WAVESHAPING FILTERS FOR SPECTRAL ESTIMATION OF SHORT SEGMENTS OF EEG SIGNALS

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Least squares waveshaping filters have been used to find the position of energy concentration in an EEG signal. It is shown that this knowledge can be used in obtaining a better spectral estimation of short segments of EEG data. The proposed method is shown to perform better than the conventional AR method.

Introduction: It has long been observed that the frequency content of an electroencephalogram (EEG), the manifestation of the brain's electrical activity, is of crucial importance in its assessment. Spectral analysis of the EEG has usually been performed on relatively long epochs or has been based on the spectral average of many epochs. However, in some applications [1, 23], it is desirable to analyse single trial, short segments because the spectral features may be changing too rapidly to be adequately studied using long or averaged segments. Conventional FFT-based estimates can suffer significant distortion due to the effects of leakage, particularly in the short segment case. Better resolution can be obtained by applying autoregressive (AR) modelling [2]. It is desirable to divide the EEG into very short segments to be able to adequately characterise the temporally rapid changes that occur. Correct estimation of these characteristics is of paramount importance because the remaining analysis is based on it. In this Letter, we suggest a method which works better than the normal AR spectral estimation method for very short segments of EEG data.

Method: A signal is said to be minimum phase if its Fourier transform is of minimum phase lag. The energy in a minimum

phase sequence will be concentrated around $n = 0$ (beginning), whereas the energy in the maximum phase sequence will be concentrated around the final point. For a mixed phase sequence, the energy will be concentrated between the initial and final points [3]. To determine the position where the energy in the signal is concentrated, waveshaping filters can be used [4]. The problem may be formulated as follows: Given a record of available data $x_n = \{x_0, x_1, x_2, \dots, x_N\}$ we want to find the best linear FIR filter with unit sample response h_n which reshapes x_n into the desired signal y_n . To do this, we first define a data matrix X , a vector of filter weights h and a vector of desired samples y as

$$X = \begin{bmatrix} x_0 & & & & 0 \\ x_0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N-1} & x_{N-2} & \dots & x_{N-M} \\ x_N & x_{N-1} & x_{N-1} & \dots & x_{N-M+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & x_N \\ & & & \dots & x_N \end{bmatrix} \quad (1)$$

$$h = [h_0 \ h_1 \ h_2 \ \dots \ h_M]^T \quad (2)$$

$$y = [y_0 \ y_1 \ y_2 \ \dots \ y_{N+M}]^T \quad (3)$$

where T denotes the transpose. The estimate of the output of the filter $\hat{y}$, the estimation error e and the total error E may then be written as

$$\hat{y} = Xh \quad e = y - \hat{y} \quad E = e^T e \quad (4)$$

It can be shown that $\hat{y} = Py$ and $E = y^T(I - P)y$ where $P = X(X^T X)^{-1} X^T$. The performance matrix P has a rank $M + 1$ and if the desired signal y is selected to be any of the $M + 1$ eigenvectors belonging to eigenvalue 1, the corresponding estimation error will be zero.

Among all possible waveshapes for y , the most important ones are spikes or impulses. In this case, the waveshaping filter is also known as a spiking filter. Here $y = u_i$ is an impulse at a delay equal to i . The actual output of the i th spiking filter is $\hat{y} = Py = Pu_i$ which is nothing but the i th column of P . The estimation error of the i th spiking filter is $E_i = u_i^T(I - P)u_i = 1 - P_{ii}$ where P_{ii} is the i th diagonal element of P . Thus, the optimum delay ($i = n_0$) is selected as that column of P which has the maximum of P_{ii} . This optimum spiking delay gives the position around which the energy in the signal is concentrated.

Spectral estimation: Any function $x(n)$ can be expressed as the convolution of a minimum phase function $x_{1min}(n)$ and an allpass function $x_{2ap}(n)$. An AR filter tries to approximate the minimum phase function and the approximation will be better if $x(n)$ itself is minimum phase. Any mixed phase function can be written as the convolution of a minimum phase function $x_{1min}(n)$ and a maximum phase function $x_{2max}(n)$. Because $x_{2max}(n) = x_{2min}(n_0 - n)$ where n_0 is the length of the maximum delay component, we can write in the z domain

$$X(z) = X_{1min}(z) \cdot X_{2min}(z^{-1})z^{-n_0} \quad (5)$$

It is this delay n_0 that we are trying to determine using the spiking filter. Knowledge of the spiking position n_0 gives us information as to how much shift is needed to make the original mixed phase signal a near-minimum-phase signal. By approximating the resulting shifted signal by an AR model, we can obtain an improved estimate of the spectrum.

Results and discussion: Application of the spiking position theory to EEG data showed that, for most of the segments of an EEG, the optimum delay position is somewhere between the initial and final positions, indicating that the EEG is a mixed delay signal (Fig. 1). Our studies have shown that knowledge of the optimum spiking position can be efficiently used in spectral estimation. For very short segments of data, we advanced the signal by the amount of spiking delay and

found the spectrum using the AR method. It was found that the spectrum of the shifted sequence gives better results.

Fig. 2 shows a comparison of spectra obtained by the two methods for simulated EEG data. The simulated data contain three sinusoids of frequencies 10, 14 and 20 Hz (with relative amplitudes 2, 5 and 0.5, respectively) in white noise. The signal was sampled at 100 Hz and a data segment of 50 samples (corresponding to 0.5 s of data) was taken for analysis. The optimum spiking delay position was found to be 15. By advancing the data sequence by 15 samples, the AR spectrum was determined (with 12 AR parameters). It is seen from Fig. 2 that the shifted sequence spectra gives better results than the spectrum of the original sequence. Fig. 3 shows similar results for an actual EEG data segment of 64 samples (0.25 s of data). Visual observation of the data

Fig. 1 Performance graph for actual EEG signal

Fig. 2 Comparison of spectra for simulated EEG data

- (i) Spectrum of shifted sequence
- (ii) Spectrum of original sequence

Fig. 3 Comparison of spectra for real EEG data

- (i) Spectrum of shifted sequence
- (ii) Spectrum of original sequence

segment showed alpha activity predominantly, superimposed on power hum. The spiking delay position was found to be 60. Comparison of curves (i) and (ii) in Fig. 3 shows that the proposed method gives better results.

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