AN ALGORITHM TO ESTIMATE MULTIPATH DELAYS OF BROADBAND SOURCES WITH ONLY TWO SENSORS.

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ABSTRACT

DOA estimation and relative delay estimation are precursors for various signal processing algorithms. In this paper a new algorithm is proposed to estimate the relative delays and the Direction Of Arrival(DOA) at the sensor for multiple sources using *only two* sensors. The properties of the sources namely, broadband width and spectral overlap are used to achieve this. Numerical simulations are presented to study the performance of the methods.

1. INTRODUCTION

Sensor array processing, using only the narrowband properties require at least as many sensors as the signal sources for DOA estimation. By spacing the sensors more apart than dictated by the Nyquist rate and making use of the broadband nature of the signal we can circumvent this limitation. Doing the DOA estimation using only two sensors can, in some sense, be motivated by the functioning of human auditory system. The relative delay of signals at the two sensors can be used to estimate the DOA if the sensor geometry is known. There is a class of methods called cross correlation methods which use this property [1],[2],[3], [4]. The proposed approach uses the whitening and interpolation of data to get the estimate. Hence it can be used to estimate the DOA of more than one source. In the case of multipaths, the relative delay between the multipath signals received at two different sensors can be estimated. Then a method is proposed to classify the delays and assign them to the proper sources. Thus for each source the method able to estimate the relative delay due to each multipath.

2. DATA MODEL.

Consider M sources $s_m(t),\ m = 1\dots M$, generating the signals and 2 sensors receiving them. Let P_m be the number of multipaths due to the m^{th} source including the direct path. Let the p^{th} multipath signal from the m^{th} source at the first sensor be denoted by $\tilde{s}_{mp}(t)$ where t is the continuous time variable. Let the delay for the signal $\tilde{s}_{mp}(t)$ from $\tilde{s}_{m1}(t)$ (direct path) be τ_{mp}. Hence $\tilde{s}_{mp}(t) = \tilde{s}_{m1}(t-\tau_{mp})$. Define Δ_{mp} as the relative delay between the first sensor and the 2^{nd} sensor for the signal $\tilde{s}_{mp}(t)$. Plane wavefront is assumed at the sensors. Define θ_{mp} as the DOA of $s_{mp}(t)$, c be the velocity and d the spacing between the sensors. The additive white noise at the two sensors are respectively $\epsilon_1(t)$ and $\epsilon_2(t)$. The output of the sensors, $\tilde{y}_1(t)$, and $\tilde{y}_2(t)$, can be expressed as

$$\tilde{y}_i(t) = \sum_{m=1}^{M}\sum_{p=1}^{P_m} \tilde{s}_{mp}(t-(i-1)\Delta_{mp}) + \epsilon_2(t),\ i = 1, 2 \quad (1)$$

where $\Delta_{mp} = d\sin(\theta_{mp})/c$. Let the sampling period be denoted by T_s. With resect to the direct path, the total delay of the signal $\tilde{s}_{mp}(t)$ at the first sensor is τ_{mp} and at the second sensor is $\tau_{mp} + \Delta_{mp}$. Hence, using the sampling theorem, in the sampled domain, this signal at the second sensor can be represented as

$$\tilde{s}_{mp}(nT_s - \Delta_{mp}) = s_{m1}(n) * sinc\left(\frac{-(\tau_{mp}+\Delta_{mp})}{T_s} + n\right). \quad (2)$$

Each of the sources, $s_{m1}(n)$ is assumed to be output of identical Auto Regressive Moving Average (ARMA) systems with input white sequences, denoted by $\eta_m(n)$, of power σ^2.

Problem Statement: Given the above scenario the aim is to estimate the relative delays between multipath signals arriving at the two different sensors for all multipaths of each source.

