$$D(s)=\begin{bmatrix} s^2+3s+2 & 0 \\ 0 & s^2+7s+12 \end{bmatrix}.$$

Let

$$F=\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}.$$

$$D(s)+FN(s)=\begin{bmatrix} (f_{11}-f_{12}+1)s^2+(2f_{11}-f_{12}+3)s+(f_{11}+2f_{12}+2) & (2f_{11}-2f_{12})s^2+(9f_{11}-5f_{12})s+(4f_{11}+3f_{12}) \\ (f_{21}-f_{22})s^2+(2f_{21}-f_{22})s+(f_{21}+2f_{22}) & (2f_{21}-2f_{22}+1)s^2+(9f_{21}-5f_{22}+7)s+(4f_{21}+3f_{22}+12) \end{bmatrix}$$

which is column proper unless

$$f_{11}-f_{12}+2f_{21}-2f_{22}+1=0. \tag{4}$$

(Note that (4) is satisfied by $F=I$ as was the case in the original paper.) Since F generically does not satisfy (4), in Theorem 3 we may take $U=I$. Then, the conditions of Theorem 3 are satisfied unless either

$$\begin{aligned} f_{11}-f_{12}+1&=0 \\ f_{21}-f_{22}&=0 \end{aligned} \tag{5}$$

or

$$2f_{11}-2f_{12}=0$$

or

$$2f_{21}-2f_{22}+1=0 \tag{6}$$

hold. So the conditions of Theorem 3 hold generically and so for the $W(s)$ defined in (3), $W(s)\,[I+FW(s)]^{-1}$ is generically proper.

Conclusion

We have shown that for all $q\times r$ rational transfer function matrices, almost all constant feedback matrices cause the resulting closed-loop transfer function matrix to be proper.

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Author's Reply[2]

M. VIDYASAGAR

I wish to thank Dr. Scott and Dr. Anderson for their interest in my paper. Their comments are correct.

I would also like to point out that in Fact 4[1] $N(s)+KD(s)$ should be changed to $D(s)+KN(s)$ throughout.

[2]Manuscript received March 4, 1976.
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Comments on "Design of Piecewise Constant Gains for Optimal Control via Walsh Functions"

N. GOPALSAMI AND B. L. DEEKSHATULU

The above paper[1] considers the use of Walsh functions in the design of piecewise constant gains for the linear optimal control problem with quadratic performance criteria. The use of Walsh functions is illustrated in the context of solving a vector differential equation. A Walsh series is assumed for each rate variable and the unknown Walsh coefficients are determined by solving the resulting set of simultaneous equations. In this correspondence it is shown that the same solution can be easily obtained by proceeding directly with the piecewise constant approximation for the rate variables.

Consider (15) of Chen and Hsiao's paper:[1]

$$C=ACP+G+BH. \tag{1}$$

Taking the transpose of (1),

$$C'=P'C'A'+G'+H'B', \tag{2}$$

i.e.,

$$[c_1c_2\cdots c_n]=P'[c_1c_2\cdots c_n]A'+G'+[h_1h_2\cdots h_n]B' \tag{3}$$

where c_i represents the Walsh series coefficients of the ith rate variable

$$c_i=[c_{i0},c_{i1},\cdots,c_{i(m-1)}]',$$

and h_i corresponds to the Walsh series coefficients of the ith input u_i,

$$h_i=[h_{i0},h_{i1},\cdots,h_{i(m-1)}]',$$

i.e.,

$$\begin{aligned} \dot{x}_i &\cong c_{i0}\phi_0+c_{i1}\phi_1+\cdots+c_{i(m-1)}\phi_{m-1} \\ u_i &\cong h_{i0}\phi_0+h_{i1}\phi_1+\cdots+h_{i(m-1)}\phi_{m-1}. \end{aligned}$$

It is well known that when a function is approximated by a Walsh series of finite order m (m=some integral power of 2), the approximated function is of piecewise constant form and the magnitude of the step in each subinterval is equal to the average value of the function in that subinterval. If f_{ij} is the average value of $\dot{x}_i$ and g_{ij} is the average value of u_i in the jth subinterval, $j=0,1,\cdots,(m-1)$, then the following relations are true.

$$c_i=\frac{1}{m}Wf_i \tag{4}$$

$$h_i=\frac{1}{m}Wg_i \tag{5}$$

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[1]C. F. Chen and C. H. Hsiao, *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 596–602, Oct. 1975.

where

$$f_i=[f_{i0},f_{i1},\cdots,f_{i(m-1)}]'$$
$$g_i=[g_{i0},g_{i1},\cdots,g_{i(m-1)}]'.$$

