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Surface Roughness Dependence of Laser Induced Damage Threshold

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Abstract—A functional relation between the rms roughness of a rough surface and the laser induced damage threshold is found. In deriving this relation it is assumed that the effective exposed area of the rough surface plays a dominant role in the damage mechanism. It is shown that the existing emperical relation between the rms roughness of a rough surface and the surface damage threshold could be derived from this functional relation under certain conditions.

I. Introduction

RECENT experimental evidence suggests that the damage threshold of optical surfaces is related to the surface roughness by an empirical relation of the form [1]

$$E\sigma^m = \text{constant} \tag{1}$$

where E is the threshold electric field and σ is the rms roughness of the surface. In general, the exponent m in (1) is observed to be around 0.5 [1]. However, the actual functional relation between the damage threshold and rms roughness of the surface is not yet established.

We demonstrate here that a functional relationship between the damage threshold and rms roughness of the surface could be derived. For this purpose we assume that the damage threshold of a rough surface is dependent upon the effective exposed area. The actual rough surface is represented by peaks and valleys which are randomly distributed over the surface and the actual area of such a surface is naturally greater than the geometrical area. In view of these facts the real surface area of a rough surface can be considered to be a random variable and its mean value should represent the effective area which depends on the rms roughness of the surface.

The assumption that the surface damage threshold depends on the effective exposed area of the surface is quite a reasonable one, because the number of defects and impurity levels over the surface will increase with the effective exposed area. As a result of this, the surface damage threshold decreases. Usually any other contribution to the surface damage threshold is masked by the roughness dependence of the damage threshold. So the ascertaining of the dependence of damage threshold on the roughness is an important endeavor.

In the next section the effective area of a rough surface is calculated which is then related to the surface damage threshold in Section III.

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II. Effective Area Calculation

If a surface is bounded by a line τ and is defined by the equation $Z = Z(x, y)$, then the area of the surface is given by

$$\eta = \iint_A \left[1 + \left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]^{1/2} dx\, dy \tag{2}$$

where A is the region on X–Y plane bounded by the line L which is the projection of the line τ on to the X–Y plane [2].

For a rough surface, the height $Z(x, y)$ and the surface gradient $[(\partial Z/\partial x)^2 + (\partial Z/\partial y)^2]^{1/2}$ are random variables. The distribution of values of these variables are referred to the mean plane of the surface. If this is taken as the X–Y plane, then the region of integration A in (2) is nothing but the geometrical area of the surface. Equation (2) involves a random variable integration. However if the mean value of η is taken as the effective area of a rough surface, then

$$A_{\text{eff}} = \langle \eta \rangle = \iint_A \left\langle \left[1 + \left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]^{1/2} \right\rangle dx\, dy. \tag{3}$$

Hence the effective area calculation amounts to calculating the mean value of the random variable

$$X = \left[1 + \left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]^{1/2} \tag{4}$$

which is done in the following analysis.

Treating the rough surface as an isotropic Gaussian random process in two dimensions, the probability density for surface gradients is given by [3]

$$f(a) = m_2^{-1} a \exp\,[-a^2/(2m_2)]\; U(a) \tag{5}$$

where

$$a = \left[\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]^{1/2} \tag{6}$$

is the surface gradient which is a random variable for a rough surface. $U(a)$ in (5) is a unit step function given by

$$U(a) = 1 \quad \text{for } a \geqslant 0$$
$$= 0 \quad \text{for } a < 0. \tag{7}$$

m_2 is one of the moments of the profile of the surface in an arbitrary direction and its value is given by [3]

$$m_2 = \Pi^2 \sigma^2 D_0^2 \tag{8}$$

where D_0 is the density of zero crossings of the profile over its mean line.