3. PROPOSED METHOD

3.1. Whitening of the signal

The first step in the proposed method is to whiten the output of the sensors. The N^{th} order LPC coefficients $\hat{c}_l$, of the signal at the first sensor is estimated and used to obtain the whitened signals at both the sensors[5]. Since the signals at each sensor are the linear combination of shifted signal sources as given by equations (1), the same whitening filter acts on each source. Whitening here is not perfect because there is a difference in time delays for different source signals at the two sensors. For this an approximate idea of the source model is required. Let ˆ denote the estimates. The whitened outputs at the two sensors $\zeta_1(n)$ and $\zeta_2(n)$ are given as

$$\zeta_1(n) = \sum_{l=0}^{N} \hat{c}_l y_1(n-l) = \sum_{m=1}^{M}\sum_{p=1}^{P_m} \hat{\eta}_m(n) * sinc\left(\frac{-\tau_{mp}}{T_s}+n\right)$$

$$\zeta_2(n) = \sum_{l=0}^{N} \hat{c}_l y_2(n-l)$$

$$= \sum_{m=1}^{M}\sum_{p=1}^{P_m} \hat{\eta}_m(n) * sinc\left(\frac{-(\tau_{mp}+\Delta_{mp})}{T_s}+n\right) \quad (3)$$

3.2. Cost function for delay estimation

The next step is to choose the cost-function for delay estimation. Note that the cross-correlation of the whitened output of the sensors is given by

$$R_{\zeta_1\zeta_2}(n) = E\{\zeta_1(n)\zeta_2^*(n)\}$$

$$= \sum_{m=1}^{M}\sum_{p=1}^{P_m}\sum_{q=1}^{P_m} \sigma^2 sinc\left(\frac{-(\tau_{mp}+\Delta_{mp}-\tau_{mq})}{T_s}+n\right) \quad (4)$$

The relative delays $\Lambda_{mpq} = (\tau_{mp}+\Delta_{mp}-\tau_{mq})$ (here Λ_{mpq} corresponds to relative delay between p^{th} path at the first sensor and q^{th} path at the second sensor for the m^{th} source) are to be estimated. There are $J = \sum_{m=1}^{M} P_m^2$ such delays. To form a cost function whose points of maxima will give all the Λ_{mpq}, consider the equation below with β_1 and β_2 being some arbitrary time values [1] [6].

$$\sum_{l=-\infty}^{\infty} \{sinc(l-\beta_1/T_s)sinc(l-\beta_2/T_s)\} = sinc((\beta_1-\beta_2)/T_s) \quad (5)$$

This will have the maxima when $\beta_1 = \beta_2$. Thus a cost function formed as $C_1 = \sum_{l=-\infty}^{\infty} R_{\zeta_1\zeta_2}(l) sinc(l-\Lambda/T_s)$ will have the prominent peaks when Λ equals some Λ_{mpq}. Based on this observation, the following cost function is proposed.

$$\Lambda_{mpq} = \arg\max_{\Lambda}\left(\sum_{l=-\infty}^{\infty} R_{\zeta_1\zeta_2}(l) sinc(l-\Lambda/T_s)\right)$$
$$m = 1\ldots M,\; p = 1\ldots P_m,\; q = 1\ldots P_m \quad (6)$$

For a given sensor spacing, the resolution of adjacent peaks of the cost function is determined by T_s as is evident from the nature of the $sinc()$ function.

3.3. DOA Estimation

When there are no multipaths all $\tau_{mp} = 0$. Then all Λ estimated are equal to some Δ_{m1}. These delays are the direct result of DOA at the sensors as given by Δ_{mp}. Hence the DOAs can be obtained from Λ using Δ_{mp}.

[1] $sinc(x) = \sin(x)/x$

3.4. Multipath Delay Partitioning

Now consider the case where there are multipaths i.e. $\tau_{mp} \neq 0$. After estimation of all J relative delays, $\{\Lambda_1, \Lambda_2, ..., \Lambda_J\}$, it is not known which of these are due to various sources. This set has to be partitioned into subsets, each subset having all delays due to multipaths from a particular source. To see how the set of Λ can be partitioned consider the following theorem.

