
11	2	2	2	2	2	2
7	2	2	2	2	2	2

Fig. 5 *TFM of example PLA*

which gives the ordered row folding pairs (4, **5**), (6, 3), (10, **9**), (2, 1), (**8,** 11), (12, 7). This is really a maximum row folding. The ordering of the top columns with the row folding cuts (/) is given by E, $Z_5/G//Z_3//Z_1$, A, C. The corresponding bottom column ordering is *F*, Z,, H, Z_4, Z_2, B, *D*. The row folded PLA is shown in Fig. 6.

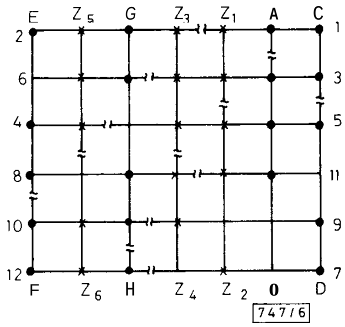

Fig. ***6 Example PLA with SRF after SCF***

Our transverse folding algorithm defines compatibility and the foldability relations on the entire set of rows (columns) of the folded PLA instead of rows (columns) in each class. Dependant on the SCF (SRF) already made which imposes the compatibility and foldability relations on the rows (columns) of the PLA, we can always find a transverse folding of maximum order, using this algorithm.

References

1 HACHTEL, G. D., NEWTON, A. R., and SANGIOVANNI-VINCENTELLI, A L.: 'Techniques for PLA folding'. 19th IEEE-DAC 1982, pp. 143–155

2 BISWAS, N. N., and BHAT, C.: 'A maximum PLA folding algorithm'. IEEE-Int. Conf. on Computer Design ICCD '87, Port Chester, N.Y. Oct. 1987

3 BHAT, C., and BISWAS, N. N.: 'A new transverse folding algorithm for PLAs'. Tech. Report No. CS-8702, Indian Institute of Science, Bangalore 560012, India, Sept. 1987