

perceptron is identical to the threshold logic gate. Consequently, all properties satisfied by a threshold gate will also apply to binary perceptrons. According to its definition, a threshold gate or a binary perceptron having n inputs x_n, x_{n-1}, \dots, x_1 with n weights w_n, w_{n-1}, \dots, w_1 on the corresponding input lines or connections, realises a Boolean function F , such that

$$\begin{aligned} F &= 1 && \text{when } \sum_{i=1}^n w_i x_i \geq T \\ &= 0 && \text{when } \sum_{i=1}^n w_i x_i < t \end{aligned}$$

where the w_i s, T , and t are real numbers, and \mathbf{x} , can assume only one of the two values 0 and 1. The quantity $\sum w_i x_i$ is called the weighted sum and T and t are the upper and lower bound, respectively, of the threshold. In our case the weights are always integers (both positive and negative), and therefore the gap between the upper and lower bound of threshold is always 1, i.e. $T - t = 1$. The actual value of threshold θ is fixed at the middle of the gap. Hence $8 = T - 0.5$. As is well known, the Boolean function realisable by a single perceptron is a linearly separable (LS) function.^{3,4} The realisation vector (RV) comprising the weight vector (WV) and the threshold θ of an LS function separates the true vertices from the false vertices of an n -dimensional hypercube. In the terminology of switching theory, it separates the on minterms whose outputs are 1s from the off minterms whose outputs are 0s.

A minterm is a product term of n variables. It is known⁵ that all product terms are LS functions. Its realisation vector is given by the following theorem. A parameter of minterms, which we shall require is defined below.

Definition 1: The number of 1s in a minterm expressed in its binary form, where the complemented and the true forms of a variable are denoted by 0 and 1 respectively, is known as its rowsum (RS).

Theorem 1: An n -variable minterm b_n, b_{n-1}, \dots, b_1 can be separated from the rest of the minterms by weight vector w_n, w_{n-1}, \dots, w_1 , where w_i is given by

$$\begin{aligned} w_i &= 1 && \text{if } b_i = 1 \\ &= -1 && \text{if } b_i = 0 \end{aligned}$$

The threshold 8 is given by $\theta = M - 0.5$, where M is the rowsum of the minterm.

Theorem 1 describes a well known result, and its proof can be found in references 3 and 4.

Theorem 2: Tuning theorem: The realisation vector ($w: 0$) realising the single n -variable minterm m_k will also realise all minterms having Hamming distance d or less from the minterm m_k if only the threshold θ in the realisation vector is reduced by d , that is, it is made $\theta - d$, the weight vector w remaining unaltered.

Proof: Let the minterm m_k have 1s at bit positions b_i and 0s at bit positions b_j . Therefore by theorem 1, in its realisation vector the corresponding w_i s are 1s and w_j s are -1 s. The weighted sum of m_k is $\sum w_i = \sigma_k$ say. Now, let a new minterm m_p having a Hamming distance of only 1 from m_k , be obtained from m_k , by changing one of its b_j s into 0. Therefore, the weighted sum of the new minterm m_p will be $\sigma_p = \sigma_k - w_j = \sigma_k - 1$. Again let a new minterm m_q , that has a Hamming distance of only 1 from m_k be obtained from m_k by changing one of its b_j s into 1. The weighted sum of the new minterm m_q will thus be $\sigma_q = \sigma_k + w_j = \sigma_k + 1$.

Therefore, irrespective of how the new minterm of Hamming distance 1 is obtained, its weighted sum will be $\sigma_k \pm 1$. This minterm can be realised by reducing the threshold θ by 1. Extending the argument it can be seen that the weighted sum of a minterm that has a Hamming distance d from m_k , can be realised by reducing θ by d . Obviously, such a reduced threshold will also realise all minterms having Hamming distance less than d . **QED**

TUNING CAPABILITY OF BINARY PERCEPTRON

Indexing term: Neural networks

A theorem called the tuning theorem, which brings out the tuning capability of perceptrons where both the inputs and outputs are binary, has been stated and proved. It has been shown how with the help of this property, an associative memory has been developed with a single layer of perceptrons working as a massively parallel machine, rather than as a fully connected iterative Hopfield network.