Theorem 1 *(Proof in Appendix) Let x_1, x_2 and $\{\epsilon_i\}_{i=1}^4$ be random variables with*

$$E\{x_i\} = 0,\; E\{\epsilon_i\} = 0,\; E\{\epsilon_i x_j\} = 0,\; E\{\epsilon_i\epsilon_j\} = \sigma_\epsilon^2\delta_{ij} \quad (7)$$

Let $\{z_i\}_{i=1}^4$ be random variables such that

$$z_1 = x_1+\epsilon_1,\; z_2 = x_1+\epsilon_2,\; z_3 = x_2+\epsilon_3,\; z_4 = x_2+\epsilon_4 \quad (8)$$

$$C = E\{(z_1z_2 - E\{z_1z_2\})(z_3z_4 - E\{z_3z_4\})\} \quad (9)$$
$$= E\{x_1^2x_2^2\} - \sigma_{x_1}^2\sigma_{x_2}^2 \quad (10)$$
$$\begin{cases} = 0 & \text{if } E\{x_1x_2\} = 0 \\ > 0 & \text{if } E\{x_1x_2\} \neq 0 \end{cases} \quad (11)$$

Let $C_2(k)$ be a cost function defined as

$$C_2(k) =$$
$$E\{(\zeta_1(nT)\zeta_2(nT+\Lambda_i) - E\{\zeta_1(nT)\zeta_2(nT+\Lambda_i)\})$$
$$(\zeta_1(nT+k)\zeta_2(nT+\Lambda_j+k) - E\{\zeta_1(nT)\zeta_2(nT+\Lambda_j)\})\}$$
$$\begin{cases} > 0 & \text{if } \Lambda_i \text{ and } \Lambda_j \text{ due to same source} \\ = 0 & \text{if } \Lambda_i \text{ and } \Lambda_j \text{ due to different sources.} \end{cases} \quad (12)$$

If pairs $\{\zeta_1, \zeta_2\}$ are substituted from (3), then it is shown below that for some pairs $\{\Lambda_i, \Lambda_j\}$, $C_2(k)$ will turn out to be of the form (8), with some terms in the summation of (3) taking the place of x_i and sum of the remaining terms the place of ϵ_i.

To explain the method, consider the scenario where there are two sources having two multipaths each. Then the relative delay estimation method would yield 8 relative delays. Let Λ_{112} and Λ_{122} be under consideration for finding $C_2(k)$. Λ_{112} is the relative delay of first source's first path at first sensor with that of second path at second sensor. Similarly Λ_{122} is the relative delay of first source's second path at first sensor with that of second path at second sensor.

The output of the first and second sensors can be written as $x_1(t)+\epsilon_1(t)$ and $x_1(t+\Lambda_{112})+\epsilon_2(t)$ respectively. $x_1(t)$ is the signal component due to the first sources first path at the first sensor. ϵ_i forms the remaining signal components, which are also white. If the output of the second sensor is shifted by Λ_{112} then the signal components of the second path of the first source are aligned in time with the corresponding components in the output of the first sensor. Then the second sensor output becomes $x_1(t)+\epsilon_2(t)$. Consider $\alpha_1(t) = (x_1(t)+\epsilon_1(t))(x_1(t)+\epsilon_2(t)) = x_1^2(t)+\epsilon_3(t)$. Similarly for Λ_{122}, $\alpha_2(t) = (x_1(t+k)+\epsilon_4(t))(x_1(t+k)+\epsilon_5(t)) = x_1^2(t)+\epsilon_6(t)$. Consider $\beta_1(t) = \alpha_1(t)-\sigma_1^2$ and $\beta_2(t) = \alpha_2(t)-\sigma_1^2$. Then $C_2(k) = E\{\beta_1(t)\beta_2(t)\} = x_1^2(t)x_1^2(t+k)-\sigma_1^4$. This would yield a peak at the shifted value k.