W is the $(m\times m)$ Walsh matrix whose (i,j)th element is

$$W_{ij}=\phi_i(j/m)=\pm 1, \qquad i,j,=0,1,\cdots,m-1.$$

Substituting (4) and (5) in (3) we get

$$\frac{1}{m}[Wf_1\ Wf_2\ \cdots\ Wf_n]=P'\frac{1}{m}[Wf_1\ Wf_2\ \cdots\ Wf_n]A' + G'+\frac{1}{m}[Wg_1\ Wg_2\ \cdots\ Wg_n]B' \tag{6}$$

or

$$[f_1\ f_2\ \cdots\ f_n]=W^{-1}P'W[f_1\ f_2\ \cdots\ f_n]A' + mW^{-1}G'+[g_1\ g_2\ \cdots\ g_n]B'. \tag{7}$$

It can be verified that

$$W^{-1}P'W=\frac{1}{m}\Sigma_{(m\times m)}=\frac{1}{m}\begin{bmatrix}\frac{1}{2} & 0 & \cdots & 0 & 0\\ 1 & \frac{1}{2} & \cdots & 0 & 0\\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 1 & 1 & \cdots & 1 & \frac{1}{2}\end{bmatrix}_{(m\times m)} \tag{8}$$

$$mW^{-1}G'=\begin{bmatrix}(Ax_0)'\\(Ax_0)'\\ \cdot\\ \cdot\\ \cdot\\(Ax_0)'\end{bmatrix}_{(m\times n)}. \tag{9}$$

Substituting (8) and (9) in (7) we get

$$[f_1\ f_2\ \cdots\ f_n]=\frac{1}{m}\Sigma[f_1\ f_2\ \cdots\ f_n]A'+\begin{bmatrix}(Ax_0)'\\(Ax_0)'\\ \cdot\\ \cdot\\ \cdot\\(Ax_0)'\end{bmatrix} +[g_1\ g_2\ \cdots\ g_n]B' \tag{10}$$

or

$$\begin{bmatrix}f_1'\\f_2'\\ \cdot\\ \cdot\\ \cdot\\f_n'\end{bmatrix}=\frac{1}{m}A\begin{bmatrix}f_1'\\f_2'\\ \cdot\\ \cdot\\ \cdot\\f_n'\end{bmatrix}\Sigma'+[Ax_0\ Ax_0\ \cdots\ Ax_0]+B\begin{bmatrix}g_1'\\g_2'\\ \cdot\\ \cdot\\ \cdot\\g_n'\end{bmatrix}. \tag{11}$$

Equation (11) can be written in Kronecker product form as

$$\begin{bmatrix}f^0\\f^1\\f^2\\ \cdot\\ \cdot\\f^{m-1}\end{bmatrix}=\frac{1}{m}[A\otimes\Sigma]\begin{bmatrix}f^0\\f^1\\f^2\\ \cdot\\ \cdot\\f^{m-1}\end{bmatrix}+\begin{bmatrix}k^0\\k^1\\k^2\\ \cdot\\ \cdot\\k^{m-1}\end{bmatrix} \tag{12}$$

where

$$f^i=[f_{1i}\ f_{2i}\ \cdots\ f_{ni}]'$$

$$k^i=Ax_0+i\text{th column of } B\begin{bmatrix}g_1'\\g_2'\\ \cdot\\ \cdot\\g_n'\end{bmatrix}.$$

The matrix $[A\otimes\Sigma]_{(mn\times mn)}$ in (12) is a lower triangular matrix and the solution of $f^0, f^1, \cdots f^{m-1}$ can hence be easily obtained.

Knowing $\dot{x}$, x can be obtained by direct integration.

$$\dot{x}'=[\dot{x}_1\ \dot{x}_2\ \cdots\ \dot{x}_n]\cong[f_1\ f_2\ \cdots\ f_n]$$

$$\therefore x'=\int_0^t[f_1\ f_2\ \cdots\ f_n]dt+x_0'\cong\frac{\Sigma}{m}[f_1\ f_2\ \cdots\ f_n]+x_0'. \tag{13}$$

Now consider

$$x=CP\phi+x_0$$

$$x'=\phi'P'C'+x_0'=\phi'P'\frac{W}{m}[f_1\ f_2\ \cdots\ f_n]+x_0'.$$

The values of ϕ' in the m subintervals when put into matrix form give the Walsh matrix W'. Then the values of x' in the m subintervals are given by

$$x'=W'P'\frac{W}{m}[f_1\ f_2\ \cdots\ f_n]+x_0'=\frac{\Sigma}{m}[f_1\ f_2\ \cdots\ f_n]+x_0'. \tag{14}$$

This is the same as in (13). Obviously the direct approach is simpler than that using Walsh functions.

