Current density on cathode surface of MPD thrusters

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Abstract: An MPD thruster formulation involving coupled aerothermodynamic-electromagnetic equations and including viscous effects is developed and solved. The effect of various input parameters on the radial current density distribution at the longitudinal cathode surface is studied. In general, radial current concentrations are found at the upstream and downstream ends of the cathode. The upstream current concentration is larger in magnitude and is attributed to large values of the Hall parameter. Higher inlet velocities and densities, lower inlet temperatures and propellants of lower molecular weight, all reduce the upstream current concentration. The downstream current concentration is attributed to large values of the inlet velocity.

1 Introduction

The magnetoplasmadynamic (MPD) thruster is an electric propulsion device with potential for application in space missions requiring high specific impulse. A typical thruster geometry is shown in Fig. 1. Propellant is

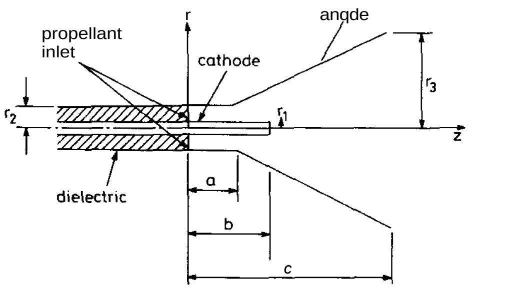

Fig. 1 *MPD thruster geometry*

injected through inlet slots in the dielectric on the left and is accelerated in the thruster, predominantly by means of the electromagnetic $J \times B$ body force. B is self-induced by J in the MPD thruster. The study of radial current density distribution on the longitudinal cathode surface is important because a relatively uniform current density distribution suppresses Joule heating loss, improves thrust efficiency and reduces cathode erosion [1]. In previous work, the author has studied the effect of plasma temperature on the current density profile [2], studied the electromagnetic force density distribution [3], and studied the phenomena of onset [4]. In the above References the aerothermodynamic field was assumed known and the electromagnetic equations were solved to obtain results.

In this paper a major generalisation of the above procedure is brought about by simultaneous solution of the coupled aerothermodynamic-electromagnetic equations.

2 Problem geometry, governing equations, solution procedure

The problem geometry is illustrated in Fig. 1. The details of the electromagnetic equations used and associated boundary conditions are given in Reference 2. The aerothermodynamic equations considered are the axial momentum equation, the radial momentum equation, the continuity equation and the state equation.

Axial momentum equation

$$\rho V_Z \frac{\partial V_Z}{\partial Z} = -\frac{\partial P}{\partial Z} + J_r B_\theta + \mu\left[\frac{4}{3}\frac{\partial^2 V_Z}{\partial Z^2} + \frac{\partial^2 V_Z}{\partial r^2} + \frac{1}{r}\frac{\partial V_Z}{\partial r}\right] \quad (1)$$

subject to:

$V_Z = 0$ on solid surfaces

$V_Z =$ specified inlet value (V_{in}) at $Z = 0, r_1 < r < r_2$

$\frac{\partial V_Z}{\partial Z} = 0$ at $Z = C, 0 \leqslant r < r_3$

where ρ = plasma density, V_Z = plasma axial velocity, Z = axial co-ordinate, P = pressure, J_r = radial current density, B_θ = azimuthal magnetic induction field, μ = plasma viscosity, r = radial co-ordinate.

The first condition follows from the inclusion of viscosity in eqn 1. The inlet velocity V_{in} will have a profile between r_1 and r_2 but here it is assumed constant. The last condition is based on the assumption that propellant acceleration is very small at the exit. The developed solution software, however, permits arbitrary profiles of V_{in} and the replacement of the third condition by any other more realistic assumption.

Radial momentum equation

$$\rho V_Z \frac{\partial V_r}{\partial Z} = -\frac{\partial P}{\partial r} - J_Z B_\theta + \frac{\mu}{3}\frac{\partial^2 V_Z}{\partial Z\,\partial r} + \mu\left[\frac{4}{3}\frac{\partial^2 V_r}{\partial r^2} + \frac{4}{3}\frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{\partial^2 V_r}{\partial Z^2} - \frac{4}{3}\frac{V_r}{r^2}\right] \quad (2)$$