Let $x_2(t)$ be the signal component of the second path of the second source. If the same process can be repeated with Λ_{222} instead of Λ_{122}. Then $\beta_2(t) = x_2^2(t+k) + \epsilon_7(t) - \sigma_2^2$. The crosscorrelation would yield 0 for all k since x_2 and x_1 correspond to 2 different uncorrelated sources.

Now considering a general case let $\Lambda_i = \Lambda_{uqr}$ and $\Lambda_j = \Lambda_{uq'r'}$ be the delays under consideration in computing $C_2(k)$. The paths r, q, r', q' are from the u^{th} source under consideration. Then $\zeta_1(nT)$ and $\zeta_2(nT + \Lambda_{urq})$ will have one of the terms for the u^{th} source to be same i.e.

$$\begin{aligned}\zeta_1(nT) &= \hat{\eta}_u(n) * sinc\left(\frac{-\tau_{ur}}{T_s} + n\right) \\ &+(1-\delta_{mu}\delta_{pr})\sum_{m=1}^{M}\sum_{p=1}^{P_m}\hat{\eta}_m(n) * sinc\left(\frac{-\tau_{mp}}{T_s} + n\right) \quad (13)\end{aligned}$$

$$\begin{aligned}\zeta_2(nT+\Lambda_{uqr}) &= \hat{\eta}_u(n) * sinc\left(\frac{-\tau_{ur}}{T_s} + n\right) + \\ (1-\delta_{mu}\delta_{pr}) &\sum_{m=1}^{M}\sum_{p=1}^{P_m}\hat{\eta}_m(n) * sinc\left(\frac{-(\tau_{mp}+\Delta_{mp})}{T_s} + n\right)\end{aligned} \quad (14)$$

This common term, $\hat{\eta}_u(n) * sinc\left(\frac{-\tau_{ur}}{T_s} + n\right)$ forms x_1. The sum of the remaining terms of $\zeta_1(nT)$ are uncorrelated to the sum of the remaining terms of $\zeta_2(nT + \Lambda_{uqr})$, because of the difference in the values of $-\tau_{mp}$ and $-(\tau_{mp} + \Delta_{mp})$. These sum terms of $\zeta_1(nT)$ and $\zeta_2(nT + \Lambda_{uqr})$ form ϵ_1 and ϵ_2 respectively, of $C_2(k)$. Similarly, for $\zeta_1(nT)$ and $\zeta_2(nT + \Lambda_{uq'r'})$ the terms corresponding to x_2, ϵ_3 and ϵ_4 can be identified. In the latter case, x_2 is a shifted version of x_1. Now this problem is mapped to that in (8). Hence checking for the relationship between x_1 and x_2 in this case, it can be seen that, since both the multipaths r and r' are from the same source, for some value of k, they become identical and $C_2(k) > 0$ will become true. In case the multipaths r, q, r', q' are from different sources, $C_2(k) = 0$ for all k. Thus searching over all pairs $\{\Lambda_i, \Lambda_j\}$ from the set already estimated, to see if $C_2(k) > 0$ or $C_2(k) = 0$ for any k, it can be determined whether the pairs are due to paths from the same source or different sources. The two cases are illustrated in figure (2). Once the pairwise relations are established within the set of Λ, this can be used to partition the set into subsets with each set containing all delays from a particular source. If two different combinations of $\Lambda = (\tau_{mp} + \Delta_{mp} - \tau_{mq})$, lead to the same value then the peaks due to these will appear at the same point in the cost function. Thus these are not distinguishable and there is an ambiguity.

Summary of the Algorithm:

1. Whiten the signals at the sensors using LPC.

2. Estimate the relative delays between all paths at the two sensors using (6). If no multipaths, obtain DOA from the delays.

3. If there are multipaths, then partition the set of relative delays into subsets, each having only relative delays of one source using (12).