In a perceptron as defined by Rosenblatt,¹ and Minsky and Papert,² the output is binary but the input may be binary or continuous. If we restrict the inputs to be only binary valued, we obtain what may be called a binary perceptron. Such a

It is now evident that a perceptron acts as a filter whose selectivity can be tuned to accept all minterms up to a Hamming distance d by adjusting its threshold. Taking this significant property and also the coding capability offered by theorem 1 as the basis, we have developed an associative memory (AM) with binary perceptrons. The AM is to recognise the correct identity of a pattern corrupted by binary noise. It stores the patterns of 26 alphabets represented on a 9×9 grid as has been done by Michel *et al.*⁵ We use 26 neurons (binary perceptrons) each having 81 weights to code each of the alphabets. Note that the 81 bit code word of an alphabet can be thought of as a minterm of an 81-variable Boolean function. It can therefore be stored with 81 weights of a neuron as given by theorem 1. The AM has 81 inputs corresponding to each bit. The 81 inputs are connected in a parallel manner to all 26 neurons, each of which stores a distinct alphabet. When a noisy pattern comes at the input, each neuron simultaneously computes its weighted sum. Each weighted sum at a neuron falls short by d from its threshold, where d is the Hamming distance of the input pattern from the stored pattern. The stored pattern has been fixed as its weights once for all when the neuron learnt the code of the alphabet. Now, a controller network connected from, and to, all the individual neurons decreases the threshold of all the neurons simultaneously in steps of 1. As soon as the decrement becomes equal to the least d , the particular neuron which is obviously closest to the pattern fires. The firing of a neuron inhibits the controller net from decreasing the thresholds further. We have simulated the working of the AM. When the AM is fed with the noisy input pattern of M as shown in Fig. 1b (taken from Fig. 3 of Reference 5) the neuron M fires. For neuron M, the d is 19, whereas the d s for other neurons having very similar pattern such as N, W and H were 22, 27 and 26, respectively.

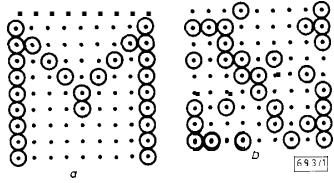


Fig. 1 *Stored pattern and noisy (binary) input pattern of alphabet M in 9 \times 9 grid*

a) Stored pattern
b) Noisy (binary) input pattern

The AM as developed by us has many advantages over those of Michel *et al.*⁵ and many other similar models which are variations of the Hopfield network.⁶ The learning which amounts to the fixing of weights according to theorem 1 is very simple. Any number of characters can be stored, the number of neurons being equal to the number of characters. The grid can be increased to improve resolution as it increases only the number of connections to each neuron, rather than increasing the number of neurons. All stored patterns are stable, and the AM is guaranteed to recognise the closest pattern. In case the noisy pattern is equidistant from more than one stored character, more than one neuron indicating the situation fire simultaneously.

References

- 1 ROSENBLATT, F.: 'The perceptron: A probabilistic model for information storage and organization in the brain', *Psychological Review*, 1958, **65**, pp. 386-408
- 2 MINSKY, M., and PAPERT, S.: 'Perceptrons' (Cambridge, MA, MIT Press, 1988), expanded edition
- 3 LEWIS, P. M., and COATES, C. L.: 'Threshold logic' (New York, Wiley, 1967)
- 4 BISWAS, N. N.: 'Introduction to logic and switching theory' (New York, Gordon and Breach, 1975)
- 5 MICHEL, A. N., SI, J., and YEN, G.: 'Analysis and synthesis of a class of discrete-time neural networks described on hypercubes', *IEEE Trans.*, 1991, **N-2**, (1), pp. 32-46
- 6 HOPFIELD, J. J.: 'Neural networks and physical systems with emergent collective computational abilities', *Proc. National Academy of Science USA*, 1982, **79**, pp. 2554-2558