Author's Reply[2]

C. F. CHEN

Gopalsami and Deekshatulu use Chen and Hsiao's operational matrix and the Walsh matrix transformation to derive the Σ matrix (8) which can be considered as the integration operational matrix of the block pulse functions [1].

Chen, Tsay, and Wu [2] recently developed a new technique for solving fractional calculus via the block pulse functions which are shown in Fig. 1.

In application, the block pulse functions approach is somewhat simpler and easier but those functions have to be arranged in the operational matrix form. Harmuth has mentioned the block pulse functions but could not apply them for solving any meaningful problems. Gopalsami and Deekshatulu, on the other hand, do not realize that (8) comes from the integration of block pulse functions. They had to use a similarity transformation on Chen–Hsiao's operational matrix.

[2]Manuscript received February 22, 1976.
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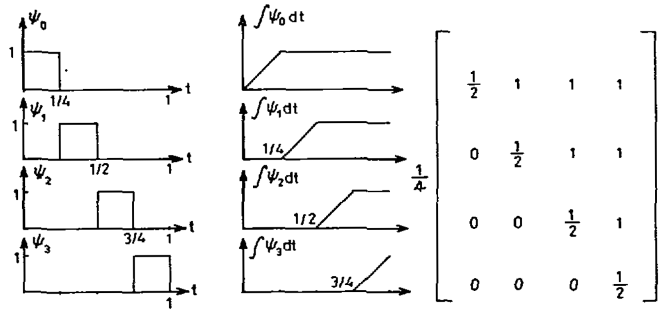

Fig. 1.

In theory, the block pulse functions do not form a complete set while the Walsh functions do. This particular point may be the advantage of the Walsh functions.

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Book Reviews

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Control Systems: Analysis, Design, and Simulation—J. W. Brewer (Englewood Cliffs, NJ: Prentice-Hall, 1974, $18.95). *Reviewed by Joseph J. Bongiorno, Jr.*

Joseph J. Bongiorno, Jr. (S'56–M'60) was born in Brooklyn, NY, on August 3, 1936. He attended the Polytechnic Institute of Brooklyn, Brooklyn, NY, and received the D.E.E. degree in 1960. He currently holds the title of Professor of Electrical Engineering at the Polytechnic Institute of New York. Over the past twenty years his research interests have been in control theory and he has published papers on adaptive systems, stability of time-varying systems, dynamical observers, and frequency-domain analytical design techniques for multivariable systems. He is also a consultant at Sperry Systems Management Division where for many years he has worked on problems related to inertial navigation. Dr. Bongiorno is a member of Eta Kappa Nu and Sigma Xi.

The author has appropriately divided the area of control systems into three parts: dynamic systems analysis and modeling (5 chapters, 154 pages), simulation (2 chapters, 68 pages), and feedback system theory and design (7 chapters, 231 pages). The text also contains a chapter on Laplace transforms (43 pages), 3 Appendixes (37 pages), and an index. Included in the Appendixes are a short table of Laplace transforms and some Fortran programs which can be helpful in design. One Appendix is devoted to a brief review of complex number algebra, the algebra of matrices and vectors, and Taylor series including linearization and Newton iteration. Most chapters contain an ample number of illustrative examples as well as problems to be worked. The text is intended for undergraduate mechanical engineering students.

The chapters on dynamic systems analysis and modeling cover electrical and magnetic circuits; mechanical network systems; hydraulic and pneumatic networks (including linearization); thermal systems and systems with delays; and prime movers, sensors, and feedback compensators. The basic elements which comprise systems and networks of the kind just mentioned are described and modeled in a commendable fashion for a text of this type. The selection of elements and devices treated is also good.

Two-dimensional vectors containing as elements the Laplace transforms of effort and flow variables are used to analyze series-shunt-connected components. Except for thermal systems, the product of the effort and flow variables in each case represents power and the vector of these variables is called the power state vector. In the chapter on thermal systems, temperature and rate of heat flow are chosen. Unfortunately, the author offers no explanation for the preferred use here of the so-called transfer state vector instead of a power state vector. The decision to present a uniform treatment of ladder-type networks, using the power or transfer state vector notion is one I would not have made. It requires each element in the network to be represented by a 2×2 transfer matrix and unnecessarily complicates most of the transfer function computations.

It is disappointing to find loop and nodal analysis omitted from the chapter on electrical and magnetic circuits. Also, the useful notion of the voltage divider is not mentioned and the derivation of several network transfer functions (the lead network for example) is overly complicated as a result. The treatment of the equivalent impedance of two series or two parallel elements is not done well. In fact, the author defines the