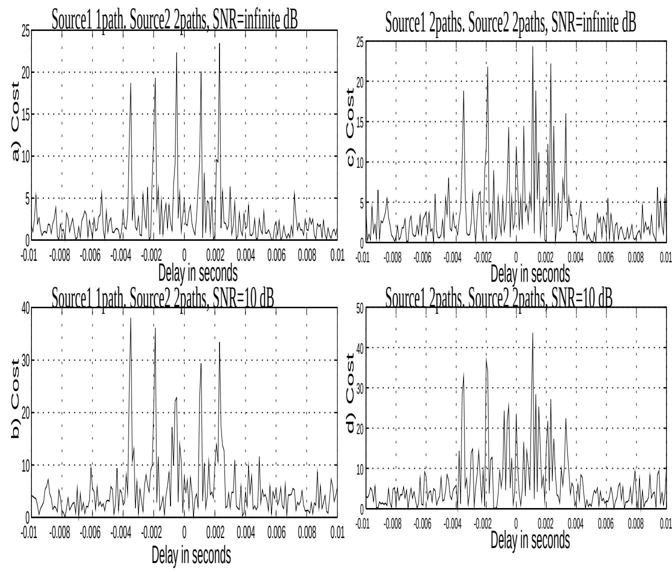

Fig. 1. Cost function for various cases with multipath. a) 2 sources, one with single path and other with 2 paths. No noise case. b) Same as in 'a)' but with 10 dB SNR at the sensor. c) 2 sources each two paths, and no noise added. d) Same as 'c)' but with 10dB SNR at the sensor.

4. SIMULATION RESULTS

The simulation results are presented to demonstrate the DOA and delay estimation, and delay partitioning in the case of ARMA sources. Time averages are used. In all cases 500 trials were used to get the statistics. In case of DOA estimation more that 2 degrees of deviation from the correct estimate was considered as outlier. In case of delays, a deviation of $0.00005sec$ was considered as an outlier. All the examples below use 2 sensors. The sources are ARMA sources with parameters $A = [1.0000\ -1.2840\ 0.4829\ -0.4829\ -0.2929]$ and $B = [0.02929\ 0\ -0.5858\ 0\ 0.2929]$ designed as bandpass filters with normalized cutoff frequencies 0.1 and 0.3 in the digital domain. The simulations were done assuming acoustic signals, traveling at velocity $330m/sec$ and sampled at a rate of $8000Hz$. The sensor spacing was 24 times the wavelength at the sampling frequency. 1000 samples were used for processing in each trial. The LPC model order used for whitening was 12. SNR (Signal to Noise Ratio) specifications are at the sensor with respect to one source signal.

Experiment 1. Multipath: Performance of Delay Estimation. In this experiment the multipath case is examined for the performance of delay estimation. There are two sources each having two multipaths. Hence $J = 8$. The DOAs and the delays of the paths in the order, [direct path of source 1, indirect path of source1 direct path of source 2, indirect path of source 2] are, $[-50\ -20\ 30\ 60]$ degrees and $[0\ -0.003\ 0\ 0.001]$ seconds.

Table(1) shows the outliers, variance and mean of the various relative delay estimates. It is evident from the table that the variance is low if the estimate is within the outlier limits. The number of outlier decreases with SNR.

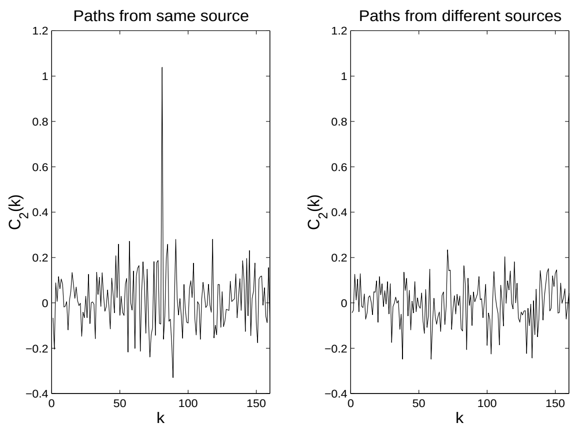

Fig. 2. The Comparison of cost function $C_2(k)$ when the paths under consideration are from the same source, (figure (2.a), and when they are from different sources. figure(2.b)

