process restricted to a subset of the state space. It is well known (e.g., [1, sect. 9]) that if the restricted subset consists only of recurrent states, then the restricted process is a well-defined Markov renewal process. ■

Let $\tilde{P}(v)$ denote the transition probability matrix of the embedded Markov chain.

For a given supervisor strategy v consider the cost process z_t, $t=0,1,\cdots$ defined by

$$z_\tau=\sum_{t=0}^{t-1} k\left(s_\tau, v^{b_t}(s_\tau)\right). \tag{5}$$

We can associate with any β in B and v^β in V^β the expected cost

$$K(\beta, v^\beta) \triangleq E_{v^\beta}\left[z_{T_{n+1}}-z_{T_n} \mid b_{T_n}=\beta\right] \tag{6}$$

and the expected transition time

$$T(\beta, v^\beta) \triangleq E_{v^\beta}\left[T_{n-1}-T_n \mid b_{T_n}=\beta\right]. \tag{7}$$

Under the strong ergodicity assumption both expressions (6) and (7) are well defined. Now consider the mean average cost:

$$Y(v) \triangleq \lim_{T\to\infty}\frac{1}{T+1}E\left[\sum_{t=0}^{T} k\left(s_t, v^{b_t}(s_t)\right)\right]. \tag{8}$$

If this expression is defined, it is also equal by (5) and (6) to

$$\lim_{T\to\infty}\frac{1}{T+1}E[Z(T)] \tag{9}$$

where if $n(t)$ is the number of jumps of b_t up to time t,

$$Z(t) \triangleq \sum_{\nu=0}^{n(t)} K\left(b_{T_\nu}, v^{b_{T_\nu}}\right). \tag{10}$$

It thus results that the supervisor problem is a very standard semi-Markov decision process for which Theorems 7.6 and 7.7 of [2] apply.

Proposition: ≪There exists a bounded function $h(\beta)$, $\beta\in B$ and a constant g such that

1) $$h(\beta)=\min_{v^\beta\in V^\beta}\left\{K(\beta, v^\beta)+\sum_{\beta'\in B}\tilde{P}_{\beta\beta'}(v^\beta)h(\beta')-gT(\beta, v^\beta)\right\}. \tag{11}$$

2) The argument of the minimization in (11) defines a stationary supervisor strategy v^* which minimizes $Y(v)$.

3) $g=Y(v^*)$.≫

Proof: Apply Theorem 7.6 and 7.7 of [2]. ■

III. Conclusion

A numerical solution of (11) is possible by using the successive approximation method of Schweitzer [3] which is not fundamentally different from the algorithm proposed in the paper.[1] The multilayer control approach is extremely promising for the solution of very large stochastic control problems. By relating more explicitly this approach to Markov renewal theory we hope to have prepared the way for further developments, in particular the extension of this scheme to a larger class of processes than Markov chains or one-dimensional diffusion processes (as in [4]).

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Comments on "Decentralization, Stabilization, and Estimation of Large Scale Linear Systems"

S. K. KATTI

***Abstract*—It is shown here that the input decentralization scheme for the large scale linear system presented by Šiljak and Vukcevic does not always result in interconnected subsystems controlled by local (scalar) inputs.**

In the above paper,[1] Šiljak and Vukcevic have proposed an input decentralization scheme which decomposes a large scale system into s number of subsystems that are controlled by distinct inputs (cf. equation 7).[1] This correspondence discusses with counterexample that the scheme of Šiljak and Vukcevic does not always lead to controllable pairs (A_i, b_i).

Example: Consider a system with the following state equations:

$$\dot{x}=\begin{bmatrix}0&1&0&0\\2&0&0&0\\0&0&0&3\\0&0&4&0\end{bmatrix}x+\begin{bmatrix}1&0\\0&1\\1&0\\0&1\end{bmatrix}u. \tag{1}$$

The system of equations may be split into coupled subsystems as follows:

$$\begin{aligned}\dot{z}_1&=\begin{bmatrix}0&1\\2&0\end{bmatrix}z_1+\begin{bmatrix}0&0\\0&0\end{bmatrix}z_2+\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}\\ \dot{z}_2&=\begin{bmatrix}0&3\\4&0\end{bmatrix}z_2+[\underline{0}]z_1+\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}.\end{aligned} \tag{2}$$

This decomposition satisfies the condition given in Šiljak and Vukcevic's paper[1] since the following two pairs:

$$\left\{\begin{bmatrix}0&1\\2&0\end{bmatrix},\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$$

and

$$\left\{\begin{bmatrix}0&3\\4&0\end{bmatrix},\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$$

are individually controllable.

Using the procedure given in the paper,[1] (2) may be rewritten as

$$\begin{aligned}\dot{x}_1&=\begin{bmatrix}0&0\\0&0\end{bmatrix}x_1+\begin{bmatrix}1&0\\0&3\end{bmatrix}x_2+\begin{bmatrix}1\\1\end{bmatrix}u_1\\ \dot{x}_2&=\begin{bmatrix}0&0\\0&0\end{bmatrix}x_2+\begin{bmatrix}2&0\\0&4\end{bmatrix}x_1+\begin{bmatrix}1\\1\end{bmatrix}u_2.\end{aligned} \tag{3}$$

It can be observed from (3) that both subsystems described by

$$\left\{\begin{bmatrix}0&0\\0&0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}$$

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[1] D. D. Šiljak and M. B. Vukcevic, *IEEE Trans. Automat. Contr.*, vol. AC-21, pp. 363–366, June 1976.

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and

$$\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$$

are not an individually controllable pair.

Hence, the decentralized-stabilization scheme proposed in the paper[1] does not always work.

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I am thankful to Dr. P. Sen for useful discussions.

Author's Reply[2]

D. D. ŠILJAK

We knew all along the more or less obvious fact that our decentralized scheme might not always succeed. The presented example is beneficial in making this fact explicit. We note that the scheme should be considered useful to the extent that it replaces the transformation of the overall system, which is required in obtaining the Luenberger canonical form, by transformations of the low-order subsystems.

Since the publication of our 1976 paper, we have developed an efficient graph-theoretic scheme [1] for decentralization of large control systems, which results in a hierarchial (lower-triangular) ordering of structurally controllable subsystems. In addition to the computational superiority of the scheme, the hierarchical structure allows one to *almost always* stabilize the overall system by stabilizing the individual subsystems using decentralized control.

[2]Manuscript received December 2, 1981.

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Correction to "Homogeneous Interconnected Systems: An Example"

M. L. EL-SAYED AND P. S. KRISHNAPRASAD

The following typographical errors should be noted for the above paper.[1]

1) In the second line after (3.3), the summation sign, Σ, should be placed right at the beginning of the following line.

2) A line is missing after (I.5). The line reads: "Since $f_1 = f_{-1}$, then letting..."

3) In the equation following (II.8), the operator $\mathfrak{L}$ in the right-hand side of the equation should actually be located before the first parenthesis.

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[1]M. L. El-Sayed and P. S. Krishnaprasad, *IEEE Trans. Automat. Contr.*, vol. AC-26, pp. 894–901, Aug. 1981.

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