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To find the effective area of the rough surface, it is required to find the probability density of the random variable X of (4). The probability density of X, using (4), is calculated to be

$$f(X) = m_2^{-1} X \exp [1/(2m_2)] \exp [-X^2/(2m_2)] \; U_1(X) \quad (9)$$

where $U_1(X)$ is a unit step function given by

$$U_1(X) = 1 \quad \text{for } X \geqslant 1$$
$$= 0 \quad \text{for } X < 1. \quad (10)$$

The mean value of X is given by

$$\langle X \rangle = \int_{-\infty}^{\infty} X' f(X')\, dX'. \quad (11)$$

From (9) and (11) we have

$$\langle X \rangle = m_2^{-1} \exp [1/(2m_2)] \cdot \int_1^{\infty} X' \exp [-X'^2/(2m_2)] \; dX'. \quad (12)$$

The above integral can be evaluated analytically to give

$$\langle X \rangle = 1 + (\pi/4)^{1/2} (2m_2)^{1/2} \exp [1/(2m_2)] \cdot \text{erfc} [(2m_2)^{-1/2}] \quad (13)$$

where erfc $[(2m_2)^{-1/2}]$ is the complementary error function of $(2m_2)^{-1/2}$ which is defined by

$$\text{erfc} [(2m_2)^{-1/2}] = (4/\pi)^{1/2} \int_{(1/2m_2)^{1/2}}^{\infty} \exp(-t^2)\, dt. \quad (14)$$

From (3) and (13), the effective area is given by

$$A_{\text{eff}} = A \; [1 + (\pi/4)^{1/2} (2m_2)^{1/2} \exp \{1/(2m_2)\} \cdot \text{erfc} \{(2m_2)^{-1/2}\}]. \quad (15)$$

Let

$$K = (2m_2)^{1/2} \quad (16)$$

and

$$g(K) = (\pi/4)^{1/2} K \exp (K^{-2}) \,\text{erfc}\, (1/K). \quad (17)$$

Then

$$A_{\text{eff}} = A \; [1 + g(K)]. \quad (18)$$

The function $g(K)$ is plotted in Fig. 1. For values of $K < 1$, the function $\exp(K^{-2})$ erfc $(1/K)$ varies linearly with K as shown in Fig. 2. Hence for small values of K, the effective area varies as σ^2 if D_0 is taken as a constant. Thus

$$A_{\text{eff}} \propto \sigma^2. \quad (19)$$

III. Damage Threshold Relation

It is shown by Bettis *et al.* [4] that the surface damage threshold is proportional to $A^{-1/4}$ where A is the area of the incident laser beam. Since for a rough surface $A_{\text{eff}} > A$, the surface damage threshold is expected to vary as $A_{\text{eff}}^{-1/4}$. For small values of K we have from (19)

$$E_{\text{th}} \propto \sigma^{-1/2}$$

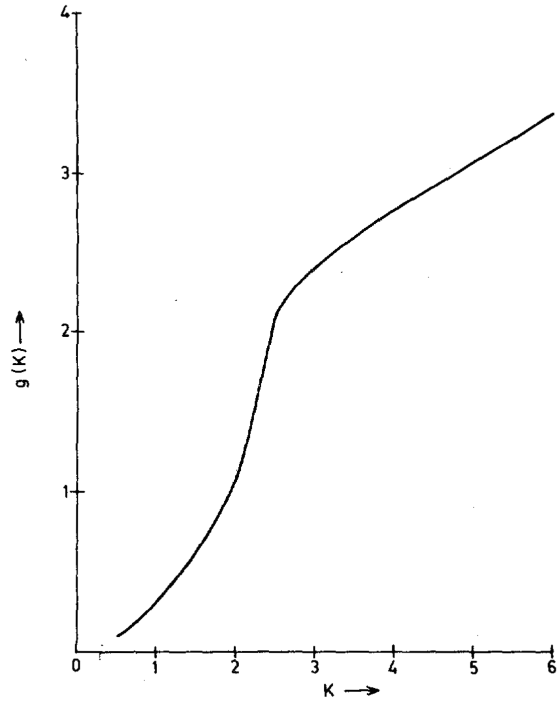

Fig. 1. The functions $g(K)$ in (17) is plotted against K.

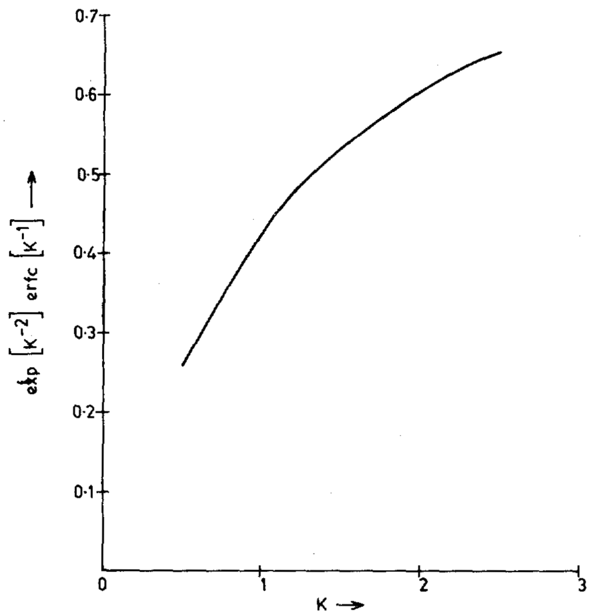

Fig. 2. The function $\exp(K^{-2})$ erfc (K^{-1}) is plotted against K.