Experiment 2: Multipath: Delay Partitioning Performance. This experiment evaluates the partitioning of the relative delays. The DOAs and the delays of the paths in the order, [direct path of source 1, indirect path of source1 direct path of source 2] are, $[-40\ \ -10\ 50]$ degrees and $[0\ \ -0.003\ 0]$ seconds. It is assumed that all paths have equal energy with respect to which SNR is calculated.

Table(2) gives the performance. The performance of delay estimation is given assuming the estimation to be correct if the estimates are within the outlier margin. This is justified from the previous experiment where it is seen that, as long as the estimate is within this range the variance is low. The performance of partitioning is given in terms of the number of correct partitioning done. It is clear from the table that the algorithm performs well in the high SNR scenario.

Experiment 3. No multipath: DOA Estimation. The sources were simulated as arriving at the sensors from angles -10, -2 and +50 degrees from the broadside of the two element array. Let λ be the wavelength at the sampling frequency. This experiment was done at sensor spacing of 24λ. To evaluate the performance of the DOA estimation algorithm, the mean, variance and outliers of estimates of DOA for various SNRs are given for all DOAs, in table (3). It is evident from the table that the variance is low if the estimate is within the outlier limits. The number of outliers decreases with increase in SNR.

Effect of sensor spacing: The experiment was done for two different sensor spacings 24λ and 8λ to show the effect of sensor spacings. Two closely placed peaks at -2 and -10 which were resolvable at a spacing of 24λ becomes unresolvable at 8λ. Hence larger the sensor spacing, better is the resolution.

5. CONCLUSION

This paper presents a new method which uses only *two sensors* for the estimation of

- relative delays between multipaths and partition them
- DOA of more than two sources for no multipath case. into subsets each belonging to one source for the multipath case.

when the sources are broadband. There is no restriction on the sources to be non-gaussian. This information can be used for spatio-temporal filter design. The method works well when noise level is low. Sensitivity to various modeling errors and thresholds are being studied.

6. REFERENCES

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Appendix: *Proof of Theorem 1:* Consider

$$\begin{aligned} C &= E\{(z_1z_2 - E\{z_1z_2\})(z_3z_4 - E\{z_3z_4\})\} && (15)\\ &= E\{z_1z_2z_3z_4 - z_1z_2E\{z_3z_4\} - && (16)\\ &\quad z_3z_4E\{z_1z_2\} + E\{z_1z_2\}E\{z_3z_4\}\} && (17) \end{aligned}$$

Substituting for z_1, z_2, z_3 and z_4 from (8)

$$C = E\{x_1^2x_2^2\} - \sigma_{x_1}^2\sigma_{x_2}^2 \qquad (18)$$

Suppose x_1 is correlated with x_2. Then they can be written as $x_1 = x_c + x_{u1}$ and $x_2 = x_c + x_{u2}$ respectively. Here x_c is the correlated component and, x_{u1} and x_{u2} are the uncorrelated components. Then

$$\begin{aligned} C &= E\{(x_c + x_{u1})^2(x_c + x_{u2})^2\} - \sigma_{x_1}^2\sigma_{x_2}^2\\ &= E\{x_c^4 + x_{u1}^2x_c^2 + 2x_c^3x_{u1} + x_c^2x_{u2}^2 +\\ &\quad x_{u1}^2x_{u2}^2 + 2x_cx_{u1}x_{u2}^2 + 2x_c^3x_{u2} +\\ &\quad 4x_c^2x_{u1}x_{u2}\} - \sigma_{x_1}^2\sigma_{x_2}^2 && (19)\\ &= E\{x_c^4\} + \sigma_c^2\sigma_{u1}^2 + \sigma_c^2\sigma_{u2}^2 +\\ &\quad \sigma_{u1}^2\sigma_{u2}^2 - \sigma_{x_1}^2\sigma_{x_2}^2 && (20)\\ \sigma_{xi}^2 &= \sigma_c^2 + \sigma_{ui}^2 \qquad i = 1,2 && (21) \end{aligned}$$

Substituting for σ_{xi}^2 in (20)

$$\begin{aligned} C &= E\{x_c^4\} - \sigma_c^4 && (22)\\ &\geq 0 \qquad \text{by Jensens inequality} && (23) \end{aligned}$$

with equality if $\sigma_c^2 = 0$

SNR at the sensor.	delay=-0.0023 sec			delay=-0.0010 sec			delay=0.0015 sec		
	mean	var.	outl.	mean	var.	out.	mean	var.	out.
No noise	-0.0023	$1.696 * 10^{-34}$	0	-0.0010	$2.000 * 10^{-11}$	0	0.0015	$2.442 * 10^{-34}$	19
0 dB	-0.0023	$1.696 * 10^{-34}$	0	-0.0010	$5.297 * 10^{-10}$	0	0.0015	$6.453 * 10^{-35}$	99
-10 dB	-0.0023	$1.585 * 10^{-34}$	7	-0.0010	$9.983 * 10^{-10}$	1	0.0015	$4.244 * 10^{-37}$	163

SNR at the sensor.	delay=0.0026 sec			delay=0.0007 sec			delay=0.0017 sec		
	mean	var.	outl.	mean	var.	out.	mean	var.	out.
No noise	0.0026	$3.818 * 10^{-34}$	128	0.0007	$7.361 * 10^{-36}$	37	0.0017	$4.82 * 10^{-35}$	0
0 dB	0.0026	$4.165 * 10^{-34}$	116	0.0007	$4.291 * 10^{-1}$	267	0.0017	$5.748 * 10^{-10}$	107
-10 dB	0.0026	$3.990 * 10^{-34}$	126	0.0007	$1.604 * 10^{-10}$	315	0.0017	$1.349 * 10^{-09}$	194

SNR at the sensor.	delay=-0.004 sec			delay=0.0036 sec		
	mean	var.	outl.	mean	var.	out.
No noise	-0.004	$6.784 * 10^{-36}$	0	0.0036	$1.088 * 10^{-33}$	0
0 dB	-0.004	$6.785 * 10^{-36}$	34	0.0036	$1.060 * 10^{-33}$	18
-10 dB	-0.0040	$6.786 * 10^{-36}$	74	0.0036	$9.502 * 10^{-34}$	65

Table 1. The sample mean, variance (var.) and outliers (out.) based on 500 trials of delay estimates for various SNRs, for experiment 1

SNR at the sensor.	Delays	Delays and segregation
No noise	423	331
10dB	264	241

Table 2. Number of correct estimates for 500 trials, of relative delays and number of correct estimates of segregation along with delay estimation in experiment 2. Each estimate being accurate to 4 decimal places.

SNR at the sensor.	DOA=-2 deg.			DOA=-10 deg.			DOA=50 deg.		
	mean	var.	out.	mean	var.	out.	mean	var.	out.
No noise	-1.910	$6.197 * 10^{-28}$	0	-9.5941	0	0	50.0555	$4.254 * 10^{-26}$	0
10 dB	-1.910	$6.197 * 10^{-26}$	0	-9.594	0	0	50.055	$4.254 * 10^{-26}$	0
0 dB	-1.910	0.044	0	-9.6174	0.0448	0	50.055	$4.254 * 10^{-26}$	1
-10 dB	-1.871	0.073	12	-9.724	0.236	23	50.055	$4.254 * 10^{-26}$	14

Table 3. The sample mean, variance (var.) and outliers (out.) based on 500 trials of DOA estimates for various SNRs, for experiment 3.