which is nothing but (1) with $m = \frac{1}{2}$. Thus the above derivation explains the $\sigma^{-1/2}$ variation of the surface damage threshold.

In the experimental investigation of surface damage threshold dependence on roughness [1], it is observed that the etched samples do not obey (1). This may be due to the variation of D_0 for the samples studied. To elaborate this point a little further for clarity we note that D_0 and σ would be expected to vary under the following conditions: 1) Rough surfaces are prepared using varying grit sizes for the final polishing compound in the conventional polishing process; 2) rough surfaces are prepared by etching the samples after conventional polishing process. Under these circumstances we would expect (since these circumstances imply the variation of D_0 in our analysis) the scaling law (1) to be inoperative. Whereas it is entirely reasonable to expect the spatial frequency D_0 to remain constant while the rms roughness changes when the surfaces are prepared using the same grit size for the final polishing compound, but the various samples are obtained by varying the polishing times. This is what was done by House *et al.* [1] in their experiments. It means, according to our analysis, D_0 is constant and hence the validity of our conclusions in regard to the applicability of the scaling law (1) for the damage threshold is shown.

Thus wherever the effective area of a rough surface plays a dominant role, the functional relation between the laser induced surface damage threshold and the rms roughness of the surface is given by

$$E_{th} = C_1 E_b \left[1 + \pi^{3/2}/2\,(\sigma D_0) \exp\{1/(2\pi^2\sigma^2 D_0^2)\} \cdot \operatorname{erfc}\{1/(\sqrt{2}\pi\sigma D_0)\}\right]^{-1/4} + C_2 \quad (20)$$

where E_b is the bulk damage threshold and C_1 and C_2 are constants. Any other contribution to the surface damage threshold other than the effective area can be taken into the constant C_2. Thus if all the parameters of the laser beam are kept constant, (20) gives a functional relation between E_{th} and σ.

IV. Conclusions

It is shown by analysis that a functional relation between laser damage threshold of an optical surface and the effective area of the surface could be established by considering the surface as a random Gaussian process in two dimensions. This involves the determination of a parameter m_2 which is the second-order moment of the power spectral density of the profile of the surface in an arbitrary direction. Experimentally m_2 could be obtained with a profilometer.

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Interference of an AlGaAs Laser Diode Using a 4.15 km Single Mode Fiber Cable

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Abstract—**Polarization characteristics in cabled single mode fibers were studied. By using 4.15 km long fibers and a single frequency AlGaAs double-heterostructure laser, interference fringes were observed.**

POLARIZATION characteristics of single mode fibers have been studied by several authors [1]-[4]. It was confirmed, by using an Nd:YAG laser, that the linear polarization state can be maintained in a 240 m long fiber [2]. It was also reported that, by using a short fiber, there are specific orientations of input polarization for which linear polarization is observed for all lengths [3]. It was derived theoretically that polarization does not change, even due to bending, only in the HE_{1m} ($m = 1, 2, \cdots$) mode [5]. When the polarization is maintained during fiber transmission, polarization dependent guided wave circuitry can be used at the receiver. Linear output polarization is also essential for polarization division multiplexing and for heterodyne detection.

This letter reports polarization and interference properties using long single mode fibers with AlGaAs double-heterostructure semiconductor laser as a light source. Ahearn *et al.* measured coherence of a CW GaAs laser cooled to 77 K by means of homodyne detection after 1 km length propagation [6]. They obtained spectral bandwidth of 150 kHz, while theoretical linewidth of a semiconductor laser is estimated to be 850 kHz [7]. Iida *et al.* studied Michelson interferometry of a single frequency AlGaAs laser [8].

The fiber cable used in the present experiment has the following features. The single mode silica fiber has nearly step

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